Lot Sizing Optimisation for Stochastic Make-to-order Manufacturing

X. J. Wang and S. H. Choi

Abstract—Lot sizing is pivotal to batch manufacturing, especially in stochastic environments. Although progress has been made in the field for some operational objectives, the optimised results are often rendered unrealistic because few studies have considered the overall business goal and the economic environment where businesses operate. This paper examines a stochastic lot sizing optimisation model for make-to-order manufacturing with a focus on the overall business goal—the maximisation of shareholder wealth. In addition to the economic objective, the effect of the economic environment is also incorporated into this model. Numerical experiments validate the importance of considering such economic and financial constraints and objectives, especially for firms with relatively high setup costs or being sensitive to lead times. The proposed model can assist the management in gaining insight into potential challenges and opportunities pertinent to the shareholder wealth.

Index Terms—batch manufacturing, CFROI, lot sizing, shareholder wealth, stochastic processes

I. INTRODUCTION

Timely provision of quality products at the lowest prices possible has become the utmost competitive edge being pursued by virtually all businesses. Firms endeavour to speed up manufacturing and delivery of goods or provision of services to customers. It was, however, estimated that only less than 15% of manufacturing time is spent on actual job processing, whereas over 85% is wasted in work-in-process (WIP) and queuing delays [1]. This warrants an imminent time-based competition, and concurrent engineering.

Despite the widespread popularity of manufacturing time minimisation in operation management [2, 3], the optimised results are often unrealistic and difficult to realize in a firm because its economic factors and financial position have not been considered. Some researchers seek to solve this problem by optimising certain economic objectives instead of operational ones. Most of them are targeted at optimising some accounting cost or profits. Ref. [4], for example, developed a cost minimisation model with several relevant costs taken into account. Ref. [5] chose to maximise accounting profits in a multi-product capacity-constrained lot sizing environment. Either minimisation of costs or maximisation of profits, in general, may not necessarily represent the full interest of equity holders, especially in some adverse economic situations, such as unexpected inflation and recessions in a business cycle. In fact, it is the shareholder wealth maximisation that have currently become the top priority of most enterprises [6-8].

Thus, it can be seen that most production optimisation models either overlook a firm’s economic conditions and financial position, or optimise some local objective functions without considering the overall business goal of maximising its shareholders’ long-term sustainable interests. Moreover, some key macroeconomic factors, such as impacts of inflation and business cycle on optimisation, have not been taken into account.

We attempt to address these problems by setting up a queuing network for the concerned stochastic make-to-order manufacturing, with an aim to maximise the long-term full interests of equity holders. Our proposed approach is different than others in the following aspects.

A. Stochastic Make-to-order Manufacturing

We concentrate on a single-item stochastic make-to-order manufacturing environment, due mainly to its widespread application and acceptance in the academia and industry. Ref. [9] established an M/M/1 queuing model with the lot-sizing policy taken into account, and then validated that the lot-sizing policy is a crucial determinant of the queuing delay for closed job shops. Ref. [10] formulated two queuing problems for the design of new systems. Not only is the lot-sizing policy involved in these two models, but the capacity issue is also examined.

However, most of these studies assumed that the interarrival of orders follows an independent Poisson process and that the processing procedure is exponentially distributed. This is, to a great extent, not true and sometimes even misleading for a great number of real manufacturing systems. Ref. [11] has argued that these factitious assumptions were extremely restrictive and unrealistic. Ref. [12] suggested an Erlang process, instead of the Poisson process, in the case of a small number of independent demand sources.

Thus, we try to formulate the proposed manufacturing scenario as a stochastic lot sizing queuing network, and characterize all random variables involved by their own statistical merits without any unrealistic assumptions on them, so as to improve the generality as well as exactness of our proposed approach.

B. Shareholder-wealth-oriented Optimisation

As stated previously, current operation optimisation objectives, focusing mainly on short-term local optimisation, such as time minimisation, accounting cost minimisation, and accounting profit maximisation, may not necessarily be beneficial to the overall business goal of maximising the
shareholder wealth, because some key determinants, such as macroeconomic factors and cash flow management, are often overlooked.

In this article, we attempt to address this problem by optimising the long-term sustainable interests of shareholders, well-known as the shareholder wealth, represented by the financial metric—cash flow return on investment (CFROI), due to its superior characteristics to other peer measures, such as net present value (NPV)[13], return on investment (ROI)[14], and economic value added (EVA)[15, 16].

