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<td>Lin, D; Tan, SC; Hui, SYR</td>
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Chromatic, Photometric and Thermal Modeling of LED Systems With Nonidentical LED Devices

Huan-Ting Chen, Member, IEEE, De-Yan Lin, Member, IEEE, Siew-Chong Tan, Senior Member, IEEE, and S. Y. (Ron) Hui, Fellow, IEEE

Abstract—With the emergence of new color-mixing LED systems based on LED devices of different color temperatures, the need for a new modeling technique for LED systems with nonidentical LED devices becomes imminent. This paper presents a modeling technique for LED systems with LED arrays comprising nonidentical LED devices that have nonidentical optical-thermal-electrical properties. Based on a general 3-D photo-electro-thermal LED node model, LED devices of different kinds can be arranged in various array forms according to their system construction and design. By linking the system matrix to the correlated-color-temperature prediction, the proposed modeling technique provides an accurate prediction of the temperature distribution, luminous flux, and correlated color temperature of the LED systems. The temperature distribution and light output of the LED systems have been measured using an infrared imaging system and a spectrophotometer with an integrating sphere. The modeling technique has been successfully demonstrated and experimentally verified on several LED systems comprising nonidentical LED devices. It is particularly useful as a modeling tool to study new color-mixing LED systems based on different types of LED devices.

Index Terms—Correlated color temperature (CCT), light-emitting diodes (LED), lighting system, nonidentical LEDs, photovoltaic, photo-electro-thermal (PET), thermal model.

I. INTRODUCTION

WITH improving luminous performance of high-power light-emitting diode (LED) devices, recent LED research works are addressing other important LED system’s issues concerning their reliability performances [1], [2]. Commercial LED products fail to meet the manufacturers’ requirement on the light output and thermal properties [3], [4], and have since aroused research interests toward their system design, which is a multidisciplinary subject involving electric power control, circuit topology, thermal management, energy efficiency, and optical performance [5], [6]. It is known that the lifetime of electrolytic capacitor is limited to only several thousand hours under the rated operating condition. This is much shorter than the potential lifetime of LEDs. Thus, eliminating the electrolytic capacitor in power supplies for LED systems is a very important topic in ballast design [7]–[10].

Heat dissipation of LED is a very important subject as it can greatly influence the electrical characteristics, optical characteristics, and reliability of the LED [11]. A higher LED junction temperature reduces its emitted luminous efficacy. The origin of this so-called efficiency-droop effect is reported in [12]. It is known that the effect of junction temperature on the light output of LEDs is strongly dependent on the device materials and structure of the LED chip [13]. In GaInN/GaN LEDs, light output is a function of forward current and dislocation density. Analysis reveals that dislocations do not strongly impact high-current performance. Instead, they contribute to increased nonradiative recombination at lower currents and a suppression of the peak efficiency [14].

With increasing power and component packaging densities in solid-state lighting, thermal management of heatsink and multichip module technology has become an important research topic [15]–[23]. For compact systems with multiple LED devices closely placed together, improper geometrical arrangements of the devices on the heatsink and uneven heat distribution could degrade the performance of the devices and the overall system. In other words, the thermal, optical, and electrical properties of the LED can be affected by their geometrical locations on the heatsink [16], [17]. Currently, there is still a lack of understanding on such aspects, which are important for the optimization of the LED system design.

LED systems with multiple LED devices essentially comprise multiple heat sources at different locations. The thermal diffusion among these heat sources significantly influences the thermal analysis of the system. Even though mounted on the same heatsink, each LED has a different junction temperature, which is dependent on its optical property, operating current, and physical location. Models that can describe such LED systems with multiple nonidentical LED devices arranged in an array structure would be useful for optimizing their system design.

Good thermal management can enhance the thermal, light output and reliability performances of the LED system. Such models will also be very useful for studying new color-mixing LED systems such as those using LED devices of different rated...
color temperatures for color control. For example, warm-white and cool-white LED devices have been used in commercial Osram and Traxon lighting products for white color control between 2700 to 5000 K.

The term nonidentical LEDs in this context is not limited to LEDs of different types (e.g., different models or manufacturers). It also refers to LEDs of the same type but with different operating conditions or physical locations. Devices placed at different physical locations or operated with a different current may result in a different junction temperature, which will affect their luminous outputs.

In this paper, a partition approach is adopted to model LED array systems using nonidentical LED devices. A 3-D photo-electro-thermal (PET) LED nodal model is proposed for the LED system with nonuniform thermal distribution of the heatsink. This LED nodal model can be used to represent the LED array structure on the same heatsink and predict the photometric, electric, and thermal behaviors of the LED system. By linking these three aspects of LED performance to the modeling technique of correlated color temperature (CCT), a useful comprehensive modeling tool becomes feasible for studying new color-mixing LED systems based on LED devices of different rated color temperature values. This new methodology offers a fast modeling and simulation solution to emerging LED systems and overcomes the limitations of existing time-consuming multiphysics simulation tools that do not have chromatic and photometric modeling functions.

