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Atom-number fluctuation and macroscopic quantum entanglement in dipole spinor condensates

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We study the spin distribution and macroscopic entanglement in spinor condensates confined in an anisotropic potential under external magnetic fields. Different types of magnetic phases can be reached by tuning the magnetic dipolar interaction strength by modifying the trapping geometry. We investigate the atom-number fluctuations of the ground state and show that the different internal hyperfine states exhibit super-Poissonian, Poissonian, and sub-Poissonian distributions with different trapping geometries and strengths of the external magnetic field. We also propose a scheme to create a macroscopic maximally entangled spin state by slowly sweeping the external magnetic field.

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I. INTRODUCTION

Bose-Einstein condensates have been studied extensively for the past ten years [1–16]. More recently, since the experimental observation of spinor condensates (condensates with spin degrees of freedom) in 23Na and 87Rb atoms [17–20], their magnetic properties have attracted much attention in both experimental [18] and theoretical studies [21–26]. Under a more careful inspection, the long-range magnetic dipolar interactions (which may originate from the intrinsic or induced dipole moments of the atoms or molecules [27–33]) may have a rather strong effect on the stability of the condensate [34–36] and significantly modify the excitation spectrum [37–40]. This is because even though the total s-wave collisional interaction strength is much larger than the dipolar interaction strength, the spin exchange interaction is not necessarily much stronger. A simple calculation shows that for the f = 1 hyperfine manifold of 87Rb atoms, the magnitude of the dipolar energy can be as large as 10% of the spin-exchange energy [41], thus making a nontrivial contribution to the total spin-dependent energy; moreover, the long-range and anisotropic nature of the dipolar interaction may further enhance its effects.

The dipolar interaction motivated a detailed investigation of spinor condensates, and some works have explored the effect of the dipolar interactions on spinor condensates confined in multiwell potentials [42–44], where only the dipolar interactions between different potential wells are taken into account. Recently, the dipolar effects in a single trapped spinor condensate with a magnetic-field-free environment was investigated [5–11]. However, those studies mainly focus on the atoms trapped in an axially symmetric potential, \( V_{\text{ext}}(r) \), with the symmetry axis chosen to be the quantization axis. In this paper, we will consider dipolar spinor condensates confined in an anisotropic potential under external magnetic fields. Such a system can be modeled as a biaxial quantum magnet. By tuning the strength of magnetic dipole interaction, the system exhibits different magnetic structures.

In the present work, we mainly consider the atom-number fluctuation of three different internal hyperfine states in the ground state. The results show that atom-number fluctuations exhibit drastically different features in the three different phases. In region Z with easy axis along the z axis (the easy axis corresponds to the direction of minimum energy), we find the \( \langle n_0 \rangle = 0 \) and \( m_F = 0 \) state exhibits super-Poissonian distributions, while the \( m_F = -1 \) state exhibits sub-Poissonian distributions and \( \langle n_1 \rangle = N/2 \). In regions X and Y (easy axis along the x and y axes, respectively), the relation between the number of different hyperfine states is given by \( \langle n_0 \rangle = 2 \langle n_{\pm 1} \rangle \simeq N/2 \). For the \( m_F = 0 \) state, it displays sub-Poissonian distributions, while the \( m_F = -1 \) state exhibits three different distributions for the different strengths of the dipole interaction. We also show that the internal hyperfine states exhibit drastically different atom-number fluctuations by changing the strength of the external magnetic field. In addition, we investigate the entanglement in this system. Based on the adiabatical theorem, we propose a scheme to create a macroscopic maximally entangled spin state by slowly sweeping the external magnetic field [5,45].

This paper is organized as follows. In Sec. II, we introduce the model of spin-1 condensates with dipole interaction confined in an anisotropic potential under an external magnetic field and derive a model which corresponds to a biaxial quantum magnet. In Sec. III, we discuss the ground-state magnetic structure of the system and investigate the atom-number fluctuation in different phases. In Sec. IV, we propose a scheme to create a macroscopic maximally entangled spin state by slowly sweeping the external magnetic field. Finally, our conclusions and some remarks on our results are presented in Sec. V.