C. Commodity Pricing Based on Economic Theory

Another critical issue is how to price the finished products. Ref. [17] stated that an appropriate price premium is allowed for a relatively short delivery time. More and more industry practices suggest that customers are willing to pay a price premium for relatively shorter delivery times than the industrial average [18-20]; and conversely for products with longer delivery times, customers are inclined to pay less or would simply go for substitutes.

Thus, it can be seen that there exists a close connection between commodity pricing and manufacturing times. In addition to these academic grounds, in our research, macroeconomic theory is also used to mathematically formulate the specific impacts of manufacturing times on prices of finished products.

To summarize, this article attempts to address the problem of the lot sizing optimisation under the stochastic make-to-order manufacturing, with an aim to maximise the long-term sustainable interests of shareholders, measured by CFROI. The uncertain manufacturing environment is formulated as a stochastic single-item lot sizing queuing network without any impractical assumption on the relevant random variables.

II. MATHEMATICAL PROGRAMMING FORMULATION

A. Supply Chain Description

<table>
<thead>
<tr>
<th>Orders</th>
<th>Gathering</th>
<th>Setup</th>
<th>Processing</th>
<th>Delivered</th>
</tr>
</thead>
</table>

Fig. 1. Total work flow profile

Fig. 1 shows the total work flow profile for the proposed make-to-order manufacturing environment. It illustrates one type of product being processed at a single machine station. Individual customer orders arrive at the gathering stage one by one, where once \( Q \) units of orders gather together, these batches of orders leave this stage and go into the setup stage for further work. Afterwards, these partially completed orders are moved to the processing stage to undergo further processing service on an individual basis to be converted into finished goods for immediate delivery to customers, without having to wait until the whole batch is completed.

All stages involved in the afore-mentioned manufacturing environment are assumed to be mutually independent. In order to improve the generality as well as the exactness of our proposed model, we characterize each random variable by its two statistic merits—expected values (or rates) and standard deviations, rather than by making any assumption on their specific theoretical distributions. In the event of competition for capacitated resources, orders are served in accordance with the first-come-first-served queuing principle. Without loss of generality, we further assume that each individual order contains only one product item, and that the manufacturer is a price taker in either the perfect or the monopolistic competition environment.

B. Stochastic Manufacturing Formulation

As stated previously, lead time optimisation has been one of the critical mainstreams in operation management. Based on Fig. 1, in our research, lead time is defined as the time that elapses after an order arrives and before being delivered, as follows:

\[
E(W) = E(W_o) + E(W'_s) + E(W'_i) + E(W'_p) + E(W'_r)
\]  
(1)

with all involved parameters defined in Table I.

According to the pioneering research works[21, 22], we have

\[
E(W_o) = \frac{1}{(2\lambda)}
\]  
(2)

\[
E(W'_i) = \frac{\rho}{2\lambda}(\epsilon_2 + \epsilon_3)
\]  
(3)

with a traffic intensity \( \rho = \lambda(\mu + \tau)/(\mu \mu) \) and,

\[
g = \left\{ \begin{array}{ll}
-2(1 - \rho)\epsilon_2 \leq 1 \\
3\rho(\epsilon_2 + \epsilon_3) \epsilon_3 \leq 1 \\
1 - \rho(\epsilon_2 + 10\epsilon_3)^2 \geq 1
\end{array} \right.
\]  
(4)

In the gathering stage, orders enter for being gathered in batches without queuing. Once placed, they can immediately go into this stage without any delay, thus,

\[
E(W_o) = 0
\]  
(5)

In addition, based on the probability theory, we can readily figure out:

\[
E(W'_i) = E(T) = \tau
\]  
(6)

\[
E(W'_r) = E(X_i) = \frac{1}{\mu}
\]  
(7)

Given \( 1 \leq i \leq Q \), representing the relative position of an order in a given batch, the expected time spent in waiting for processing service by it is:

\[
E(W_{o,i}) = E(X_{i+1} + X_{i+2} + ... + X_{i+|X|}) = (i - 1)/\mu
\]  
(8)

Thus, the expected queuing time for processing all customer orders should be:

\[
E(W_o) = E(E(W_{o,i})) = E\left(\frac{1}{\mu}(1 - \frac{1}{\mu})\right) = 1 \frac{\mu}{1 - \frac{1}{\mu}}
\]  
(9)

Hence, Eq. (1) can be rewritten as follows:

\[
E(W) = \frac{1}{2\lambda} + \frac{\rho}{2\lambda}(\epsilon_2 + \epsilon_3) + \frac{\rho}{2\lambda}(1 - \rho) + \frac{Q - 1}{2\mu} + \frac{1}{2\mu}
\]  
(10)

