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Monitoring of Multiple Loads in Wireless Power Transfer Systems Without Direct Output Feedback

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Abstract—In this paper, a computational method that uses only the input voltage and current to identify the impedances of multiple loads in wireless power transfer systems without any direct load measurements is proposed. The method has been practically realized in a wireless power domino-resonator system comprising 8 resonators. A good degree of accuracy has been achieved in the practical verification. Although the method is demonstrated in a relatively complex system, the principle applies to any wireless power transfer system with 3 or more coil-resonators. Without the requirements for direct load measurements, control circuitry of a wireless power transfer system with multiple loads can avoid the needs for using wireless communication system for feedback purposes.

I. INTRODUCTION

Nicola Tesla investigated the fundamental principles of wireless power transfer based on the magnetic resonance and near-field coupling a century ago [1]. With the availability of power electronics as an enabling technology, wireless power transfer has re-emerged as a hot research topic in recent years, covering biomedical implant devices [2-9], inductive power transfer [10-12] and wireless charging techniques [13-17]. Due to the wireless nature of the system, wireless communication system has been proposed to feedback the load measurements for closed-loop control purpose [18]. It has however been pointed out that, for a system with a single load, the output load information can be derived from the measurements of input voltage and current only [19].

In this project, the computational approach is extended to cover multiple loads [20], [21]. The uniqueness of the proposed method includes the ability to manage nonlinear and time-varying input information to determine the multiple output load conditions without using any direct output load feedback. This concept would be powerful and efficient especially for large systems with more magnetic resonators and longer transmission distance. With the dynamic monitoring of the multiple load conditions, various control schemes can be applied to meet the power flow control requirements. This computational approach is explained and practically verified with measurements obtained from an 8-coil wireless power transfer system. The proposal can greatly reduce the cost and complexity for input power control by eliminating the requirements of any output feedback circuitry, which may not be allowed or available in some applications.

II. MATHEMATICAL MODEL

Fig. 1 illustrates a general wireless power domino-resonator system consisting of \( n \) coils, where an AC sinusoidal voltage source with a root-mean-square amplitude \( V_S \) and an angular frequency \( \omega \) is applied to the transmitter coil-1 on the input side. Without losing generality one load \( Z_{l1} \) with equivalent impedance \( R_{l1} + jX_{l1} \) is connected to coil-\((n-1)\) and the other load \( Z_{l2} \) with equivalent impedance \( R_{l2} + jX_{l2} \) is connected to coil-\( n \) on the output sides. In principle, the load

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can be connected to any coil in the system except the transmitter coil-1. Therefore, coil-1 is the transmitter coil, coil-(n - 1) and coil-n are the receivers, and the other coils from 2 to n - 2 are the relay coils. A capacitor is connected in series with each coil to form a resonator so as to enhance the power transfer capability. The capacitance value of each 

\[
\begin{bmatrix}
Z_1 & j\omega M_{12} & \cdots & j\omega M_{1(n-1)} & j\omega M_{1n} \\
0 & Z_2 & \cdots & j\omega M_{2(n-1)} & j\omega M_{2n} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & j\omega M_{(n-1)1} & j\omega M_{(n-1)2} & \cdots & Z_{n-1} + Z_{L1} \\
0 & j\omega M_{n1} & j\omega M_{n2} & \cdots & j\omega M_{(n-1)n} + Z_n + Z_{L2}
\end{bmatrix} = 
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_{n-1} \\
I_n
\end{bmatrix}
\]  

where \( Z_i = R_i + j\left(\omega L_{ij} - \frac{1}{\omega C_i}\right) \) is the total impedance of coil-i, \( R_i \) is the equivalent resistance of coil-i, \( L_i \) is the self-inductance of coil-i, \( C_i \) is the capacitance of the externally connected capacitor on coil-i, \( I_i \) is the current phasor flowing along coil-i, \( M_{ij} \) is the mutual inductance between coil-i and coil-j \((M_{ji} = M_{ij})\), 1 \( \leq i \leq n, 1 \leq j \leq n \).