II. SPINOR CONDENSATE WITH DIPOLAR INTERACTION

We consider a dilute gas of trapped bosonic atoms with hyperfine spin \( f = 1 \). The Hamiltonian without the dipolar interaction and under a uniform magnetic field \( B \), in the second
reads \[3\] single-mode approximation (SMA) in which atoms in different atoms in the combined symmetric channel of total spin with the atom in the hyperfine spin state assumed to be spin independent. The nonlinear coefficients are introduced into the vacuum magnetic permeability. The total Hamiltonian is then

\[
H = \frac{3}{2} \mathbf{L}^2 - D \mathbf{L}^2 + E (\mathbf{L}_z^2 - \mathbf{L}_y^2) - \mathbf{H} \cdot \mathbf{L},
\]

where

\[
D = -3\kappa^2/|c_2|, \quad E = -3\kappa^2/|c_2|, \quad \text{and} \quad \mathbf{H} = g_F \mu_B B/|c_2|.
\]

To get some idea of the interaction parameters, we then choose the Gaussian wave packet for the condensate wave function

\[
n(r) = \pi^{-3/2}(q_x, q_y, q_z)^{-1} e^{-x^2/q_x^2 - y^2/q_y^2 - z^2/q_z^2},
\]

which yields

\[
D(\kappa_x, \kappa_y) = -4\pi c_d \left( \frac{\kappa_x \kappa_y}{|c_2|} \right) \int_0^\infty dt t e^{-(\kappa_x^2 + \kappa_y^2)t^2}/t,
\]

\[
E(\kappa_x, \kappa_y) = -4\pi^{3/2} c_d \left( \frac{\kappa_x \kappa_y}{|c_2|} \right) \int_0^\infty dt e^{-(\kappa_x^2 + \kappa_y^2)t^2}/t^2 \times \left[ \left( \kappa_x^2 - \kappa_y^2 \right)^2/2 \right] e^{i \kappa_{x,y} x} [1 - \text{erf}(t)],
\]

with \(\kappa_{x,y} \equiv q_{x,y}/q_z, I_0[x] \) and \(I_1[x] \) being the modified Bessel functions of the first kind, and erf being the error function. Here we notice that when \(E = 0\), Hamiltonian (10) reduces to the uniaxial magnet [4,5].

### III. SPIN STRUCTURE IN THE GROUND STATE

Equation (10) is reminiscent of the Hamiltonian describing an anisotropic quantum magnet. The ground state of the system can be found by numerically diagonalizing Hamiltonian (10). As shown in Fig. 1, the \(\kappa_x\) and \(\kappa_y\) parameter plane is divided into regions X, Y, and Z based on the ground-state wave function in the absence of magnetic field [45]. Regions X, Y, and Z represent the ferromagnetic phase with the ground states \(|G\rangle = |N, \pm N\rangle_x, |N, \pm N\rangle_y, \text{ and } |N, \pm N\rangle_z\) respectively, where we have introduced the standard angular momentum basis states \(|l, m\rangle\) that are common eigenstates of \(\mathbf{L}^2\) and \(L_z\). In the parameter region \(-1 \leq \log_{10} \kappa_x/\Lambda_x \leq 1\) under an arbitrary magnetic field, we find that the value of the total angular momentum is given by \((\mathbf{L}^2) \geq N(N + 1)\) for \(N \geq 5\), which indicates the condensate always remains in the ferromagnetic phase with a sufficiently large number of atoms and total spin \(I \geq N\) [45]. In addition we also find that the contribution of the linear terms \([-D\mathbf{H}_0 - E(\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_1)\)] in Hamiltonian (10) is small enough to be neglected.

First, we consider the case that a field is applied longitudinally along the positive \(z\) axis, i.e., \(\mathbf{H} = H \mathbf{\hat{z}}\). After dropping the unimportant linear terms and the constant terms,
the classical version of the Hamiltonian is
\[ \mathcal{H}(\theta,\phi) = -D N^2 \cos^2 \theta + EN^2 \sin^2 \theta \cos 2\phi - H_c N \cos \theta, \]
where \((\theta,\phi)\) denotes the direction of \(\mathbf{L}\). The ground state \((\theta_0,\phi_0)\) is obtained by minimizing the energy. In region Z, the ground state is given by \(\theta_0 = 0\). In region Y, for \(H_c < 2(E - D)N\), \(\mathcal{H}(\theta,\phi)\) has two degenerate minima at
\[ \theta_0^{(\pm)} = \pi/2 \pm \left( \pi/2 - \cos^{-1} \frac{H_c}{2(E - D)N} \right), \quad \phi_0 = \frac{\pi}{2}. \]
When \(H_c > 2(E - D)N\), the degeneracy is removed, and the system is completely polarized by the external magnetic field. In region X, for \(H_c < 2(E + D)N\), the ground state is given by
\[ \theta_0^{(\pm)} = \pi/2 \pm \left( \pi/2 - \cos^{-1} \frac{H_c}{2(E + D)N} \right), \quad \phi_0 = 0. \]
When \(H_c > 2(E + D)N\), the spin direction of the ground state is polarized along the \(z\) axis.

