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<td>Author(s)</td>
<td>Wang, Y; Zhang, F</td>
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<td>Citation</td>
<td>Physical Review B (Condensed Matter and Materials Physics), 2014, v. 89 n. 9, article no. 094519</td>
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<td>Issued Date</td>
<td>2014</td>
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<td>URL</td>
<td><a href="http://hdl.handle.net/10722/199199">http://hdl.handle.net/10722/199199</a></td>
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Momentum-resolved electronic relaxation dynamics in d-wave superconductors

Yong Wang and Fu-Chun Zhang

1Department of Physics, The University of Hong Kong, Hong Kong SAR, China
2Department of Physics, Zhejiang University, China

(Received 13 October 2013; revised manuscript received 5 February 2014; published 25 March 2014)

Motivated by recent development in time-resolved angle-resolved photoemission spectroscopy (trARPES) for d-wave superconductors, we analyze the nonequilibrium relaxation dynamics of the laser pulse excited sample within the scenario of a two-temperature model. The main features reported in the trARPES technique may be understood within this phenomenological picture. The momentum dependencies of the excited quasiparticle density and the relaxation rate are associated with the dynamics of the nodal d-wave superconducting gap, and the fluence dependence of the relaxation rate is related to the recombination process of quasiparticles into Cooper pairs.

DOI: 10.1103/PhysRevB.89.094519

PACS number(s): 74.25.Jb, 74.40.Gh, 74.72.–h, 78.47.jh

I. INTRODUCTION

Study of temporal dynamics has the potential to advance our understanding of correlated electron systems, since such dynamics may reveal some essential energy scales associated with the elementary excitations and collective modes in these systems [1]. The newly developed time-resolved angle-resolved photoemission spectroscopy (trARPES) is a powerful technique to directly measure momentum-resolved electronic dynamics [2], and has been applied to study ultrafast dynamics in charge density wave materials [3,4], high-temperature superconductors (HTSCs) [5-7], topological insulators [8,9], and heavy fermion systems [10]. Smallwood et al. [7] have used the technique to study gap and quasiparticle (QP) population dynamics in optimally doped d-wave HTSC Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$. Their results clearly show that the QP’s relaxation rate is dependent on the momentum and fluence. These intriguing observations have challenged the understanding of nonequilibrium dynamics in the superconducting (SC) state, and called for theoretical explanations for further explorations on HTSCs with trARPES technique. In this paper we apply a two-temperature model to theoretically study the transient energy distribution and QP relaxation dynamics of a d-wave superconductor measured in trARPES. Our theory explains the momentum-resolved dynamics of the photoexcited QPs, and is in good agreement with experiments.

II. THEORETICAL MODEL

A. Two-temperature scenario

In a typical trARPES experiment, a pump laser pulse is first applied to excite the investigated sample into a nonequilibrium state, and ARPES technique is then used to measure the temporal electronic dynamics of the sample. Presently, trARPES techniques with a time-resolution $\tau_{\text{ARPES}} \approx 100$–300 fs have been applied to study the HTSCs [5-7]. In the relaxation process of the laser-excited d-wave superconductors, there are two essential time scales determined by the intrinsic interactions, namely, $\tau_{\text{ee}} \sim 0$ (fs) due to the electron-electron (e-e) scattering and $\tau_{\text{ep}} \sim 0$ (ps) due to the electron-phonon (e-p) scattering. We consider a typical case where $\tau_{\text{ee}} \ll \tau_{\text{ARPES}} \ll \tau_{\text{ep}}$, which is suitable for the present experiment resolution of $\tau_{\text{ARPES}}$. In this case, the e-e scattering thermalizes the excited electronic subsystem into a quasiequilibrium state in the time scale of $\tau_{\text{ee}}$. A theoretical analysis to consider such type of processes has been given by Howell et al. [11]. Then the detailed nonequilibrium processes of the electronic subsystem will be washed out during the time $\tau_{\text{ARPES}}$, and the quasiequilibrium state can be characterized by the effective temperature $T_{\text{e}}$. In the longer time scale $\tau_{\text{ep}}$, the extra energy in the electronic subsystem will be dissipated to the lattice subsystem through electron-phonon interaction, and $T_{\text{e}}$ will decrease to the lattice temperature $T_{\text{l}}$. Thus the observed gap and QP dynamics of the superconductors will be associated with the time-dependent electronic temperature $T_{\text{e}}$. This simple two-temperature scenario or its more sophisticated generalizations has been applied to describe the relaxation dynamics in HTSCs [2,12,13], and will be applied to study the trARPES results here. The applicability of the two-temperature model to the cuprate experiments of our interests will be discussed in the section of the conclusion and discussions.