II. THREE-DIMENSIONAL ARRAY THERMAL RESISTANCE MODEL OF AN LED SYSTEM WITH NONIDENTICAL LED DEVICES IN AN ARRAY STRUCTURE

A. Basic Concept of a Partitioned Grid Model

The original PET theory reported in [5] assumes that all the LED devices mounted on the same heatsink are identical in terms of their device characteristics and their case and junction temperatures. In practice, the temperature distribution on the heatsink’s base may not be uniform, and therefore the LED devices may operate dissimilarly even if they are selected from the same bin. In this section, a 3-D PET LED node model is developed to describe the nonuniform thermal distribution of the LEDs on the heatsink in an XYZ-space system, where X, Y, and Z are coordinate axes that are perpendicular to one another. The modeling method is first illustrated on a 3 × 3 array as shown in Fig. 1. It will then be generalized to an n × m array.

In the proposed model, the heatsink is first partitioned into small square or rectangular grids of which each has a size equal to or slightly larger than an LED device. The LED devices are placed on some or all of the square/rectangular grids. Fig. 2 shows a figure of a 32 × 17 LED array system with 544 partitioned square grids and with 30 LEDs (circle shapes) placed on 30 of the square grids.

In the following analysis, it is assumed that each LED device is assigned to one grid on the base surface of the heatsink. For the example in Fig. 1, nine LED devices are mounted on the base surface, with each of the LED placed on one of the nine grids in the 3 × 3 array arrangement.

Here, $R_{s1}, R_{s2},$ and $R_{s3}$ are defined as the thermal resistances between the heatsink grid and the ambient along the X-axis, Y-axis, and Z-axis, respectively, and $R_x$ and $R_y$ are defined as the thermal resistances between any adjacent interconnected heatsink grids along the X-axis and Y-axis, respectively. It should be noted that the heat transfer process of $R_{s1}, R_{s2},$ and $R_{s3}$ is based on thermal convection from node to ambient, and this is dependent on the size of the contact area to the ambient. $R_c$ and $R_n$ model the thermal coupling effect between adjacent interconnected grids, which is a function of grid distance. The general nodal thermal structure (without the heat source) comprising $R_{s1}, R_{s2}, R_{s3}, R_x,$ and $R_y$ for each grid is shown in Fig. 3(a). Note that with the LED mounted on the heatsink grids and that the thermal resistance of LED device being $R_{jc},$ then the overall thermal resistance from the LED device to the ambient along the Z-axis will be $R_{s3} + R_{jc}.$ As the thermal resistance values are different for LED devices mounted on different grids, therefore the junction temperature values of the LEDs mounted on the heatsink are dissimilar. For example, for an LED mounted on grid point 1 of Fig. 1, its thermal resistance include $R_{s1}, R_{s2}, R_{s3}, R_x,$ and $R_{jc-1}.$ For an
LED mounted on grid point 5, the thermal resistance includes $R_{s3}$, $2R_x$, $2R_y$, and $R_{jc3,5}$.

Next, $P_{h_{j,k}}$ and $T_{hs_{j,k}}$ are, respectively, defined as the heat dissipation of the LED device and the heatsink temperature at the $i$th grid point. $T_a$ is the ambient temperature. The thermal node for the $i$th grid is illustrated in Fig. 3(a) for the 3-D heat flow and its equivalent thermal model is shown in Fig. 3(b).

Here, $R_{eq}$ represents the equivalent thermal resistance from the heatsink surface to the ambient, and it comprises all five thermal resistors in parallel connection and can be expressed as

$$R_{eq} = \frac{1}{\frac{1}{R_{s1}} + \frac{1}{R_{s2}} + \frac{1}{R_{s3}} + \frac{1}{R_x} + \frac{1}{R_y}}. \quad (1)$$

Using the nodal energy balance concept, the thermal model of the LED system in Fig. 1 can be mathematically represented as (2) shown at the bottom of the page.

It has been found that more than 90% of the total heat generated by the LED is dissipated to the ambient through the heatsink. The small remaining portion of the heat is dissipated to the ambient through other heat flow paths. Therefore, the heat flow $q$ generated by the LED can be assumed to be the heat flow through the heatsink, which obeys the following equation:

$$q = \frac{T_j - T_a}{R_{s3} + R_{jc}} = \frac{T_{hs} - T_a}{R_{s3}}. \quad (3)$$

Based on the previous nodal thermal energy balance equations, the thermal matrix equation for the $3 \times 3$ array system can be expressed as (4) shown at the bottom of the next page, where $\Delta T_{hs_{j,k}}$ represents the temperature rise of heatsink at the $i$th grid as shown in Fig. 3(b), and $Z_1 = 1/R_{s1} + 1/R_{s2} + 1/R_{s3} + 1/R_x + 1/R_y$; $Z_2 = 1/R_{s2} + 1/R_{s3} + 2/R_x + 1/R_y$; $Z_3 = 1/R_{s1} + 1/R_{s3} + 2/R_y$; and $Z_4 = 1/R_{s1} + 2/R_x + 2/R_y$.