Now we investigate the number of atoms for different internal hyperfine states under the external magnetic field. For simplicity, here we consider a weak magnetic field. As shown in Fig. 2, in region Z (i.e., \(-1 \leq \log_{10} \kappa_{x,y} \leq 0\)), the atomic number of the \(m_F = 0\) component is given by \(\langle n_0 \rangle = 0\), and that of the \(m_F = -1\) component is \(\langle n_{-1} \rangle = N\), which corresponds to the ground state \(|G\rangle = |N,N\rangle\). In the X and Y regions, we can find \(\langle n_0 \rangle \simeq N/2\) and \(\langle n_{-1} \rangle \simeq |\langle n_1 \rangle| \simeq N/4\).

Next, we consider the atom-number fluctuation in terms of the Mandel \(Q_a\) parameter
\[ Q_a = \frac{\langle \hat{n}_a^2 \rangle - \langle \hat{n}_a \rangle^2}{\langle \hat{n}_a \rangle} - 1, \]
with \(Q_a < 0\), \(Q_a = 0\), and \(Q_a > 0\) specifying sub-Poissonian, Poissonian, and super-Poissonian distributions, respectively.

In particular, when \(Q_a = -1\), no atom-number fluctuations exist. In Eq. (17), \(\alpha = 0,1\) correspond to three different components. As shown in Fig. 3, the numerical result shows that in regions X and Y, \(Q_0 < 0\), and the \(m_F = 0\) state has a sub-Poissonian distribution. However, in region Z, \(Q_0 > 0\), which means that in this region the \(m_F = 0\) state has a super-Poissonian distribution. At the boundaries between the different regions (Z and X, Y), \(Q_1\) attains maximum values, which indicates the system exhibits the maximum atom-number fluctuation at the boundaries between different phases. The \(m_F = -1\) state displays a sub-Poissonian distribution in region Z. At the boundaries between regions Z and X, Y, \(Q_{-1}\) attains maximum values. In regions X and Y, \(Q_{-1}\) displays different Poissonians for different \(\kappa_x, \kappa_y\). In region X, for a fixed value of \(\kappa_y\), \(Q_{-1}\) exhibits a transition from super-Poissonian to sub-Poissonian distribution as \(\kappa_x\) increases. However, in region Y, the spin distribution of \(Q_{-1}\) transfers from super-Poissonian to sub-Poissonian for a fixed value of \(\kappa_x, \kappa_y\) increases.

Now we consider the effect of the external magnetic field on the atom-number fluctuations. For simplicity, here we focus on region X in which the easy axis is along the \(x\) axis. First, we define the parameter
\[ H_c^0 = 2|E + D|N - H_c, \]
With the help of Eq. (16), we can see that when \(H_c^0 > 0\), two degenerate ground states exist; if \(H_c^0 < 0\), the system is fully polarized along the \(z\) axis. In Fig. 4, we plot \(H_c^0\) as a function of external magnetic field and \(\log_{10} \kappa_y\) with \(\log_{10} \kappa_x = 1\). We
we focus on the region with magnetic field along the hard axis. Without loss of generality, we investigate the ground state of the system under an external cavity quantum electrodynamics [46]. Here we adopt a scheme works in quantum computation and quantum information. Quantum entanglement has been used to perform many useful tasks, such as atom-number fluctuation.

Before discussing the process of the scheme, we first consider the dynamic creation of maximally entangled spin states by using the condensate in region Z. When the field further reduces and equals zero, the minimal-energy states are given by $|\theta = 0, \pi, \pi|$, which corresponds to the quantum state $|N, \pm N\rangle_z$, and the state of the system is the maximally spin entangled Greenberger-Horne-Zeilinger (GHZ) state,

