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Finite-Temperature Conductivity and Magnetoconductivity of Topological Insulators

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The electronic transport experiments on topological insulators exhibit a dilemma. A negative cusp in magnetoconductivity is widely believed as a quantum transport signature of the topological surface states, which are immune from localization and exhibit the weak antilocalization. However, the measured conductivity drops logarithmically when lowering temperature, showing a typical feature of the weak localization as in ordinary disordered metals. Here, we present a conductivity formula for massless and massive Dirac fermions as a function of magnetic field and temperature, by taking into account the electron-electron interaction and quantum interference simultaneously. The formula reconciles the dilemma by explicitly clarifying that the temperature dependence of the conductivity is dominated by the interaction, while the magnetoconductivity is mainly contributed by the quantum interference. The theory paves the road to quantitatively study the transport in topological insulators, and can be extended to other two-dimensional Dirac-like systems, such as graphene, transition metal dichalcogenides, and silicene.

Introduction.—The experiments on the electronic transport in topological insulators [1–4] present a dilemma. A negative cusp in weak-field magnetoconductivity [solid curve in Fig. 1(a)] was measured in various topological insulators [5–9] and commonly regarded as a signature of the weak antilocalization (WAL) [10] of the surface states. For topological surface states, WAL stems from a $\pi$ Berry phase [11] acquired by electrons after circling around the spin-momentum-locked Fermi surface [12]. When lowering temperature, the $\pi$ Berry phase induces a destructive interference between backscattered electrons, then enhances the conductivity [dashed line in Fig. 1(b)] [13,14]. A magnetic field can destroy the interference and the conductivity enhancement, showing the negative magnetoconductivity cusp as the signature of WAL. The dilemma is, opposite to the enhancement expected from WAL, the conductivity was observed to decrease logarithmically with decreasing temperature [15–20] [solid line in Fig. 1(b)], indicating a behavior of the weak localization (WL) [10]. However, WL should exhibit a positive magnetoconductivity [dashed curve in Fig. 1(a)]. It has been suggested [15,16] that the electron-electron interaction could be the possible mechanism [21], but has not been fully appreciated [22], mainly because the quantitative comparison so far [15–20] was using the theories established for the conventional electrons [23–25]. While in topological insulators, it is well accepted that the topological surface electrons are massless Dirac fermions [1–3,22,26,27] and the bulk electrons have to be described by a massive Dirac model to account for the topological properties properly [4,28].

In this Letter, we resolve the dilemma by calculating the corrections to the conductivity from both the electron-electron interaction and quantum interference for disordered massless and massive Dirac fermions in two dimensions. We derive a formula of conductivity as a function of temperature and magnetic field, using the diagrammatic technique (Fig. 2). The formula reveals explicitly that in topological insulators such as Bi$_2$Se$_3$ and Bi$_2$Te$_3$ (i) the interaction always suppresses the conductivity with a strength stronger than the enhancement from the quantum interference of the surface states, leading to the WL-like temperature dependence in the conductivity, (ii) both the interaction and quantum interference of the surface electrons produce negative magnetoconductivity, but the portion from the interaction is at least 1 order smaller, so the signature of WAL in magnetoconductivity mainly comes from the quantum interference, (iii) both phenomena are attributed to a small screening factor of interaction resulting from a large permittivity in these materials and (iv) the results agree well with the experiments [15–20] at comparable temperatures (0.1 to 10 K) and magnetic fields (0 to 5 T). We quantitatively compare the theory with a set of experiments.
for the slope of the conductivity vs temperature, by using the Dirac mass as the fitting parameter for both the gapless surface and gapped bulk states. The theory is developed for massless and massive Dirac fermions; hence, it paves the road towards the quantitative study of the electronic transport in topological insulators, and can be extended to other Dirac-like systems, such as graphene, transition metal dichalcogenides \[29–32\], and silicene \[33–35\] after intervalley scattering and interaction are taken into account.

**Model.**—We start with the two-dimensional (2D) Dirac model

\[
H = \begin{pmatrix} \Delta/2 & i\gamma(k_x - ik_y) \\ -i\gamma(k_x + ik_y) & -\Delta/2 \end{pmatrix},
\]

where \( \gamma = v/\hbar \), \( v \) is the effective velocity, \( \hbar \) is the reduced Planck constant, and \( (k_x, k_y) \) is the wave vector. \( H \) describes two energy bands with strong spin-orbit coupling, separated by a gap opened by the Dirac mass \( \Delta \) [see Fig. 3(a)]. We assume that the Fermi energy \( E_F \) crosses the higher band. The model has two limits: one is the massless limit with \( \Delta/2E_F = 0 \), e.g., for the surface states in topological insulators; the other is the large-mass limit, which has a finite gap and the Fermi level at the band bottom such that \( \Delta/2E_F \to 1 \), and is applicable to the bulk electrons in topological insulator thin films near the band edges \[38,39\].