**B. Generalized Partitioned Model for an $n \times m$ LED Array Structure**

In general, any LED array system can be partitioned with a large number of grid points for enhanced resolution and precision, as shown in Fig. 2. For an $n \times m$ LED array system, where $n$ represents the number of grids in each row, and $m$ represents the number of grids in each column, the thermal resistance can be written in matrix form as follows:

$$\begin{bmatrix}
P_{h_{1,1}} & P_{h_{1,2}} & \cdots & P_{h_{1,m}} \\
P_{h_{2,1}} & P_{h_{2,2}} & \cdots & P_{h_{2,m}} \\
\vdots & \vdots & \ddots & \vdots \\
P_{h_{n,1}} & P_{h_{n,2}} & \cdots & P_{h_{n,m}}
\end{bmatrix}
\begin{bmatrix}
\Delta T_{hs_{1,1}} \\
\Delta T_{hs_{2,1}} \\
\vdots \\
\Delta T_{hs_{n,m}}
\end{bmatrix}
= 
\begin{bmatrix}
1/R_{s1} & 1/R_{s2} & \cdots & 1/R_{s3} \\
1/R_{s1} & 1/R_{s2} & \cdots & 1/R_{s3} \\
\vdots & \vdots & \ddots & \vdots \\
1/R_{s1} & 1/R_{s2} & \cdots & 1/R_{s3}
\end{bmatrix}
\begin{bmatrix}
\Delta T_{hs_{1,1}} \\
\Delta T_{hs_{2,1}} \\
\vdots \\
\Delta T_{hs_{n,m}}
\end{bmatrix}. \quad (5)$$

For $n \neq m$, the nondiagonal element $R_{nm}$ in the thermal resistance matrix represents the thermal resistance between the interconnected node $n$ and node $m$, either in the $X$-axis or in the $Y$-axis. (Note: For interconnected grids, the thermal resistance
is $R_x$ along the $X$-axis and $R_y$ along the $Y$-axis. On the other hand, $R_{mn}$ for noninterconnected grids is 0). For $n = m$, the diagonal element $R_{mn}$ represents the thermal resistance for node $n$, which includes the five thermal resistors as shown in Fig. 3. The generalized heat flow model of an $n \times m$ LED array system can be expressed as $\Delta$

$$
\begin{bmatrix}
\Delta T_{hs,1} \\
\Delta T_{hs,2} \\
\vdots \\
\Delta T_{hs,mn}
\end{bmatrix} =
\begin{bmatrix}
P_{h,1} \\
P_{h,2} \\
\vdots \\
P_{h,mn}
\end{bmatrix}
\begin{bmatrix}
1 & 1 & \cdots & 1 \\
\frac{1}{R_{11}} & \frac{1}{R_{12}} & \cdots & \frac{1}{R_{1n}} \\
\frac{1}{R_{21}} & \frac{1}{R_{22}} & \cdots & \frac{1}{R_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{R_{m1}} & \frac{1}{R_{m2}} & \cdots & \frac{1}{R_{mn}}
\end{bmatrix}^{-1}
$$

(6)

C. Simplified Generalized Partitioned Model for an $n \times m$

Array Structure Based on Grid Clustering

In the case of very large array structures, it is proposed in this paper to adopt the method of grid clustering in developing the system model in order to reduce the required intensive calculation. Instead of adopting the direct approach presented earlier, a two-level approach can be adopted to derive: 1) a lower level but smaller grid cluster model; and 2) a higher level model describing the relationship between all the grid clusters. For example, the complexity of calculating (6) can be significantly reduced via a lower level nine-grid cluster model as described in the following. By applying a nine-grid cluster configuration into (4) give

$$
\begin{bmatrix}
P_{hw,1} \\
P_{hw,2} \\
\vdots \\
P_{hw,mn}
\end{bmatrix} =
\begin{bmatrix}
Z_u & \Delta T_{hs,u,1} \\
\Delta T_{hs,u,2} \\
\vdots \\
\Delta T_{hs,u,mn}
\end{bmatrix}
\begin{bmatrix}
P_{hw,1} \\
P_{hw,2} \\
\vdots \\
P_{hw,mn}
\end{bmatrix}
$$

(11)

$$
\begin{bmatrix}
P_{hw,1} \\
P_{hw,2} \\
\vdots \\
P_{hw,mn}
\end{bmatrix} =
\begin{bmatrix}
1 & 1 & \cdots & 1 \\
\frac{1}{R_{11}} & \frac{1}{R_{12}} & \cdots & \frac{1}{R_{1n}} \\
\frac{1}{R_{21}} & \frac{1}{R_{22}} & \cdots & \frac{1}{R_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{R_{m1}} & \frac{1}{R_{m2}} & \cdots & \frac{1}{R_{mn}}
\end{bmatrix}^{-1}
\begin{bmatrix}
\Delta T_{hs,u,1} \\
\Delta T_{hs,u,2} \\
\vdots \\
\Delta T_{hs,u,mn}
\end{bmatrix}
\begin{bmatrix}
P_{hw,1} \\
P_{hw,2} \\
\vdots \\
P_{hw,mn}
\end{bmatrix}
$$

(12)

where (8)–(10), as shown at the bottom of the next page.

Equations (7)–(10) give the equivalent thermal calculation of a nine-grid cluster within the $n \times m$ array. This model has to be

$$
\begin{bmatrix}
P_{h,u} \\
P_{h,\cdot u}
\end{bmatrix} =
\begin{bmatrix}
\begin{bmatrix}
Z_1 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
1 & Z_2 & \frac{1}{R_x} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & Z_3 & \frac{1}{R_x} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & \frac{1}{R_y} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & \frac{1}{R_y} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & \frac{1}{R_y} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{R_y} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{R_y} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & \frac{1}{R_y}
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\Delta T_{hs,u,1} \\
\Delta T_{hs,u,2} \\
\Delta T_{hs,u,3} \\
\Delta T_{hs,u,4} \\
\Delta T_{hs,u,5} \\
\Delta T_{hs,u,6} \\
\Delta T_{hs,u,7} \\
\Delta T_{hs,u,8} \\
\Delta T_{hs,u,9}
\end{bmatrix}
$$

(4)
By using this approach, the matrix given in (6) can be significantly reduced, thereby greatly simplifying the analysis and calculation for very large array systems.