The dependence of $T_{\text{e}}$ on the time $t$ in the two-temperature model is not known a priori, but may be determined from the experimental measurements or derived from the microscopic theory. For simplicity we assume that the dominant process for the electronic system to dissipate energy is the pairwise recombination of QPs into Cooper pairs, and the Rothwarf-Taylor equation [14] for the total QP number still holds, which is supported by the previous THz conductivity measurement [15]. Further exploiting the fact that the total QP density is approximately proportional to the square of electronic temperature $T_{\text{e}}$, then the equation of the electronic temperature $T_{\text{e}}$ is estimated as (see the Appendix)

$$\frac{dT_{\text{e}}^2}{dt} = -r(T_{\text{e}}^4 - T_{l}^4).$$ (1)

Here the pump laser is assumed to be applied at time $t = 0$, which promotes the electronic temperature from $T_{l}$ to $T_{e}(0)$. For given $T_{l}$, higher laser fluence will drive the sample into a higher excited state and lead to larger $T_{e}(0)$. Then Eq. (1) describes the decay of the electronic temperature from $T_{e}(0)$ to $T_{l}$, and the parameter $r$ is proportional to the recombination rate of QPs (see the Appendix) and will be determined by fitting to the experimental results later on.
B. Band structure, gap function, ARPES intensity

Now we consider the quasiequilibrium electronic state characterized by temperature $T_e$ and the resulting consequences in trARPES measurements. Since the trARPES has been reported on the optimally doped Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (Bi-2212) samples [5–7], our calculations below will be specified to this compound although the main conclusions are expected to apply to general $d$-wave superconductors. In high $T_e$, cuprate, electrons are highly correlated and it is generally believed that the electron-electron repulsion leads to the high angular momentum $d$-wave superconductivity. Here our focus is to study the electronic relaxation dynamics in the optimally doped SC state. For simplicity we shall treat the correlation effect to the kinetic energy by a simple renormalization factor [16], and SC pairing by using a $d$-wave SC gap function. With this in mind, we use a simple tight-binding model below suggested in Ref. [17] to describe the normal state energy dispersion, which takes into account the kinetic energy renormalization,

$$
e_k = c_0 + \frac{c_1}{2}(\cos k_x + \cos k_y) + c_2 \cos k_x \cos k_y$$

$$-\frac{c_3}{2}(\cos 2k_x + \cos 2k_y) + c_5 \cos 2k_x \cos 2k_y$$

$$+\frac{c_4}{2}(\cos 2k_x \cos 2k_y + \cos k_x \cos k_y),$$

(2)

where the coefficients given in the unit eV are $c_0 = 0.1305$, $c_1 = -0.5951$, $c_2 = 0.1636$, $c_3 = -0.0519$, $c_4 = -0.0510$, $c_5 = 0.94519 - 2014$. The Fermi surface (FS) determined by $e_k = 0$ is shown in Fig. 1(a). A momentum cutline denoted by an angle $\phi$ for ARPES measurements is also shown.

We consider a simple $d$-wave SC gap function for Bi-2212, $\Delta_k = \frac{\Delta(\nu)}{\delta_k}$$(\cos k_x - \cos k_y)$, and further assume a simple BCS form [18] for the temperature dependence of $\Delta(T_e)$, which is determined by the self-consistent equation [19]

$$\frac{1}{N(0)V} = \int_0^{h_{\text{Fermi}}} \frac{\phi}{\pi} \tanh\left[\frac{\phi}{2\Delta(\nu)}\frac{(\xi^2 + \Delta^2)^{1/2}}{(\xi^2 + \Delta^2)^{1/2}}\right] d\xi.$$  

(3)

Here $k_0$ is the Boltzmann constant, $N(0)$ denotes the density of states at the Fermi level of one spin orientation, and $V$ characterizes the strength of pair potential for the electrons within the cutoff energy $h_{\text{Fermi}}$. With the weak-coupling condition $h_{\text{Fermi}} \gg k_b T_e$ and the relation $\Delta(0) = 1.764$, where $T_e$ is the SC critical temperature, Eq. (3) gives the universal function of $\Delta(T_e)$/$\Delta(0)$ on $T_e/T_c$ [19], as shown in Fig. 1(b). While this form is for a conventional $s$-wave superconductor, the deviation from $d$-wave superconductors is not significant to change the qualitative results below.