C. Commodity Pricing

As mentioned previously, scholars have suggested a negative relationship between commodity prices and lead times. Here we further illustrate this point from the perspective of the macroeconomic theory.
As illustrated in Fig. 2, the supply and the demand firstly balance at point $A$, where the manufacturer produces a quantity $Q_1$ of products and sells them at the price $p_1$. The decreasing lead time produces an increased market demand for the products, and therefore the demand curve shifts upward from $D_1$ to $D_2$. Then, a new balance between supply and demand sets up at point $B$. If there is no constraint on capacity, the manufacturer would choose to produce $Q_2$ and sells them at the sales prices of $p_2$ to take all potential profits. Subject to the capacity constraint, however, it has no extra capability to produce more products than $Q$. Consequently, it can only make use of this competitive advantage by asking for a price, as high as possible. Eventually, the real new balance builds at point $C$, rather than $B$. Thus, a decrease in lead time directly gives rise to a corresponding linear increase in sales price, and vice versa.

![Fig. 2 The supply-demand curve analysis under macroeconomic theory](image)

Based on the above supply-demand curve analysis and suggestions from pioneering studies [17, 18, 20], we derive a negatively linear relationship between sales prices and the expected lead time:

$$p = -\kappa(E(W) - E(W)_{\text{avg}}) + p_{\text{avg}} \quad (\text{Floor} \leq p \leq \text{Cap})$$  \hfill (11)

The parameter $\kappa$ indicates the level of customer sensitivity to the delivery, and hence the lead time spread, of a product. A large $\kappa$ means that customers are knowledgeable of the market information and have a strong desire to acquire the product soon. It is difficult to determine $\kappa$ theoretically for a firm because of various complicated factors. Nevertheless, it can be heuristically set between the range of 0 and 100.

**D. Shareholder Wealth Derivation**

Firstly, the operating income can be estimated as the revenue minus the total cost of goods sold (COGS), as in

$$OI = p_1 \frac{1}{P} - \frac{DA}{L} - FOC, - a_1, \frac{1}{P} - s, \frac{1}{P} - Q - E(W) \frac{1}{P}$$  \hfill (12)

where $P_1 \frac{1}{P}$ represents sales revenue, and $\frac{DA}{L}$ denotes the depreciation expense when using the straight-line depreciation method; $a_1, \frac{1}{P} - s, \frac{1}{P} - Q$ and $E(W) \frac{1}{P}$, respectively mean the total purchasing cost, setup cost, and inventory cost. Thus,

$$TFC = \frac{1}{P} + s, \frac{1}{P} - Q - E(W) \frac{1}{P} + v_1, \frac{1}{P} + OI, x_t, \frac{1}{P}$$  \hfill (13)
Finally, the period cash flow can be estimated as the net income plus noncash expenses [23], as in:

$$CF = R - TVC - TFC + \frac{DA}{L}$$  \hspace{1cm} (14)

Subsequently, we have to relate these parameters for maximising the shareholder wealth in terms of CFROI. As stated previously, CFROI is a real, cross-sectional internal rate of return (IRR) calculated at a time point from aggregate data for a firm. It is one of economic performance metrics, focusing on the real rate of return earned on the entire assets. The basic valuation of CFROI is based on DCF. So the conception of IRR and DCF can be applied to calculate the CFROI [23], as follows:

$$T_{d} = \frac{1}{\sum_{j} \frac{CF}{(1+CFROI)^{j}}} + \frac{NA}{(1+CFROI)^{j}}$$  \hspace{1cm} (15)

with the following constraint conditions:

$$Q \geq 1$$
$$\rho < 100\%$$
$$F \leq p \leq C$$  \hspace{1cm} (16)

III. NUMERICAL EXPERIMENTS

This article seeks to optimise the single-item lot sizing manufacturing under uncertainty, aimed to maximise the shareholder wealth. The proposed approach incorporates some real industrial practices. In manufacturing of specialised bicycles, for example, orders for bicycles arrive on an individual basis and are gathered by the sales department, and then some operations, such as electroplating, are conducted on a batch basis, like the setup service. Finally, components are assembled into finished bicycles one by one for delivery to end customers. Another typical example is in the metal treatment industry, where metal workpieces arrive individually at furnaces for heat treatment. As soon as a given number of metal workpieces are batched, they are loaded as a whole for heat treatment. Subsequently, they are sandblasted on an individual basis before delivery.