The choice of using either the direct approach or the two-level approach for calculating the thermal distribution of the LED system should depend on the grid size. In very large systems, the two-level approach significantly increases the computational efficiency, at the expense of accuracy. Therefore, a balance should be struck between calculation time and accuracy. Generally, the direct approach is applicable to most real-life LED systems for its better accuracy and acceptable calculation time. For example, the computational time of a 32 × 17 LED array system with 544 partitioned square grids and 30 LEDs is only 1 min using the direct approach. The two-level approach will be useful for very large LED panel, such as that for large display screens.

III. PET MODELING OF NONIDENTICAL LED DEVICES IN ARRAY ARRANGEMENTS

The nonuniform temperature distribution on the heatsink leads to the nonidentical optical behavior of the LEDs. For each LED device in the system, the thermal characteristic can be expressed as follows:

\[
T_{j,i} = T_{hs,i} + R_{jc,i}k_{h,i}P_{d,i} = \Delta T_{hs,i} + T_a + R_{jc,i}P_{h,i}
\]  

(13)

where \( T_{j,i} \) is the junction temperature, \( T_{hs,i} \) is the heatsink temperature, \( R_{jc,i} \) is the junction-to-case thermal resistance, \( k_{h,i} \) is the heat dissipation coefficient, \( P_{d,i} \) is the input electrical power, and \( P_{h,i} \) is the heat dissipation power of the \( i \)th LED of the system.

Fig. 4(a) shows a thermal model unit of an LED in the \( i \)th grid and (b) its equivalent model. With this, the thermal nodal model of the entire LED array system can be formulated using the array arrangement of multiple thermal nodal models.

Using (6), (13), and the equivalent model shown in Fig. 4(b), the junction temperature of the LEDs in the array structure can be calculated as follows:

\[
Z_u = \begin{bmatrix}
    Z_1 & -\frac{1}{R_x} & 0 & -\frac{1}{R_y} & 0 & 0 & 0 & 0 & 0 \\
    -\frac{1}{R_x} & Z_2 & 0 & 0 & -\frac{1}{R_y} & 0 & 0 & 0 & 0 \\
    0 & -\frac{1}{R_x} & Z_1 & 0 & 0 & -\frac{1}{R_y} & 0 & 0 & 0 \\
    -\frac{1}{R_y} & 0 & 0 & Z_3 & -\frac{1}{R_x} & 0 & -\frac{1}{R_y} & 0 & 0 \\
    0 & -\frac{1}{R_y} & 0 & -\frac{1}{R_x} & Z_4 & -\frac{1}{R_x} & 0 & -\frac{1}{R_y} & 0 \\
    0 & 0 & -\frac{1}{R_y} & 0 & Z_3 & 0 & 0 & -\frac{1}{R_y} & 0 \\
    0 & 0 & 0 & -\frac{1}{R_y} & 0 & Z_1 & -\frac{1}{R_x} & 0 & 0 \\
    0 & 0 & 0 & 0 & Z_2 & -\frac{1}{R_x} & 0 & -\frac{1}{R_y} & Z_1
\end{bmatrix}
\]  

(10)

\[
P_{h,i} = \frac{P_{h,1} + P_{h,2} + P_{h,3} + P_{h,4} + P_{h,5} + P_{h,6} + P_{h,7} + P_{h,8} + P_{h,9}}{9}
\]  

\[
\Delta T_{hs,i} = \frac{\Delta T_{hs,1} + \Delta T_{hs,2} + \Delta T_{hs,3} + \Delta T_{hs,4} + \Delta T_{hs,5} + \Delta T_{hs,6} + \Delta T_{hs,7} + \Delta T_{hs,8} + \Delta T_{hs,9}}{9}
\]  

(8)
be expressed as follows:

\[
\begin{bmatrix}
T_{j,1} \\
T_{j,2} \\
\vdots \\
T_{j,m,n}
\end{bmatrix}
= k_{h,j} P_{d,j} + T_a +
\begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix}
\begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix}

\times
\begin{bmatrix}
\frac{1}{R_{11}} & \frac{1}{R_{12}} & \cdots & \frac{1}{R_{1n}} \\
\frac{1}{R_{21}} & \frac{1}{R_{22}} & \cdots & \frac{1}{R_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{R_{m1}} & \frac{1}{R_{m2}} & \cdots & \frac{1}{R_{mn}}
\end{bmatrix}^{-1}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
- T_0.
\]

(14)

Thus, the luminous flux and the luminous efficacy for the \( i \)th LED can be written, respectively, as follows:

\[
\phi_{\nu,j} = E_i P_{d,j} = E_0 [1 + k_{e,j} (T_{j} - T_0)] P_{d,j}
\]

(15)

\[
E_i = E_0 [1 + k_{e,j} (T_{j} - T_0)] \text{ for } T_j \geq T_0 \text{ and } E_i \geq 0
\]

(16)

where \( \phi_{\nu,j} \) is the luminous flux, \( E_i \) is the luminous efficacy, \( k_{e,j} \) is the relative rate of reduction of luminous efficacy with increasing temperature, of the \( i \)th LED; and \( E_0 \) is the rated efficacy at the rated temperature \( T_0 = 25^\circ C \).