Before solving the equations, some assumptions are made as follows:

- The coils are magnetically coupled to each other and such coupling does not interact with the loads, which means that the load, including both the resistance and the reactance, is independent of the coils.
- All the parameters of the coils can be measured or calculated and are constant. Since the positions of coils are fixed, the mutual inductance of every pair of coils is also constant and can be measured accurately or calculated mathematically.
- The input voltage phasor \( V_S \) and current phasor \( I_1 \) in the coil-1 can be measured in real time.
- In the following analysis, inductive-resistive loads are assumed, i.e. \( Z_L = R_L + j\omega L \). But in principle, the method applies to any combination of inductive, resistive, capacitive elements.

III. COMPUTATIONAL APPROACH FOR DOUBLE LOAD APPLICATION

A. General Steps to Solve the System Equations

Now (1) can be rearranged in the following steps:

\[
\begin{bmatrix}
V_S - Z_{L1} I_1 \\
-j\omega M_{12} I_1 \\
\vdots \\
-j\omega M_{(n-1)1} I_1 \\
-j\omega M_{n1} I_1
\end{bmatrix} = 
\begin{bmatrix}
 j\omega M_{12} & j\omega M_{13} & \cdots & j\omega M_{1(n-1)} & j\omega M_{1n} \\
 Z_2 & j\omega M_{23} & \cdots & j\omega M_{2(n-1)} & j\omega M_{2n} \\
 \vdots & \vdots & \ddots & \vdots & \vdots \\
 j\omega M_{(n-1)2} & j\omega M_{(n-1)3} & \cdots & j\omega M_{(n-1)(n-1)} & j\omega M_{(n-1)n} + Z_{L1} \\
 j\omega M_{n2} & j\omega M_{n3} & \cdots & Z_n + Z_{L2}
\end{bmatrix} \begin{bmatrix}
I_1 \\
\vdots \\
I_{n-1} \\
I_n
\end{bmatrix}
\]

The unknowns in above equations are \( I_1, I_2, \ldots, I_{n-1}, Z_{L1} \) and \( Z_{L2} \). Transfer all the known terms, which are the terms with \( I_1 \), from the right hand sides of the equations to the left hand sides and leave only unknown terms on the right hand side as shown in (3).

\[
\begin{bmatrix}
V_S - Z_{L1} I_1 = j\omega M_{12} I_2 + \cdots + j\omega M_{1(n-1)} I_{n-1} + j\omega M_{1n} I_n \\
-j\omega M_{12} I_1 = Z_2 I_2 + \cdots + j\omega M_{2(n-1)} I_{n-1} + j\omega M_{2n} I_n \\
\vdots \\
-j\omega M_{(n-1)2} I_1 = j\omega M_{(n-1)3} I_2 + \cdots + j\omega M_{(n-1)(n-1)} I_{n-1} + Z_{L1} I_{n-1} \\
-j\omega M_{n2} I_1 = j\omega M_{n3} I_2 + \cdots + Z_n I_n + Z_{L2} I_n
\end{bmatrix}
\]

It is obvious that (3) are nonlinear due to the terms of products of two unknowns \( Z_{L1} I_1 \) and \( Z_{L2} I_1 \). If we regard the load terms \( Z_{L1} I_{n-1} \) and \( Z_{L2} I_{n-1} \) as new unknowns, the original n+1 unknowns \( I_2, I_3, \ldots, I_n, Z_{L1} \) and \( Z_{L2} \) will become \( I_2, I_3, \ldots, I_n, Z_{L1} I_{n-1} \) and \( Z_{L2} I_{n-1} \). Then (3) can be transformed into linear form as shown in (4). However the solutions to (4) are not unique, because there are \( n-1 \) equations with \( n-1 \) unknowns. Now assume \( Z_{L1} I_{n-1} \) is also known. By putting it on the left hand side of the equation in the form of (5), then the unknowns of (5) can be placed on the left hand side of (6).
where $P$ is the inverse matrix of the coefficient matrix in (5), i.e.