To further validate our proposed model, three numerical experiments are performed. The first one compares the proposed shareholder wealth maximisation model to the traditional operation optimisation. The second one explores the impacts of the unrealistic statistical distributions on shareholder wealth. In the last one, we test the hedging capability of our model to provide insights into how possible and at what level these risks affect the long-term sustainable interests of investors, especially equity holders.

A. Comparison of Optimisation Objectives

In order to better examine the difference between operational and financial optimisations, we firstly determine the optimal lot size that can minimise the total lead time, with expected times for each manufacturing step shown in Table II.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gathering Queuing</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Gathering</td>
<td>11.500</td>
<td>16.000</td>
</tr>
<tr>
<td>Setup Queuing</td>
<td>1.466</td>
<td>0.013</td>
</tr>
<tr>
<td>Setup</td>
<td>10.000</td>
<td>10.000</td>
</tr>
<tr>
<td>Processing Queuing</td>
<td>5.750</td>
<td>8.000</td>
</tr>
<tr>
<td>Processing</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Fig. 3 Lead time as a function of lot size

Fig. 4 Shareholder wealth as a function of lot size

It can be seen that the expected collecting time increases by about 39.13%, from 11.500 minutes to 16.000 minutes, while the expected queuing time for the processing service jumps from 5.750 minutes to 8.000 minutes. Conversely, the expected queuing time for the setup service drops drastically by 99.12%, from 1.466 minutes to 0.013 minutes. To a great extent, the decrease of time spent in the setup service offsets the time increase in the gathering service and processing delay. In comparison with the lead time optimisation, the total lead time increases only slightly under the financial optimisation when the fixed lot size changes from 24 to 33.

B. Shareholder Wealth under Theoretical Assumptions

Here we consider a theoretical case to validate the extensibility of our proposed model. Several theoretical assumptions will be made. Interrival of orders follows a Poisson process, while the processing time is exponentially distributed. We also assume that the setup time is completely deterministic, such that $c_{s} = \frac{z}{Q}$, $c_{a} = Q/(\mu + Q)^{2}$. Hence, Eq.(10) can be reorganized by substituting these new expressions for the older ones, as follows:
\[ E(W) = \frac{Q-1}{2} + E(W_0) + \tau + \frac{Q-1}{2\mu} \]

where,

\[ E(W_0) = \frac{\lambda(\mu + Q)^2 + Q^2}{2\mu Q[\lambda Q - \lambda(\mu + Q)]} \]

\[ \times \exp \left[ -2\left[ Q - \lambda(\mu + Q) \right] Q - \lambda^2(\mu + Q) \right] \]

(17)

(18)

Next, we need to recalculate the optimal lot sizes under the theoretical case. Fig. 5 shows the relation between the lot size and the lead time under the above assumptions. The minimum total lead time of 29.938 corresponds to the lot size of 25 with a traffic intensity of 90.00%. The time consumptions for each manufacturing step are given in Table III. Here the shareholder value optimisation leads to an optimal lot size of 33 with a maximum CFROI value of 47.93%. Fig. 6 illustrates the effects of various lot sizes on the shareholder wealth. It can be seen that the time consumed on gathering increases from 12,000 to 16,500 minutes, approximately an increase of 37.50%. In the processing stage, more 2,250 minutes are spent in the processing service. Queuing time for setup service drops from 1.438 to 0.041 minutes. Similarly, mutual offset of these manufacturing times only lead to a slight increase of the total lead time, from 29.938 up to 34.559, under the theoretical case when the optimal lot size changes from 25 to 33.

It is worth noting that the shareholder wealth decreases from 48.18% without statistical assumptions on random variables to 47.93% under the theoretical assumptions. This further demonstrates that incorrect distributions assumptions are always unrealistic and misleading.

**TABLE III EXPECTED TIMES FOR EACH MANUFACTURING STEP UNDER THE THEORETICAL ASSUMPTIONS (MINS)**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Gathering Queuing</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>Setup</td>
<td>12.000</td>
<td>16.500</td>
</tr>
<tr>
<td>Setup Queuing</td>
<td>1.438</td>
<td>0.041</td>
</tr>
<tr>
<td>Processing</td>
<td>6.000</td>
<td>8.250</td>
</tr>
<tr>
<td>Processing Queuing</td>
<td>0.500</td>
<td>0.500</td>
</tr>
</tbody>
</table>

**C. Sensitivity Analysis**

Finally, we take a closer look at how sensitive the shareholder wealth is to two key parameters \( \kappa \) and \( s_s \).

