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Analysis of Tooth-Tip Flux Leakage in Surface-Mounted
Permanent Magnet Linear Vernier Machines

Wenlong Li, K. T. Chau, Chunhua Liu, Shuang Gao, and Diyun Wu
Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong, China

The permanent magnet linear vernier (PMLV) machine is becoming attractive for low-speed and direct-drive applications. Due to its vernier structure and multipole configuration, the flux leakage especially the tooth-tip flux leakage is severe. In order to analyze its leakage flux, the magnetic equivalent network of the PMLV machine is applied for analytical modeling. During modeling, the fringing effect in the air-gap is taken into consideration. The key for the flux leakage calculation is to calculate the permeances at different tooth positions. Finally, in order to evaluate the proposed analytical model of the PMLV machine, the finite element method is employed for numerical calculation. The comparison between the analytical and numerical results verifies that the proposed analytical modeling approach is valid and accurate.

Index Terms—Analytical modeling, flux leakage, linear machine, permanent magnet machine, vernier machine.

I. INTRODUCTION

P ERMANENT MAGNET (PM) vernier machines are becoming more and more popular in recent years, since they are capable of developing high torque [1]–[3]. Particularly, they are very promising for linear low-speed and direct-drive applications [4]. By using a set of stator teeth, they can realize the so-called “magnetic gearing effect” and exhibit high torque density for low-speed operation without using any mechanical intermediate. However, due to the toothed-pole stator and multipole PMs on the rotor or mover, they suffer from severe flux leakage. Especially, the flux leakage via the tooth tips of the stator predominates the total leakages. This phenomenon decreases the utilization of PM materials and also results in a low power factor of the machine. Consequently, the machine performance is deteriorated.

For the conventional PM machines, the tooth-tip flux leakage is not so severe and does not attract many attentions [5]–[7]. For instance, the analytical modeling of tooth-tip flux leakage is generally established without considering the fringing effect; or the model is not position-dependent which can not be applied to the PM vernier machines.

The purpose of this paper is to present an analytical modeling approach for analyzing the tooth-tip flux leakage of a surface-mounted PM linear vernier (SPMLV) machine. In essence, the analytical model is deduced by the determination of the permeances corresponding to the tooth-tip leakage flux and the air-gap flux.

II. ANALYSIS

In order to analytically express the leakage flux of the SPMLV machine, the magnetic equivalent network (MEN) is adopted. For simplifying the mathematical modeling, some assumptions have been made: the saturation of the iron core is ignored; the longitudinal length of the machine is assumed to be infinite so that the longitudinal end-effect is negligible; the machine stack length is equal to or greater than the machine thickness so that the transverse end-effect, namely the 3-D fringing flux, is also negligible; and the relative recoil permeability is assumed to be unity.

Fig. 1 shows the topology of the SPMLV machine, consisting of a vernier structure stator and a surface-mounted PM mover, where the possible flux paths are identified. There are several leakage fluxes for the SPMLV machine, namely the tooth-tip leakage flux, the air-gap leakage flux and the magnet-end leakage flux.

The MEN of a half magnet pole-pair to a single stator tooth is shown in Fig. 2, where $\Phi_s$ is the flux source of one magnet pole, $\Phi_g$ is the air-gap flux of the air-gap cross-sectional area facing to one magnet pole, $R_{m,s}$ and $R_{g,s}$ are the reluctances corresponding to $\Phi_s$ and $\Phi_g$ respectively, $R_{ receptors}$ is the reluctance corresponding to the tooth-tip leakage flux, $R_{m,m}$ is the reluctance corresponding to the air-gap leakage flux, $R_{m,l}$ is the reluctance corresponding to the magnet-end leakage flux, $R_{r}$ is the reluctance of the stator back-iron and $R_{r}$ is the reluctance of the rotor back-iron. When the saturation is negligible under no-load condition, $R_{r}$ and $R_{r}$ can be ignored. Therefore, the MEN can be further simplified as shown in Fig. 3. For the surface-mounted and multipole PMs and open-slot stator, the magnet-end leakage flux reluctance $R_{m,l}$ and the air-gap leakage flux reluctance $R_{m,m}$ also can be ignored.