$$P = \begin{bmatrix} j\omega M_{12} & j\omega M_{13} & \cdots & j\omega M_{1n} \\ Z_2 & j\omega M_{23} & \cdots & j\omega M_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ j\omega M_{2(n-1)} & j\omega M_{3(n-1)} & \cdots & j\omega M_{n(n-1)} \\ j\omega M_{2n} & j\omega M_{2n} & \cdots & Z_n \end{bmatrix}^{-1}$$

Now extract the last 3 equations of the matrix equation (6) as:

$$I_{n-1} = a + bZ_{L1}I_{n-1} \tag{8}$$

$$I_n = c + dZ_{L1}I_{n-1} \tag{9}$$

$$Z_{L2}I_n = e + fZ_{L1}I_{n-1} \tag{10}$$

where,

$$a = P_{n-2}V$$
$$b = -P_{n-2(n-1)}$$
$$c = P_{n-1}V$$
$$d = -P_{n-2(n-1)}$$
$$e = P_nV$$
$$f = -P_{n(n-1)}$$

Substituting (13) into (9),

$$I_n = c + \frac{adZ_{L1}}{1-bZ_{L1}} \tag{14}$$

Substituting (13) and (14) into (10), the $Z_{L1}Z_{L2}$ relationship is:

$$AZ_{L1}Z_{L2} + BZ_{L1} + CZ_{L2} + D = 0 \tag{15}$$

where,

$$A = ad-bc = \alpha_a + j\beta_a$$
$$B = be-af = \alpha_b + j\beta_b$$
$$C = c = \alpha_c + j\beta_c$$
$$D = -e = \alpha_d + j\beta_d$$

(16)

As the load model is assumed to be

$$Z_{L1} = R_{L1} + j\omega L_{L1}$$

$$Z_{L2} = R_{L2} + j\omega L_{L2}$$

one can express (15) as (17) and then separate the real and imaginary parts respectively as (18) and (19).

$$\begin{align*}
(\alpha_a + j\beta_a)(R_{L1} + j\omega L_{L1})(R_{L2} + j\omega L_{L2}) + (\alpha_b + j\beta_b)(R_{L1} + j\omega L_{L1}) + (\alpha_c + j\beta_c)(R_{L2} + j\omega L_{L2}) + \alpha_d + j\beta_d &= 0 \tag{17} \\
\alpha_aR_{L1}R_{L2} - \alpha_a\omega L_{L1}L_{L2} - \beta_a\omega L_{L1}L_{L2} + \beta_a\omega L_{L1}L_{L2} + \alpha_bR_{L1} - \beta_b\omega L_{L1}L_{L2} + \beta_b\omega L_{L1}L_{L2} - \alpha_d &= 0 \tag{18} \\
\beta_aR_{L1}R_{L2} - \beta_a\omega L_{L1}L_{L2} + \alpha_a\omega L_{L1}L_{L2} + \beta_a\omega L_{L1}L_{L2} + \beta_bR_{L1} + \alpha_b\omega L_{L1}L_{L2} + \beta_b\omega L_{L1}L_{L2} - \beta_d &= 0 \tag{19}
\end{align*}$$
where (18) is the real part equation of (17) and (19) is the imaginary part equation of (17).

There are seven equivalent unknowns \( R_{L1}L_{L2}, L_{L1}L_{L2}, R_{L1}, L_{L1}, R_{L2}, \) and \( L_{L2} \) in the equations above. To solve them together we need at least four different input frequencies to obtain eight real and imaginary equations in the general form of (18) and (19).