By substituting (14) into (16), the column matrix of the luminous efficacy can be expressed as follows:

\[
\begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_{m,n}
\end{bmatrix}
= E_0 \left[ 1 + k_{e,j} \left( k_{h,j} P_{d,j} + T_a + \begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix} \begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix} \begin{bmatrix}
\frac{1}{R_{11}} & \frac{1}{R_{12}} & \cdots & \frac{1}{R_{1n}} \\
\frac{1}{R_{21}} & \frac{1}{R_{22}} & \cdots & \frac{1}{R_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{R_{m1}} & \frac{1}{R_{m2}} & \cdots & \frac{1}{R_{mn}}
\end{bmatrix}^{-1} \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix} - T_0 \right]
\]

(17)

and the total luminous flux of LED array system can be calculated as

\[
\begin{bmatrix}
\phi_{\nu,1} \\
\phi_{\nu,2} \\
\vdots \\
\phi_{\nu,m,n}
\end{bmatrix}
= \begin{bmatrix}
E_1 \\
E_2 \\
\vdots \\
E_{m,n}
\end{bmatrix}
\begin{bmatrix}
P_{d,1} \\
P_{d,2} \\
\vdots \\
P_{d,m,n}
\end{bmatrix}.
\]

(18)

Finally, the total luminous flux of the LED array system with LED devices of nonidentical electrical, thermal, and optical characteristics can be determined as follows:

\[
\sum_{i=1}^{m,n} \phi_{\nu,j} = \sum_{i=1}^{m,n} E_i P_{d,j}
\]

\[
= \sum_{i=1}^{m,n} E_0 \left[ 1 + k_{e,j} \left( k_{h,j} P_{d,j} + T_a + \begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix} \begin{bmatrix}
k_{h,j} P_{d,j} \\
k_{h,j} P_{d,j} \\
\vdots \\
k_{h,j} P_{d,j}
\end{bmatrix} \begin{bmatrix}
\frac{1}{R_{11}} & \frac{1}{R_{12}} & \cdots & \frac{1}{R_{1n}} \\
\frac{1}{R_{21}} & \frac{1}{R_{22}} & \cdots & \frac{1}{R_{2n}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{R_{m1}} & \frac{1}{R_{m2}} & \cdots & \frac{1}{R_{mn}}
\end{bmatrix}^{-1} \begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix} - T_0 \right]
\]

(19)

The general expressions of (5)–(19) provide a new set of LED system equations for analyzing LED systems with nonidentical devices mounted in an array structure.

IV. CHROMATIC MODELING OF LED ARRAY SYSTEMS WITH NONIDENTICAL LED DEVICES

Using the proposed array thermal resistance model, the color-mixing characteristic of an LED array system with nonidentical LED devices can also be derived. In this section, a bicolor white LED array comprising both warm-white LEDs (e.g., with CCT = 2700 K) and cool-white LEDs (e.g., with CCT = 5000 K) is used as a case study example for illustrating the concept.

The relationship between the tristimulus values \((X,Y,Z)\) and the chromaticity \((x,y,z)\) of a light source is

\[
\begin{align*}
X &= \frac{X}{Y} = \frac{X + Y + Z}{Y} \\
y &= \frac{X}{Y} = \frac{X + Y + Z}{Z} \\
z &= \frac{X}{Y} = \frac{X + Y + Z}{Z}.
\end{align*}
\]

Equation (20) can be rewritten as

\[
\begin{align*}
X &= xY \\
y &= yY \\
z &= \frac{1 - x - y}{y}.
\end{align*}
\]

If the tristimulus values for the warm-white LED source is \((X_W, Y_W, Z_W)\) and the tristimulus values for the cool-white LED source is \((X_C, Y_C, Z_C)\), the overall tristimulus values \((X_M, Y_M, Z_M)\) of the total light mixture emitted from the LED array, which is the sum of the respective sources [26], are

\[
\begin{align*}
X_M &= X_W + X_C \\
Y_M &= Y_W + Y_C \\
Z_M &= Z_W + Z_C.
\end{align*}
\]

Then, the chromaticity coordinate of the light mixture system is given by

\[
\begin{align*}
y_M &= \frac{Y_M}{X_M + Y_M + Z_M} = \frac{Y_W + Y_C}{X_W + X_C + Y_W + Y_C + Z_W + Z_C} \\
&= \frac{x_W y_W + x_C y_C + y_W + Y_C + \frac{1 - x_W - y_W}{y_W}}{y_W + Y_C} \\
&= \frac{Y_W + Y_C}{y_W + Y_C}.
\end{align*}
\]
Note that the tristimulus value $Y$ represents the luminance, which can be assumed to be a constant proportion of the luminous flux $\Phi_v$, and chromaticity $y$ is proportional to the CCT. Thus, (24) can be rewritten as follows:

$$\text{CCT}_M = \frac{\phi_W + \phi_C}{\text{CCT}_W} = \frac{\phi_W + \phi_C}{\text{CCT}_C}.$$

According to the PET model as given in (19), the mixture CCT for nonidentical LED array system can be given as

$$\text{CCT}_M = \frac{\phi_W + \phi_C}{\text{CCT}_W} = \frac{\phi_W + \phi_C}{\text{CCT}_C} = \frac{\sum_{i=1}^{n(w)} E_i P_{d,\phi, i}}{\text{CCT}_W} + \frac{\sum_{i=1}^{n(c)} E_i P_{d,\phi, i}}{\text{CCT}_C}$$

where $n(w)$ is the number of warm-white LEDs, $n(c)$ is the number of cool-white LEDs, and $mn$ is the total number of LEDs. Equation (26) is the key equation linking the CCT, input power, and luminous efficacies of the individual and overall LED sources together for the LED array system.