Based on the simplified MEN, the flux leaving one magnet $\Phi_m$ can be expressed as

$$\Phi_m = \frac{R_{m,o}R_{t,p} + 4R_{m,e}R_g}{R_{m,o}R_{t,p} + 4R_{m,e}R_g + R_{t,p}R_g} \Phi_r$$

(1)
where \( R_{\text{m}} = h_m/\mu_0 w_m L_{ef}, \) \( \Phi_r = B_r w_m L_{ef}, \) \( h_m \) is the height of the magnet, \( \mu_0 \) is the vacuum permeability, \( w_m \) is the magnet width, and \( L_{ef} \) is the machine stack length. The associated tooth-tip leakage flux \( \Phi_{Lt} \) can also be determined by:

\[
\Phi_{Lt} = \frac{2R_g}{R_{tp} + 4R_g} \Phi_m
\]

(2)

According to (2), the tooth-tip leakage flux due to one half of a PM pole-pair can be estimated. However, for the SPMLV machine, the tooth-tip leakage flux of one magnet pole-pair varies according to its position versus the magnet pole-pair. It means that the reluctances \( R_{tp} \) and \( R_g \) are position-dependent variables. Due to the salient feature of the stator, the flux patterns can be more complicated. Therefore, in order to accurately model the leakage flux, the fringing effect in the air-gap should be taken into consideration.

In PM vernier machines, the stator tooth number \( N_s \), the armature pole-pair number \( N_p \), and the PM pole-pair number in the rotor \( P_f \) should obey the following equation which enables the magnetic gearing effect:

\[
P_f = N_s \pm P_a
\]

(3)

where the minus case is generally selected to achieve higher torque or force density [8]. The magnetic gear ratio is defined as \( G_r = P_f/P_a \). As a linear machine, the stator tooth pitch \( \tau_t \) and the PM pole-pitch \( \tau_m \) should satisfy the following relationship:

\[
N_s \tau_t = 2P_f \tau_m
\]

(4)

where the stator tooth pitch \( \tau_t \) equals the sum of the tooth width \( b_t \) and slot width \( b_s \), and the PM pole-pitch equals the sum of the PM width \( w_m \) and its adjacent width \( w_f \) as shown in Fig. 1. Since \( N_s \) is larger than \( P_f \), the relationship of \( \tau_t \) and \( \tau_m \) satisfies

\[
\begin{align*}
\tau_m &< \tau_t < 2\tau_m, \quad \text{for } G_r > 1 \\
\tau_m &\geq \tau_t, \quad \text{for } G_r \leq 1
\end{align*}
\]

(5)

Taking into account the fringing effect, there are three cases of the relative position between the stator tooth and the magnet: namely the exact overlapping, partial overlapping, and nonoverlapping as shown in Fig. 4 [9], [10].

For the exact overlapping, the fringing effect is ignored. Therefore, the permeance corresponding to the tooth-tip leakage flux is expressed as

\[
P_{\text{ex}} = \frac{\mu_0 x L_{ef}}{g_0}
\]

(6)

where \( g_0 \) is the air-gap length.

For the partial overlapping, the flux path can be approximately divided into the nonfringing part “\( x \)” and the fringing part “\( x_2 \)”. The total permeance is the sum of the permeances of these two parts as given by

\[
P_{\text{pp}} = P_{x1} + P_{x2} = \frac{\mu_0 x_1 L_{ef}}{g_0} + \mu_\ell L_{ef} \int_0^{x_2} \frac{1}{g_0 + \pi x/2} dx
\]

\[
= \frac{\mu_\ell x_1 L_{ef}}{g_0} + \frac{2\mu_0 L_{ef}}{\pi} \ln \left( 1 + \frac{\pi x_2}{2g_0} \right).
\]

(7)

For the nonoverlapping, the permeance can also be obtained in a similar way as given by

\[
P_{\text{nn}} = \frac{2\mu_0 L_{ef}}{\pi} \ln \left( 1 + \frac{\pi x}{2g_0 + \pi x_0} \right).
\]

(8)

In order to calculate the tooth-tip leakage flux for a half of magnet pole-pair to one stator tooth, the values of \( R_{tp} \) and \( R_g \) should be calculated first. Because of its variable nature, the two extreme situations (namely the positions for the minimum and maximum tooth-tip leakage fluxes) are investigated. As shown in Fig. 5(a), when the left edge of the magnet “2” aligns with the slot center line, the flux lines from the magnet “2” unlikely go through the tooth “A” because of the ferromagnetic feature of the stator tooth. The value of \( R_{tp} \) is assumed to be infinite. Therefore, the tooth-tip leakage flux is nearly zero. The MEN for this situation is shown in Fig. 5(b).