With the bare band dispersion relation $e_k$ and the gap function $\Delta_k$, the QP excitation energy $E_k$ is determined as $E_k = \sqrt{e_k^2 + \Delta_k^2}$. The key quality measured in the trARPES is the spectral function $A(\omega, k)$ in terms of the energy distribution curves, which takes the form below with a Lorentzian line shape [20]

$$A(\omega, k) = \frac{1}{\pi} \left[ \frac{\mu^2 k^2}{(\omega - E_k)^2 + \Gamma^2} + \frac{\nu^2 k^2}{(\omega + E_k)^2 + \Gamma^2} \right],$$

(4)

where $\mu$ and $\nu$ are the coherence factors. In our calculations we set $T_e = 90$ K, $\Delta(0) = 0.03$ eV, and $\Gamma = 0.01$ eV. When the electronic temperature $T_e$ changes, the spectrum function $A(\omega, k)$ also changes due to the temperature-dependent SC gap.

Several physical qualities based on the spectral functions are usually given in the trARPES experiments, and will be calculated below in order to compare with the experimental results. One is the line-momentum-integrated ARPES intensity $I(\omega)$ (ARPES intensity hereafter) along a momentum cutline $L$ [the choice in our calculations is shown in Fig. 1(a)]

$$I(\omega) = f(\omega) \int_{L} d\mathbf{k} A(\omega, \mathbf{k}),$$

(5)

where $f(\omega)$ is the Fermi-Dirac (FD) distribution function, which depends on the electronic temperature $T_e$. Note that

FIG. 1. (Color online) (a) Fermi surface in a tight-binding model [17] studied in the present paper for optimally doped Bi-2212. $\phi$ denotes the angle to describe a momentum cutline in ARPES measurements. (b) Temperature dependence of SC gap, determined from Eq. (3). (c) and (d) Calculated change in the line-momentum-integrated ARPES intensity $\delta I(\omega)$ defined in Eq. (6), between $T_e = 90$ K and $T_e = 20$ K, along the momentum cutlines $\phi = 45^\circ$, and $\phi = 31^\circ$, respectively. (e) and (f) Experimental result of the intensity change $\delta I(\omega)$ along the momentum cutlines $\phi = 45^\circ$ and $\phi = 31^\circ$, respectively, taken from Ref. [7] for pump fluence 5$\mu$J/cm$^2$ with copyright granted by the original authors. Blue color represents intensity gain and red color intensity loss.
A(ω,k) is also a function of $T_c$ via the SC gap function. In
the two-temperature scenario, as $T_c$ decreases from $T_c(0)$
to the equilibrium temperature $T_L$, the SC gap increases and
the thermal distribution of QPs is suppressed, then the number
of excited QPs is reduced. This process is reflected in the time
evolution of the ARPES intensity $I(ω)$ [5–7].

A more relevant quantity is the change of the ARPES
intensity between the two temperatures $T_c$ and $T_L$,

$$\delta I(ω) \equiv I(ω; T_c) - I(ω; T_L).$$  \hspace{1cm} (6)

It is convenient to introduce the line-momentum-integrated
density of states (DOS hereafter) along the momentum cutline
$L$, $D_L(ω) = \int L d\k A(ω,k)$, then $\delta I(ω)$ is decomposed into two
terms,

$$\delta I(ω) = D_L(ω; T_c)\delta f(ω) + \delta D_L(ω) f(ω; T_c),$$  \hspace{1cm} (7)

where $\delta f(ω) \equiv f(ω; T_c) - f(ω; T_L)$ is the change of the
distribution function and $\delta D_L(ω) \equiv D_L(ω; T_c) - D_L(ω; T_L)$ is
the change of the DOS along $L$. This decomposition is vital
for us to understand the results below. In Figs. 1(c) and 1(d)
we show the results of $\delta I(ω)$ for the system for two sets of FS
angles, i.e., a diagonal cutline $ϕ = 45^\circ$ (nodal cutline) and an
off-nodal cutline $ϕ = 31^\circ$, respectively. The two temperatures
are $T_c = 90$ K and $T_L = 20$ K. $\delta I$ is strongly dependent
on the cutline. For the nodal cutline, $\delta I$ is approximately
antisymmetric with respect to $ϕ$, similar to the change of the
Fermi distribution function $\delta f(ω)$. In particular, $\delta I = 0$
at $ω = 0$. For the non-nodal cutline, the shape is far from the
antisymmetric, and $\delta I(ω = 0)$ is finite and positive, and $\delta I$
occurs at $ω < 0$. Our calculations are in good agreement with
the experimental data [7], which are reproduced in Figs. 1(e)
and 1(f) for comparison. The strong FS angle dependence of
the ARPES intensity is associated with the symmetry of the SC
gap function. For the nodal cutline of ($ϕ = 45^\circ$), the second
term in Eq. (7) vanishes. Furthermore, $\delta f(−ω) = −\delta f(ω)$ and
$\delta f(ω)$ is of substantial value only within a window of $|ω| < 3k_BT_c$. Since $D_L(ω; T_L)$ does not vary sharply near $ω = 0$,
we have $\delta I(ω)$ approximately antisymmetric as numerically
shown in Fig. 1(c). For the off-nodal case, the second term in
Eq. (7) plays an important role, and $\delta I(ω)$ is no longer
antisymmetric. At $ω = 0$ we have $\delta D_L(0) > 0$ according to
Eq. (4) and $f(0) = 1/2$, thus $δI(0) = 1/2δD_L(0) > 0$.