Similarly if the load is assumed to be capacitive-resistive, i.e.

\[
\frac{1}{\omega^2} \frac{1}{C_{L1}C_{L2}} + \frac{1}{\omega} \left( \frac{R_{L1}}{C_{L1}} \right) + \frac{R_{L2}}{C_{L1}} + \alpha_g R_{L1} + \alpha_c R_{L2} + \beta_C \frac{1}{\omega C_{L2}} = -\alpha_D
\]

(20)

\[
\frac{1}{\omega^2} \frac{1}{C_{L1}C_{L2}} - \frac{1}{\omega} \left( \frac{R_{L1}}{C_{L2}} \right) + \frac{R_{L2}}{C_{L2}} + \beta_g R_{L1} - \alpha_g \frac{1}{\omega C_{L1}} + \beta_c R_{L2} - \alpha_c \frac{1}{\omega C_{L1}} = -\beta_D
\]

(21)

B. Simplification Method for Resistive Load Conditions

If the loads are purely resistive, i.e. \( Z_{L1} = R_{L1}, Z_{L2} = R_{L2}, \) (18) and (19) can be simplified into

\[
\alpha_g R_{L1}R_{L2} - \frac{1}{\omega^2} \frac{1}{C_{L1}C_{L2}} + \beta_g \left( \frac{R_{L1}}{C_{L1}} \right) + \frac{R_{L2}}{C_{L1}} + \alpha_g R_{L1} + \beta_g \frac{1}{\omega C_{L1}} + \beta_c R_{L2} + \beta_C \frac{1}{\omega C_{L2}} = -\alpha_D
\]

(22)

\[
\beta_g R_{L1}R_{L2} - \frac{1}{\omega^2} \frac{1}{C_{L1}C_{L2}} - \beta_g \left( \frac{R_{L1}}{C_{L2}} \right) + \frac{R_{L2}}{C_{L2}} + \alpha_g \frac{1}{\omega C_{L1}} + \beta_c R_{L2} - \alpha_c \frac{1}{\omega C_{L1}} = -\beta_D
\]

(23)

There are three equivalent unknowns \( R_{L1}, R_{L2}, R_{L1}, R_{L2} \). So we need two different frequencies at most to get four real and imaginary equations in the general form of (22) and (23) so as to find the three unknowns together.

The equivalent unknowns in (22) and (23) are not independent, since the term \( R_{L1}R_{L2} \) is the product of the other two unknowns. Therefore only \( R_{L1} \) and \( R_{L2} \) are independent unknowns. After some linear transformation steps the original (22) and (23) can be combined into one linear equation where the term \( R_{L1}R_{L2} \) is cancelled, i.e.

\[
\alpha_g \beta_a - \alpha_g \beta_a = 0
\]

\[
\alpha_g \beta_a - \alpha_g \beta_a + (\alpha_c \beta_a - \alpha_g \beta_c)R_{L2} = \alpha_g \beta_b - \alpha_g \beta_a
\]

(24)

In (24) there are two independent unknowns \( R_{L1} \) and \( R_{L2} \) with coefficients related to the measurements of \( V_S \) and \( I_I \) under one specific frequency. If we input another different frequency, a new set of coefficients will be obtained in the similar form of (24). Finally, only two sets of \( V_S \) and \( I_I \) measurements under different frequencies are sufficient theoretically to find the two load resistance values.

In most applications the loads in a practical wireless power transfer system can be considered purely resistive which are preceded by AC to DC rectifier stages, such as battery chargers. Therefore (24) is practically realistic.

C. A Simple Case: Three-Coil System

The proposed method can be illustrated with a three-coil wireless power transfer system. For such a system, the inverse matrix of \( P \) in the form of (7) can be expressed as (25):

\[
P^{-1} = \begin{bmatrix}
0 & \frac{M_{13}}{M_{12}} & \frac{M_{13}}{M_{12}} \\
\frac{M_{23}}{M_{12}} & \frac{1}{j\omega M_{23}} - \frac{M_{13}Z_{L2}}{M_{12}} & \frac{1}{j\omega M_{23}} - \frac{M_{13}Z_{L2}}{M_{12}} \\
\frac{Z_{L1}Z_{L2}}{j\omega M_{23}} - \frac{(j\omega M_{23})^2}{M_{12}} & \frac{Z_{L1}Z_{L2}}{j\omega M_{23}} - \frac{(j\omega M_{23})^2}{M_{12}} & 1
\end{bmatrix}
\]

(25)
The results of (25) are essential to the proposed calculation procedure. For example, the intermediate variables \( a, b, c, d, e, f \) in (11) and \( A, B, C, D \) in (16) are all obtained from with help of (25). Finally the two load impedances in (15) can be found.