We firstly examine the effects of customer sensitivity levels to lead time, \( \kappa \), on the shareholder wealth. Table IV lists the optimal CFROI values and corresponding lead times as \( \kappa \) deceases from 100 to 0. The lower its value, the more indifferent the customers are to the changes in lot sizes and lead times. Fig. 7 shows the curves of the lead time and the CFROI metric for a various range of \( \kappa \) values. The figure illustrates that different values have no effects on the curve shape of the lead time, but it distorts the curve shape of the shareholder wealth.

**TABLE IV EFFECTS OF CUSTOMER SENSITIVITY TO LEAD TIME ON SHAREHOLDER WEALTH**

<table>
<thead>
<tr>
<th>( \kappa )</th>
<th>Lot Size</th>
<th>CFROI</th>
<th>Lead Time (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>25</td>
<td>360.46%</td>
<td>29.938</td>
</tr>
<tr>
<td>10</td>
<td>28</td>
<td>73.60%</td>
<td>31.148</td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>49.91%</td>
<td>33.835</td>
</tr>
<tr>
<td>0.1</td>
<td>33</td>
<td>48.11%</td>
<td>34.559</td>
</tr>
<tr>
<td>0</td>
<td>33</td>
<td>47.91%</td>
<td>34.559</td>
</tr>
</tbody>
</table>

**Fig. 7 Lead time and shareholder wealth against lot sizes under various customer sensitivities**

Table IV and Fig. 7 have two important implications. The first is that the optimisation results under both the operational
and financial cases are much closer when the $\kappa$ value is large enough. Second, the proposed model has almost the same optimisation result regardless of the $\kappa$ value when it is small enough.

The second implication can be directly reflected in (11). When the $\kappa$ value is small enough, $-\kappa(E(W) - E(W)_{opt})$ is nearly zero and can be ignored. In other words, the selling price approximately equals $p_{opt}$. Thus, in this case, the changes of $\kappa$ almost have no effect on the shareholder wealth, just as illustrated in Table IV and Fig. 7.

Another concern is how the setup cost can affect the shareholder wealth. Table V lists the optimal fixed lot sizes, the corresponding CFROI values and lead times for a various series of setup costs. This table shows that when setup costs are low enough, the financial optimisation results in the similar optimal lot sizes to the operational model. For example, when $s_i = 10$, the optimal fixed lot size in the first numerical experiment is 24, while the resulting optimal lot size is 25 under the financial optimisation. However, as soon as the setup cost increases to a certain degree, its impact on the shareholder wealth becomes more substantial.

**TABLE V: EFFECTS OF SETUP COST ON SHAREHOLDER WEALTH**

<table>
<thead>
<tr>
<th>$s_i$</th>
<th>Lot Size</th>
<th>CFROI</th>
<th>Lead Time (mins)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>25</td>
<td>189.25%</td>
<td>29.938</td>
</tr>
<tr>
<td>100</td>
<td>25</td>
<td>177.61%</td>
<td>29.938</td>
</tr>
<tr>
<td>1000</td>
<td>31</td>
<td>69.50%</td>
<td>33.124</td>
</tr>
<tr>
<td>11000</td>
<td>32</td>
<td>58.64%</td>
<td>33.835</td>
</tr>
<tr>
<td>12000</td>
<td>33</td>
<td>47.91%</td>
<td>34.559</td>
</tr>
</tbody>
</table>

IV. CONCLUSION

This article presents research in attempt to address the infeasibility issues of traditional manufacturing optimisation based on operational objectives. It proposes a stochastic lot-sizing optimisation model for make-to-order manufacturing to maximise a financial objective. The proposed model incorporates financial and economic parameters relevant for a firm’s business goal of maximising the shareholder wealth, measured by CFROI. It assumes individual arrivals and departures and general distribution of random variables. This treatment enables the proposed model to deal with relatively more realistic demand patterns, and enhances its generality and extensibility.

Numerical experiments show that when the setup costs are low and the customer sensitivity to lead time is high, there is no significant difference between operational optimisation and financial optimisation. However, if the setup costs are high or the customer sensitivity to lead time is low, the optimal lot size for operational optimisation is generally much smaller than for maximisation of the shareholder wealth. This validates that traditional operational optimisation is not necessarily in line with the overall business goal of a firm, and thus highlights the importance of considering financial and economic parameters for optimising manufacturing decisions.

A limitation of the proposed model is that it focuses only on a single-item, single-machine stochastic lot-sizing scenario. It would therefore be worthwhile to extend it for dealing with relatively more complicated manufacturing environments. Moreover, further research work would be needed to examine the relationship between the lead time spread and the selling prices.

**REFERENCES**