Another important quality given in trARPES experiments
is the angle-resolved QP density. Following the experiments
[5,7] we define $I$ as the integral of the ARPES intensity $I(ω)$
over $ω$ above the Fermi energy along the momentum cutline
$L$ at temperature $T_L$, and $\Delta I$ is the change of $I$ when
the electronic temperature increased from $T_L$ to $T_c$, i.e.,

$$I = \int_0^\infty dω I(ω), \quad \Delta I = \int_0^\infty dω \delta I(ω).$$  \hspace{1cm} (8)

Below we analyze the dynamics of $\Delta I$ to further explain the
trARPES experiments.

III. CALCULATION RESULTS

A. Angle dependence of photoexcited quasiparticle density

It has been observed that the photoexcited QP density
is strongly dependent on the FS angle $ϕ$ similar to the $d$-wave
SC gap function [5]. According to our discussions above, the
electronic temperature has been increased from $T_L$ to $T_c(0)$ after
the laser pumping, because the detailed electronic dynamics to
reach such a quasiequilibrium state has been washed out due to
the large time resolution of the ARPES measurements. Then
the SC gap is reduced and even closed if $T_c(0) \geq T_c$ and more
QPs are also excited. Thus we can calculate the photoexcited
QP density according to Eq. (8), and the results of $\Delta I/I$ for the
temperatures $T_c(0) = 90, 80, 70, 60, 50$ K and $T_L = 20$ K are
given in Fig. 2(a). The calculation results reproduce the angle-
dependence relation observed in experiments [5], as shown
in Fig. 2(c). Notice that higher pump fluence corresponds to higher electronic temperature $T_e(0)$.

To understand the results, we recall that $\Delta I$ has both the contributions from thermal broadening and gap reduction according to Eq. (7). We denote $\Delta I \equiv \Delta I_f + \Delta I_g$, where $\Delta I_f = \int_0^\infty d\omega D_L(\omega; T) \delta f(\omega)$, $\Delta I_g = \int_0^\infty d\omega \delta D_L(\omega) f(\omega; T)$. The angle dependence of $\Delta I_f / I$ is expected to be weak as long as $D_L(\omega; T)$ does not vary sharply near the Fermi energy. Specially if $D_L(\omega; T)$ is approximated by its value $D_L(0; T_l)$ at Fermi energy, $\Delta I_f / I$ will be independent on the FS angle $\phi$. While for $\Delta I_g / I$, its angle dependence comes from the change of DOS $\delta D_L(\omega)$ which is strongly dependent on the $d$-wave gap function. For the nodal case ($\phi = 45^\circ$), $\delta D_L(\omega)$ vanishes at the Fermi energy $\Delta(\omega; 0) = 0$, while when the temperature $T_e$ is below $90 \, K$ and lattice temperature $T_l = 20 \, K$, its angle dependence is weak [7] and away from the nodal region, the gap reduction becomes smaller, and the angle dependence of $\Delta I / I$ becomes weak [7]. This tendency is shown in Fig. 2(a), and is also found in the trARPES experiments in Fig. 2(c).