As the mover travels towards left, the tooth-tip leakage flux begins to increase along with the motion. When the quadrature-axis aligns with the tooth center line, the tooth-tip leakage flux reaches its maximum value. The value of \( R_g \) tends to be infinite.
At this situation, the total flux leaving the magnet is the tooth-tip leakage flux as depicted in Fig. 6.

For other situations, the tooth-tip leakage flux is in the range of the minimum and maximum values. Therefore, the permeance corresponding to the tooth-tip leakage flux and the air-gap flux can be expressed as (9) and (10), shown at the bottom of the page.

Based on (1), (2), (9), and (10), the tooth-tip leakage flux due to a half of the PM pole-pair can be calculated. By adding up every portion of the tooth-tip leakage flux of the magnet pole-pairs to the same stator tooth, the total tooth-tip leakage for a single tooth can be obtained. It should be emphasized that for the PM vernier machines, $G_v$ usually is far greater than unity, while the magnet width and the tooth width usually are not less than a half of the magnet pole-pitch and a half of tooth-pitch, respectively [1].

III. VERIFICATIONS

Based on the above analytical modeling, the tooth-tip leakage flux of the SPMLV machine can be calculated directly. Due to the aforementioned assumptions, the 2-D finite element method (FEM) is employed to compute the leakage flux numerically.

Fig. 7 shows the flux calculation model for the 2-D FEM. Fig. 7(a) and (b) describe the positions for the maximum and minimum tooth-tip leakage fluxes, respectively, while Fig. 7(c) and (d) describe the positions in between the above two circumstances. The points 1, 2, 3, and 4 are selected for calculating the anticipated fluxes. Meanwhile, the points 3 and 4 are the positions along the magnet quadrature-axis at the tooth bottom edge and the border of leakage flux, respectively. The flux leaving the magnet can be calculated by

$$\Phi_{m} = A_1 - A_2 |L_{ef}|$$  \hspace{1cm} (11)

where $A_1$ and $A_2$ are the magnetic vector potentials at point 1 and 2, respectively.

The total tooth-tip flux leakage can be calculated by

$$\Phi_{L, f} = |A_3 - A_4| L_{ef}$$  \hspace{1cm} (12)

where $A_3$ and $A_4$ are the magnetic vector potentials at point 3 and 4, respectively. The positions of point 3 and 4 are selected based on practical circumstances which may be influenced by one or two PM pole-pairs.

For verification, practical machine parameters are selected as listed in Table I, which correspond to the configuration as shown in Fig. 1.
Based on the aforementioned assumption, the longitudinal length of the machine is infinite. Therefore, the tooth-tip flux leakage is periodic. According to (4), it can be deduced that this period is the greatest common divisor (GCD) of \( N_p \) and \( 2P_f \), namely GCD \( (N_p, 2P_f) \). For this case, the period becomes 2. So, only the tooth-tip leakage fluxes of 5 teeth are calculated.

Table II and Table III compare the analytical and numerical results of the tooth-tip leakage fluxes of 5 adjacent teeth with the first tooth locating at the minimum and maximum positions, respectively. It can be observed that the maximum error is within 5% which confirms the accuracy of the proposed analytical approach. It can also be observed that the tooth-tip leakage flux varies with the tooth number due to the variation of \( R_{1p} \) and \( R_g \).

Therefore, each tooth has the same tooth-tip leakage flux profile but with a phase shift. Fig. 8 provides the tooth-tip leakage flux variation of a single tooth within a magnet pole-pitch. It should be noted that although at the minimum position the targeted half magnet pole-pair has no tooth-tip leakage flux, its adjacent magnet pole-pair does have leakage flux; therefore, the total leakage flux associated with this tooth at the minimum position is not null.

The proposed analytical modeling unveils the relationship between the tooth-tip flux leakage and the key dimension of the SPMLV machine. Hence, it can provide a much faster estimation of the tooth-tip leakage flux for SPMLV machines than that using the FEM, while the accuracy can be maintained. Moreover, this modeling approach can provide a relatively accurate assessment of the magnet leakage coefficient and machine power factor which are particularly important during the machine design stage.

IV. CONCLUSION

In this paper, an analytical modeling approach for the tooth-tip leakage flux in the SPMLV machine has been proposed and verified. Due to the multipole PM mover and toothed-pole stator, the tooth-tip flux leakage varies from tooth to tooth. The flux fringing effect is taken into consideration for permeance calculation. The analytical results are quantitatively compared with the FEM results, hence verifying the validity and accuracy of the proposed modeling approach. In addition, this analytical modeling approach can readily be extended to analyze the tooth-tip flux leakage of other linear vernier machines or magnetic gears.

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