V. RESULTS AND DISCUSSIONS

The proposed models are tested on several LED systems using the same LED types and that using a mixture of two different types of LEDs. The temperature profiles of the LED array systems obtained from numerical calculation of the model are compared to that obtained from experiment.

Experiments are conducted at an ambient temperature of 26 °C. Thermal imaging of samples is obtained by using the infrared imaging system NEC THS100. Thermal images for different LED arrangements are recorded after 50 min of operation at different electrical power levels. The luminous flux is measured using a PMS-50 spectrophotocolorimeter under the same experimental conditions. Note that the junction temperature of the LED is not directly measurable by the thermal imaging system. The thermal images provide only the surface temperature of the LED packages, which is slightly lower than the junction temperature.

A. LED System With 30 × 8 W LEDs (Sharp GW5BWF15L00)

In this test, 30 × 8 W LEDs (Sharp GW5BWF15L00) are mounted on a heatsink with thermal resistance of 0.4 °C/W in a regular array arrangement with distances between adjacent LED devices in the X-axis and Y-axis being 32 and 58 mm, respectively.

Fig. 5 shows the temperature distribution of the system obtained from thermal imaging (more specifically, through infrared radiation measurement). The highest temperature obtained from calculation and experiment is 87.9 and 83.2 °C, respectively. Once the temperature distribution of the heatsink is obtained, the temperature response of the heatsink can be calculated. Then, the heat dissipation power of the LED devices is calculated using optical and electrical measurements following the experiment described in [25]. According to the predefined grid area on the heatsink, the average temperature of each grid can be calculated. Substituting the average temperature of each grid and heat dissipation power into (6), the coefficients of $R_{s1}, R_{s2}, R_{s3}, R_c,$ and $R_y$ in the thermal resistance matrix can be calculated through a genetic algorithm program, which is designed to minimize the fluctuation between the measured and calculated temperature distribution on heatsink. The extracted values of $R_{s1}, R_{s2}, R_{s3}, R_c,$ and $R_y$ corresponding to the thermal characteristic of the heatsink are $R_{s1} = 157.0$ °C/W, $R_{s2} = 242.0$ °C/W, $R_{s3} = 331.0$ °C/W, $R_c = 11.0$ °C/W, and $R_y = 4.5$ °C/W.

To verify the proposed PET model, the temperature distribution of the system is calculated using (14) and the total luminous flux is calculated using (19) with $R_{j, \phi} = 6.0$ °C/W, $P_d = 3.6$ W, $k_h = 0.75$, and $P_h = 3.0$ W. It should be noted that $k_h$ is non-constant and is a function of the electrical power and junction temperature [25]. The given $k_h$ is for the LED operating with $P_d = 3.6$ W under ambient temperature. Variation of $k_h$ with $T_j$ is considered in the proposed model and calculation. The variation range of $k_h$ with $T_j$ is shown in Table I.

![Fig. 5. Practical layouts and temperature distribution of the 30 × 8 W LED system obtained from thermal imaging.](image)

<table>
<thead>
<tr>
<th>LED SYSTEMS PARAMETERS</th>
<th>$E_0$ (lm/W)</th>
<th>$k_a$ ($T_j$=45°C-140°C)</th>
<th>$k_e$</th>
<th>$R_c$ (°C/W)</th>
<th>N</th>
<th>$T_s$ (°C)</th>
<th>$T_j$ (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sharp</td>
<td>96</td>
<td>0.72 to 0.86</td>
<td>-0.0039</td>
<td>6.0</td>
<td>8</td>
<td>25</td>
<td>26</td>
</tr>
<tr>
<td>Cree</td>
<td>87</td>
<td>0.67 to 0.84</td>
<td>-0.0023</td>
<td>12.0</td>
<td>8</td>
<td>25</td>
<td>26</td>
</tr>
</tbody>
</table>
Fig. 6 shows the calculated LED junction temperature (based on the proposed PET model) and measured LED package temperature distributions of the 30 LED devices in the LED array at different electrical power levels. As expected, the profile of the package temperature is slightly lower than that of the junction temperature. These temperature distribution results are in good agreement, thereby verifying the thermal part of the modeling. The temperature of each LED increases with the increase in electrical power. When the electrical power is relatively small at 4.0 W, the difference in temperature between individual LED is small. At 6.5 W, the difference in temperature between the hottest and the coolest LED is 14.0 °C. This signifies that the difference in the temperature distribution on the heatsink is large at high power levels.

According to the PET theory, a change in the LED’s temperature results in a change of its light output. The luminous flux emitted from each LED is different since the temperature distribution is nonuniform. Using the proposed PET model, the luminous flux for each LED can be calculated using (18) and (19). The designated parameters for the calculation are $E_0 = 96 \text{ lm/W}$, $k_e = -0.0039$, $T_0 = 25 ^\circ C$, $T_a = 26 ^\circ C$, $R_{jc} = 6.0 ^\circ C/W$, and $n = 30$.