**D. Exceptional Case**

The aforementioned computational method is about how to estimate two load impedances in a wireless power transfer system only from the measured voltage and current at the input terminals. It should be noted that this method cannot work under the conditions of a symmetric geometrical structure and identical load circuit parameters. In other words, the proposed method does not apply to a wireless power transfer system with two identical and symmetric loads unless some more sensors for monitoring are added to other coils. It is easy to explain in a perceptual sense. Take a three-coil system, depicted in Fig. 2, as the example, where coil-1 is the terminal for the AC voltage input, and coil-2 and coil-3 are connected to coil-2 and coil-3 respectively, a pair of input terminals. It shows that the proposed method cannot differentiate load coil-2, there will not be any change in the input electrical quantities. In this special case, we cannot estimate the load impedances in (15) can be solved mathematically.

Substituting \( M_{12}=M_{13} \) and \( Z_2=Z_3 \) into (25), we can get the following results.

\[
P^{+} = \begin{bmatrix}
\frac{M_{23}}{M_{12}(j\omega M_{23}-Z_2)} & 0 & 0 \\
\frac{-1}{j\omega M_{23}-Z_2} & 1 & 0 \\
\frac{-Z_2 + j\omega M_{23}}{j\omega M_{12}} & \frac{1}{j\omega M_{23}-Z_2} & 1
\end{bmatrix}
\]

Figure 2. 3-coil symmetric wireless power transfer system with double loads.

In the final results, \( B=C \), so the equivalent unknowns in (15) are \( Z_{21}, Z_{22} \) and \( Z_{42}, Z_{42} \). Therefore we cannot find a unique pair of \( Z_{21} \) and \( Z_{22} \), which are the practical load impedances.

**IV. EXPERIMENTAL VERIFICATION**

To verify the proposed ideas for identifying the load impedances, an 8-coil wireless power domino-resonator system is adopted in an experimental study. Each coil has the same wiring manufacture, i.e. \( R_1 \approx s=0.9998262 \, \Omega, L_1 \approx 82.03468 \, \mu\text{H} \). The system parameter details are provided in Tables I and II. To reduce the error we sweep the input frequency from 480 to 570 kHz and take the average of the estimation values as the final result. The resistive bank consists of many resistors with nominal resistance values. Note that the actual resistance values are not precisely the identical to their nominal values as they vary with temperature.

The practical (nominal) values of the resistive loads under different settings are listed and compared with the computed results in Table III, where the reference values are for the standard resistors connected practically, the simulated values are from the simulated input electrical quantities, and the estimated values are averaged from the practically measured input electrical quantities. It shows that the proposed method can achieve a good degree of accuracy.

The simulated results indicate that the proposed method can estimate the load resistances with good accuracy. The estimated values calculated from the practically measured input voltages and currents have some errors due to the measurement and parameter errors. But it shows that the proposed method can achieve a good degree of accuracy by taking the average.
In this paper, a new computational approach is presented to use only the input voltage and input current to determine the impedances of two different loads in a wireless power transfer system. Such principles have been successfully demonstrated in a relatively complex 8-coil wireless power transfer system. However, the same methodology can be applied to relatively simple structures such as 3-coil and 4-coil systems that are recently investigated by many researchers. The proposal is particularly suitable for applications in which feedback circuitry may not be available in the receiver coils.

V. CONCLUSION

In this paper, a new computational approach is presented to use only the input voltage and input current to determine the impedances of two different loads in a wireless power transfer system. Such principles have been successfully demonstrated in a relatively complex 8-coil wireless power transfer system. However, the same methodology can be applied to relatively simple structures such as 3-coil and 4-coil systems that are recently investigated by many researchers. The proposal is particularly suitable for applications in which feedback circuitry may not be available in the receiver coils.

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