**Conductivity formula.**—We reexamine the finite-temperature conductivity for 2D Dirac fermions in magnetic field. Disorder scattering and electron-electron interaction are considered when calculating the conductivity (see Sec. S2 of Ref. \[37\] for details). With the help of the diagram techniques (see Fig. 2), we find that the temperature and magnetic field dependent conductivity can be written into two parts \( \sigma = \sigma^{ei} + \sigma^{ee} \): (i) the conductivity correction from the quantum interference,

\[
\sigma^{ei} = e^2/\pi\hbar \sum_{i=0,1} \alpha_i |\psi(1/2 + \ell_i^2/\ell_B^2)| - \ln(\ell_B^2/\ell^2),
\]

and (ii) the conductivity correction from the electron-electron interaction

\[
\sigma^{ee} = \rho_n/(C_138),
\]
where \(e^2/h\) is the conductance quantum, \(\psi\) is the digamma function, and \(\ell'\) is the mean free path. We find for the Dirac model, the screening factor of the total slope is also positive in the large-mass limit. The change of the conductivity \(\delta\sigma\) becomes completely suppressed [Fig. 4(d) and 4(e)] due to a large relative permittivity \(\varepsilon_r\) (typically \(\sim 100\)). In the large-mass limit \(F\) approaches 1, meanwhile the \(F\)-independent part of \(\delta\sigma\) becomes completely suppressed [Fig. 4(a)]. As a result, \(\delta\sigma\) is at least one order smaller than \(\delta\sigma^{0}\). Above, we show that even in the presence of interaction the negative magnetoconductivity observed in topological insulators is mainly contributed by the quantum interference.

Slope vs magnetic field.—The \(\ln T\) and magnetic field dependence of the conductivity is characterized by the slope \(\kappa\) as a function of \(B\), which is summarized in Fig. 5 for

\[ \sigma^{ee} = \frac{e^2}{\pi h} (1 - \eta_M F) \ln \frac{2\ell'^2}{\ell'^2 + \frac{e^2}{\pi h} \eta F \psi\left(\frac{1}{2} + \frac{\ell'^2}{\ell''_{\phi}}\right)}, \]  

(3)

where \(e^2/h\) is the conductance quantum, \(\psi\) is the digamma function, and \(\ell'\) is the mean free path.
The screening factor of interaction $F$ as a function of $\Delta/2E_F$ for different $\varepsilon_r$, the relative permittivity. (c) $F$ as a function of $\varepsilon_r$ for different $\Delta/2E_F$. The parameters are the same as those in Fig. 3 except that $F = 0.5$ in (c) for a better demonstration. $\varepsilon_r$ is about 100 in Bi$_2$Se$_3$ and Bi$_2$Te$_3$ [40].

Fitting experiments.—As an application of our theory, we fit the experimental slopes in Fig. 5. Take a 80 nm Cu-doped Bi$_2$Se$_3$ thin film [18], for example. Its sheet carrier density is about $6.7 \times 10^{12}$/cm$^2$. At this carrier density, the Fermi energy is estimated to cross not only the surface band but also the very bottom of the bulk conduction band [51], where $\Delta/2E_F \rightarrow 1$. The experiment was originally fitted with the formula $\kappa = 1 - (3/4)\bar{F}$ for the conventional electrons [25]. However, $\kappa = 1.67$ at high $B$ yielded a negative $\bar{F}$, which by definition should be positive, and $\kappa$ should range between 0 and 1. Moreover, the slope change was $\delta\kappa = 0.3$, differing from the theoretical value 0.5 for a gapless Dirac cone of the surface states [inset of Fig. 3(b)].

The inconsistencies imply the possibility of two channels, channel 1 with $\Delta/2E_F = 0$ and $\delta\kappa_1 \sim 0.5$ for the gapless surface states and channel 2 with a large $\Delta/2E_F$ and $\delta\kappa_2 \sim -0.2$ for the gapped band-edge bulk states, then $\delta\kappa = \delta\kappa_1 + \delta\kappa_2 = 0.3$ and $\kappa = \kappa_1 + \kappa_2 \in (1.2)$. With $\Delta/2E_F$ as a fitting parameter, the experimental slope as a function of magnetic field can be fitted by two channels very well (• in Fig. 5). Furthermore, Fig. 5 shows that other experiments also fall in the vicinity of the $\kappa - B$ curves calculated from the Dirac model. We find that thicker films (80 nm in [18] and 65 nm in [19]) and thinner films (10 nm in [17], 4 nm in [20]) are better fitted by two channels and one massless channel, respectively, implying the dominance of the surface states in thinner films.

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