According to the original thermal model [5], the heatsink temperature and junction temperature of each LED are identical. The junction temperature of all LEDs with different electrical power levels (4.0, 5.1, and 6.5 W) is 79, 94, and 113 °C, respectively. Putting these values into the original PET model, the luminous flux of each LED will be identical since the temperature distribution and $k_e$ is uniform. The results are plotted in Fig. 7.

Fig. 7 shows the measured and calculated total luminous flux of the 30 × 8 W LED system. The average deviation between the proposed model and the measurement is about 4.6%. The maximum deviation between the proposed model and the measurement is 6.6% and that between the original model and the measurement is 10.2%. The calculated values using the proposed model are generally more consistent with the measurements.

B. LED System With 8 × 8 W LEDs (Sharp GW5BWF15L00) and 8 × 3 W LEDs (Cree XREWHT-L1-0000-00C01)

In this second example, 16 LEDs comprising eight Sharp LEDs (GW5BWF15L00) and eight Cree LEDs (XREWHT-L1-0000-00C01) are mounted on the heatsink with thermal resistance of 1.3 °C/W. The Sharp LEDs are of the same model as that previously described. The physical dimensions of the Cree LED are length (7 mm), width (9 mm), and thickness (1.5 mm) and that of the heatsink are base length (160 mm), width (150 mm), thickness (4 mm), fins number (15), fin height (10 mm), and fin width (2 mm).

Fig. 8 shows the measured package surface temperature distribution of the 16 LED array system. Fig. 9 shows the calculated junction temperature and measured package surface temperature distribution of the same system for different electrical power levels. The measured averaged temperature values of the LED systems are 71.7, 100.3, and 115.9 °C corresponding to the electrical power levels of 2.0, 3.0, and 3.5 W, respectively.

Fig. 10 shows the calculated (using (18) and (19)) and measured total luminous flux of the 16-LED system for different electrical power levels. The parameters used are shown in Table I. The average deviation between the proposed model and the measurement is about 4.7%. The maximum deviation between the proposed model and the measurement is 8.9% and that between the original model and the measurement is 14.3%. Again, the calculated values are in good agreement with the measurements.
Fig. 8. Practical layouts and thermal distribution of the 16 LED array system obtained using thermal imaging.

Fig. 9. Calculated junction temperature and measured package surface temperature distribution of the 16-LED array system at different electrical power levels.

Fig. 10. Measured and calculated total luminous flux of the 16 LED system.

C. LED System With 8 × 4.4 W Cool-White LED (Sharp GW5BNC15L02) and 8 × 6.4 W Warm-White LED (Sharp GW5BTF27K00)

The third example is related to emerging color-mixing LED systems. A bicolor white LED system with 8 × 4.4 W cool-white LED (Sharp GW5BNC15L02) with a nominal CCT of 5000 K and 8 × 6.4 W warm-white LED (Sharp GW5BTF27K00) with a nominal CCT of 2700 K mounted on a heatsink with thermal resistance of 1.3 °C/W is used to validate the proposed CCT model. Note that the CCT values of both the warm-white and cool-white LEDs are usually assumed to be constant (according to datasheet), but such an assumption is not true in practice. It should also be mentioned that deviations of ±145 K for a nominal CCT of 2700 and ±283 K for a nominal CCT of 5000 K are cited as acceptable tolerances in solid-state lighting and are considered as unnoticeable to human eyes according to ANSI Standard C78.377 [24].

The temperature and luminous flux of each LED and the mixed CCT of the LED system are calculated using (14), (19), and (26). The parameters of the cool-white LEDs are $E_0 = 80.5 \text{ lm/W}$, $k_e = -0.0027$, $R_{jc} = 6.5 \degree\text{C/W}$, and $k_h = 0.69-0.87 \left(T_j = 38-145 \degree\text{C}\right)$, and that of the warm-white LEDs are $E_0 = 62.2 \text{ lm/W}$, $k_e = -0.0012$, $R_{jc} = 5.5 \degree\text{C/W}$, and $k_h = 0.77-0.88 \left(T_j = 36-143 \degree\text{C}\right)$.

Fig. 11 shows the measured temperature distribution of the bicolor 16-LED array system. Fig. 12 shows the calculated junction temperature and measured package surface temperature distributions of the same system for different electrical power levels. The measured averaged temperatures of LED systems are 60.3, 94.5, and 122.3 °C for 1.7, 2.7, and 4.4 W, respectively.

Using the calculated temperature distribution, the luminous flux for each LED can be calculated using (18) and (19). The overall mixed CCT of the bicolor white LED system with non-identical LEDs is calculated using (26). For this system, the theoretical overall mixed CCT is adjustable and can be changed from 2700 to 5600 K when a different combination of duty cycles is applied to the pair of cool-white and warm-white LEDs.
Fig. 11. Practical layouts and thermal distribution of the bicolor 16 LED array system obtained using thermal imaging.

Fig. 12. Calculated junction temperature and measured package surface temperature distribution of the bicolor 16-LED array system at different electrical power levels.

Fig. 13. Measured and calculated values of the overall mixed CCT versus different ratios of luminous flux between the cool-white and the warm-white LEDs.

Fig. 14. Measured and calculated total luminous flux of the bicolor 16 LED system.