### B. Angle dependence of quasiparticle decay rate

We further analyze the angle-dependent decay rate of QPs. It has been observed that the QPs in the off-nodal region decay faster than the ones in the nodal region [7]. In the two-temperature scenario, the ARPES measures the quasiequilibrium electronic state characterized by the temperature $T_e(t)$. We first calculate the normalized $\Delta I$ for different temperatures at given FS angle $\phi$, and the results are shown in Fig. 3(a). Here we have fixed $T_l = 20 \, K$, and varied the temperature $T_e$ from 90 to 20 K, and then normalize $\Delta I$ by its value at 90 K. It is found that for given temperature $T_e$, the normalized $\Delta I$ decreases when the angle $\phi$ moves from the nodal region to the off-nodal region. Then with the help of Eq. (1), the temperature dependence of the normalized $\Delta I$ in Fig. 3(a) is mapped to the corresponding time-dependence relation, as shown in Fig. 3(b). Here we have set $r = 1.5 \times 10^{-4} \, K^{-2} \, ps^{-1}$ to fit the experimental results. Our results here reproduce the observations that the decay of normalized $\Delta I / I$ depends on the FS angle [7] shown in Fig. 3(c).

The results in Fig. 3 can also be understood from the two contributions $\Delta I_f$ and $\Delta I_g$ in $\Delta I$. For the nodal case ($\phi = 45^\circ$) we have $\Delta I_g = 0$, while $\Delta I_f \sim D_L(0; T_l) k_b(T_e - T_l)$ if the DOS $D_L(\omega; T)$ is approximated by its value at Fermi energy. This explains the nearly linear dependence of the normalized $\Delta I$ on the temperature for $\phi = 45^\circ$ in Fig. 3(a). In the off-nodal case, $\Delta I_g$ due to the gap variation contributes to $\Delta I$ in addition to $\Delta I_f$. When the temperature $T_e$ decreases, the gap becomes larger and $\delta D_L(\omega)$ decreases. Thus in the off-nodal region the normalized $\Delta I$ deviates from the linear temperature-dependence relation and decays faster, as shown in Fig. 3(a). After mapping to the time scale, the normalized $\Delta I$ then shows the angle-dependent decay rate of QPs.

### C. Fluence dependence of quasiparticle decay rate

It is also observed that the QPs at fixed angle relax faster if the sample is pumped by higher fluence laser [7]. The reason however is not the same as the angle-dependence case discussed above, considering that the superconducting gap is

![FIG. 3. (Color online) (a) The temperature dependence of normalized $\Delta I$ for several FS angles with initial electronic temperature $T_e(0) = 90 \, K$ and lattice temperature $T_l = 20 \, K$. (b) Decay curves of normalized $\Delta I$ for several FS angles by mapping the curves in (a) to the time scale with the help of Eq. (1). The fitting parameter $r = 1.5 \times 10^{-4} \, K^{-2} \, ps^{-1}$. (c) Experimental decay curves of normalized $\Delta I$ at $\phi = 45^\circ$ and $\phi = 31^\circ$, respectively, taken from Ref. [7] with copyright granted by the original authors.](094519-4)
IV. CONCLUSION AND DISCUSSIONS

In conclusion, we show that the main features of the trARPES observations for $d$-wave superconductors so far can be explained within a simple two-temperature scenario. In this picture, the effective electronic temperature affects both the thermal distribution of the quasiparticles and the superconducting gap. The angle dependence of the photoexcited quasiparticle density and the quasiparticle decay rate are associated with the $d$-wave gap dynamics, while the fluence-dependent quasiparticle decay rate is due to the pairwise recombination of quasiparticles into Cooper pairs.

APPENDIX: EQUATION FOR ELECTRONIC TEMPERATURE

Here we explain the derivation of Eq. (1), which describes the decay of the electronic temperature $T_e$. This is based on two basic starting points. One is the two-temperature scenario, which has been explained in the body text. Another is that the dynamics of total quasiparticle (QP) number obeys the Rothwarf-Taylor (RT) model [14].

The RT model was originally proposed to phenomenologically describe the bidirectional transformation process between the QPs and Cooper pairs in accompanying with emission or absorption of phonons in $s$-wave superconductors [14]. In the photoexcited superconductors, the extra energy in refer to the details of the microscopic scattering processes, which have been washed out due to the thermalization of e-e scattering in the time scale $\tau_{ee}$. Our results suggest that the phenomenological two-temperature model could be a good starting point to analyze the trARPES experiments before extracting other interesting microscopic dynamics processes.