$$|\text{GHZ}\rangle_z = \frac{1}{\sqrt{2}}(|N, N\rangle_z + |N, -N\rangle_z).$$

In Fig. 6, the overlap between the maximally spin entangled GHZ state and the generated state is plotted as a function of the time-dependent magnetic field $H_s$. Figure 6 clearly shows that when the strength of the field reduces to zero, the overlap $|\langle \Psi(t)|\text{GHZ}\rangle_z|^2 = 1$, which means the system generates a maximally spin entangled GHZ state. For the scheme discussed above, we can see that the dynamical process does not rely on precise knowledge of system parameters, such as the dipole interaction strength, particle numbers, and the evolution time. As shown in Fig. 6, we just set the total particle number $N = 10$. We can also notice that over the whole process, the system is always in the minimal-energy state, which makes the system immune to the spontaneous emission-induced decoherence suffered by the electronically excited states [5]. Moreover, this scheme is rather robust; if the adiabaticity is not strictly obeyed during the evolution process, the generated state may have slightly different weights on $|N, N\rangle_z$ and $|N, -N\rangle_z$, but it remains a significantly entangled state. Also we notice that if the first apply a magnetic field along the $x$ axis, i.e., $H = H_s \hat{x}$. Since we focus on the case of the ferromagnetic, we treat the total spin $\hat{L}$ as a classical magnetization vector with length $|\hat{L}| = N$. After neglecting the unimportant linear terms and the constant terms, the energy of the system becomes

$$\epsilon(\theta, \phi) = -DN^2 \cos^2 \theta + E N^2 \sin^2 \theta \cos 2\phi - H_s N \sin \theta \cos \phi,$$

where $(\theta, \phi)$ denotes the direction of $\hat{L}$. Apparently, when $H_s < H_s^\ast = 2N(D + E)$, the ground state is doubly degenerate, and they are located at

$$\theta_0^{(\pm)} = \frac{\pi}{2} \pm \left( \frac{\pi}{2} - \sin^{-1} \frac{H_s}{2N(D + E)} \right), \quad \phi_0 = 0.$$  

When $H_s > H_s^\ast$, the two degenerate states collapse into one, the system is fully polarized by the external field, and the two minima merge along the $x$ axis.

Now we discuss the details of the scheme to create maximally entangled spin states. First, we introduce a time-dependent magnetic field $H_s(t)$ which is swept linearly at a constant rate $v < 0$; that is,

$$H_s(t) = H_s^{(i)} + vt$$

for $t \geq 0$, and $H_s^{(i)} > 0$ is the initial field. Initially, we prepare the initial magnetic field to be sufficiently large, $H_s^{(i)} > H_s^\ast$, which means the initial state roughly stays at the $L_z = N$ level. The corresponding energy has a single minimum at $\theta = \pi/2, \phi = 0$. Then we slowly reduce the strength of the magnetic field; when $H_s < H_s^\ast$, the ground state is doubly degenerate, which corresponds to the states in Eq. (20). Then the ground state of the system becomes the symmetric quantum superposition of the two corresponding minimal-energy states. If the external field further reduces and equals zero, the minimal-energy states are given by $\theta_0 = 0$ and $\pi$, which corresponds to the quantum state $|N, \pm N\rangle_z$, and the state of the system is the maximally spin entangled Greenberger-Horne-Zeilinger (GHZ) state, $|\text{GHZ}\rangle_z = 1 \sqrt{2} (|N, N\rangle_z + |N, -N\rangle_z)$.
magnetic field \( \mathbf{H} \) is misaligned such that it forms an angle \( \delta \phi \) to the \( z \) axis, it will degrade the entanglement. Finally, the evolution time can be adjusted by accelerating the sweeping rate beyond the region near the point \( H_x = H_x^* \). In addition, we note that this scheme may be also applied to other biaxial magnetic materials to create an entangled state.

V. CONCLUSION

To conclude, we have studied a spinor condensate with dipole interaction confined in an anisotropic potential under external magnetic fields. Using the single-mode approximation, we show the system can be modified as a model which corresponds to a biaxial quantum magnet. Different magnetic structures can be reached by tuning the dipole interaction strength by modifying the trapping geometry. We have studied the atom-number fluctuation for the three internal hyperfine states and found that in region \( Z \), the \( m_F = 0 \) state has a super-Poissonian distribution, while the \( m_F = -1 \) state just displays a sub-Poissonian distribution. In regions \( X \) and \( Y \), \( \langle \hat{n} \rangle \simeq 2 \langle \hat{n}_{\pm 1} \rangle \simeq N/2 \), and the \( m_F = 0 \) state displays sub-Poissonian fluctuation, while the \( m_F = -1 \) state exhibits three different distributions for the different \( \kappa_x \) and \( \kappa_y \). We also show that by increasing the strength of the external magnetic field, the internal hyperfine states exhibit drastically different quantum fluctuations.

Finally, we discuss the entanglement state generated in this system. We propose a scheme to create a macroscopic maximally entangled spin state by slowly sweeping the external magnetic field. We note that this scheme can also be applied to other biaxial magnetic materials to create an entangled state.

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