Fig. 13 shows the calculated and measured values of the overall mixed CCT of the system with various ratios of luminous flux between the cool-white and warm-white LEDs. The mixed CCT reduces with an increasing warm-white LEDs’ luminous flux and increases with an increasing cool-white LEDs’ luminous flux. The total luminous flux of LEDs system is calculated and measured with different electrical powers, as shown in Fig. 14. From the figures, the calculated values are generally consistent with the measurements. The average deviation between the proposed model and the measurement is about 5.9%. The maximum deviation between the proposed model and the measurement is 7.8% and that between the original model and the measurement is 13%. These results confirm the accuracy of the CCT prediction based on the proposed LED modeling technique for nonidentical LED devices.

VI. CONCLUSION

A modeling technique that combines the use of the PET LED nodal model and CCT prediction is proposed for modeling emerging LED systems with nonidentical LED devices. The modeling method links the chromatic, photometric, electric, and thermal characteristics of an LED system altogether. The LED nodal model can be arranged in different array forms according to the practical layouts of the LED devices on the same heatsink. The proposed modeling method has been
experimentally verified with three practical LED array systems. The good agreements between the theoretical and experimental values indicate that the proposed method can provide good predictions of the thermal distribution, total luminous output, and CCT of the LED systems. This method is particularly suitable for use as a modeling tool to study emerging LED systems with color mixing functions.

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REFERENCES


Huan-Ting Chen (M’13) received the Ph.D. degree in radio physics from Xiamen University, Xiamen, China, in 2010. He was a Joint Ph.D. student at the Light and Lighting Laboratory, Catholic University College Gent, Belgium, from November 2009 to May 2010. He was a Senior Research Associate in the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, in 2011. He is currently a Postdoctoral Fellow in the Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam, Hong Kong, His research interests include solid-state lighting theory and technology.

De-Yan Lin (M’09) was born in China in 1972. He received the B.Sc. and M.A.Sc. degrees from the Huazhong University of Science and Technology, Wuhan, China, in 1995 and 2004, respectively, and the Ph.D. degree from the City University of Hong Kong, Kowloon, Hong Kong, in 2012. From 1995 to 1999, he was a Teach Assistant in the Electrical Engineering Department at Jianghan University, Wuhan, China, where he became a Lecturer later. From 2008 to 2009, he was a Senior Research Assistant with the City University of Hong Kong. He is currently a Research Associate in the Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam, Hong Kong. His current research interests include memristor, and modeling, control, simulation of gas-discharge lamps, and wireless power transfer.
Siew-Chong Tan (S’00–M’06–SM’11) received the B.Eng. (Hons.) and M.Eng. degrees in electrical and computer engineering from the National University of Singapore, Singapore, in 2000 and 2002, respectively, and the Ph.D. degree in electronic and information engineering from the Hong Kong Polytechnic University, Hung Hom, Hong Kong, in 2005.

From October 2005 to May 2012, he was a Research Associate, Postdoctoral Fellow, Lecturer, and Assistant Professor in the Department of Electronic and Information Engineering, Hong Kong Polytechnic University. From January to October 2011, he was a Senior Scientist in Agency for Science, Technology and Research (A*Star), Singapore. He is currently an Associate Professor in the Department of Electrical and Electronic Engineering, The University of Hong Kong, Pokfulam, Hong Kong. He was a Visiting Scholar at Grainger Center for Electric Machinery and Electromechanics, University of Illinois at Urbana-Champaign, Champaign, from September to October 2009, and an Invited Academic Visitor of the Huazhong University of Science and Technology, Wuhan, China, in December 2011. He is a coauthor of the book *Sliding Mode Control of Switching Power Converters: Techniques and Implementation* (Boca Raton, FL, USA: CRC, 2011). His research interests are focused in the areas of power electronics and control, LED lightings, smart grids, and clean energy technologies.

Dr. Tan serves extensively as a Reviewer for various IEEE/IET Transactions and Journals on power, electronics, circuits, and control engineering.


He has previously held academic positions at the University of Nottingham (1987–1990), University of Technology Sydney (1991–1992), University of Sydney (1992–1996), and City University of Hong Kong (1996–2011). Presently, he holds the Chair Professorship at the University of Hong Kong and Imperial College London. He has published more than 200 technical papers, including more than 170 refereed journal publications and book chapters. More than 55 of his patents have been adopted by industry.

Dr. Hui has been an Associate Editor of the IEEE TRANSACTIONS ON POWER ELECTRONICS since 1997 and an Associate Editor of the IEEE TRANSACTIONS ON INDUSTRIAL ELECTRONICS since 2007. He has been appointed twice as an IEEE Distinguished Lecturer by the IEEE Power Electronics Society in 2004 and 2006. He served an AdCom member of the IEEE Power Electronics Society and was the Chairman of its Constitution and Bylaws Committee from 2002–2010. He received the Excellent Teaching Award in 1998 and the Earth Champion Award in 2008. He won an IEEE Best Paper Award from the IEEE IAS Committee on Production and Applications of Light in 2002, and two IEEE Power Electronics Transactions Prize Paper Awards for his publications on Wireless Charging Platform Technology in 2009 and on LED system theory in 2010. His inventions on wireless charging platform technology underpin key dimensions of Qi, the world’s first wireless power standard, with freedom of positioning and localized charging features for wireless charging of consumer electronics. In Nov. 2010, he received the IEEE Rudolf Chope R&D Award from the IEEE Industrial Electronics Society, the IET Achievement Medal (The Crompton Medal) and was elected to the Fellowship of the Australian Academy of Technological Sciences & Engineering.