Below we discuss various approximations we have used in our theory. First, we have used a two-temperature model, which assumes $\tau_{ee} \ll \tau_{ep}$. The validity of this assumption strongly depends on the materials and also temperature in some cases. It has been proposed that $\tau_{ep}$ is shorter in some simple metallic systems at low temperatures [21,22]. We argue that the assumption should apply to high $T_c$ cuprates for the strong electron-electron scattering as evidenced in the resistivity measurement, where the linear resistivity has been observed in a large temperature region down to above $T_c$, which is dominated by the electron-electron scattering. The good qualitative agreement between our theory and experiments in Ref. [7] is also a further support to this assumption. Second, we have neglected the dynamics of phonon, hence the bottleneck effect in our theory [23]. In the experiment of Ref. [7], the laser pump is relatively weak, the neglect of the bottleneck effect should not change the qualitative physics, especially the momentum dependence of the relaxation dynamics we report here. Third, the linewidth of the spectral function can be momentum dependent due to the anisotropic scattering rate [24], but this will not change our results qualitatively since the relative change of the spectral intensity is measured in the experiments. Finally, we remark that the trARPES technique with $\tau_{ARPES} = 30$ fs has recently been reported [25], which may reveal more details of the electronic relaxation dynamics. It will be important to test our theory in future trARPES experiments and to develop our theory further along this direction.

ACKNOWLEDGMENTS

We thank Jianqiao Meng for indicating us to the trARPES technique, and thank Weiqiang Chen, Zijian Yao, and Hongmin Jiang for helpful discussions. We also thank P. J. Hirschfeld and T. Durakiewicz for useful comments. This work is supported by the Hong Kong grants of University Grant Council AoE/P-04/08 and GRC HKU707010, and by NSFC 11274269 in China.
the electron subsystem will be dissipated into the lattice subsystem, which is governed by the electron-phonon interaction. In the superconducting states, there are two types of processes for the energy dissipation. One process is that one quasiparticle (QP) is scattered from one energy state into another by emitting one phonon, which is similar to the normal scattering process in simple metals; another process is that two QPs are combined into one Cooper pair by emitting one phonon. The time-reversal process can also happen due to the existence of phonons. The former process will cause the redistribution of the energy states of the QPs without changing their total number; while the latter will change the total number of QPs and is responsible for the dynamics of the superconducting gap. We consider the pairwise recombination-breaking process to be dominant and expect that the RT model is also applicable for the energy dissipation. One process is that one quasiparticle (QP) is scattered from one energy state into another by coupling to the environment. For the case studied in this paper, \( I_0 \) comes from the initial pump laser pulse. For the low fluence pump laser, we assume that the generated phonons during the pairwise recombination process decay fast and thus neglect the deviation of \( N_\omega \) from its equilibrium value \( N_{\omega T} \), i.e., \( N_\omega = N_{\omega T} \). This implies that the bottleneck effect has been neglected in this treatment. Then the relaxation dynamics of QP number \( N \) after the excitation is simplified as

\[
\frac{dN}{dt} = -RN^2 + \beta N_{\omega T}. \tag{A3}
\]

The equilibrium value \( N_{\tau} \) for QP number satisfies the relation \( RN_{\tau}^2 = \beta N_{\omega T} \). Thus Eq. (A3) can be rewritten as

\[
\frac{dN}{dt} = -R(N^2 - N_{\tau}^2). \tag{A4}
\]

Equation (A4) describes the decay of the total number of QPs during the relaxation of the electronic subsystem to the equilibrium state. As we also take the two-temperature scenario, then the decay equation for the effective electronic temperature \( T_e \) can be estimated from Eq. (A4) by utilizing the relation between the effective electronic temperature \( T_e \) and the total number of QPs \( N \). When the temperature \( T_e \) is low, the density of states for QPs in \( d \)-wave superconductors is approximately linear in the electronic temperature, thus the total number of QPs is approximately proportional to the square of the temperature, i.e., \( N = bT_e^2 \). Substituting this relation into Eq. (A4), and note that \( N_{\tau}^2 = bT_{\tau}^2 \), we get the equation for the electronic temperature \( T_e \) as

\[
\frac{dT_e}{dt} = -r(T_e^4 - T_{\tau}^4), \tag{A5}
\]

where \( r = bR \). This gives an estimation of the decay of the electronic temperature \( T_e \), which finally reaches \( T_{\tau} \) when \( t \to \infty \).

---

[18] High \( T_c \) cuprates are highly correlated systems, where the correlations have profound effect. Here we focus on the \( d \)-wave superconducting state in the optimal doping region. Within this region, the superconducting state may be described by a renormalized BCS mean field theory. The renormalization is to change the parameters in the kinetic and pairing terms from the
original model such as the $t$-$J$ model. Therefore, for a fixed hole doping, we could use BCS mean field theory with the renormalized parameters to discuss some of the dynamic effects of the system.


