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Comparison of Chaotic PWM Algorithms for Electric Vehicle Motor Drives

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Abstract—This paper presents a comparison of two chaoized PWM algorithms for motor drives in the electric vehicle (EV), which are the chaotic sinusoidal pulse width modulation (SPWM) and the chaotic space-vector pulse width modulation (SVPWM). The SPWM scheme can be chaoized by three modulation methods, including the chaotically amplitude-modulated frequency modulation (CAFM), the chaotically position-modulated position modulation (CPPM), and the hybrid chaotic frequency modulation (HCFM), while the chaotic SVPWM can be fulfilled by the chaotically frequency-modulated frequency modulation (CFFM) and the CAFM methods. The performance indexes used in the comparative analysis are the electromagnetic interference (EMI) and the mechanical resonance (MR). The chaotic PWM algorithm is designed and implemented to increase the electromagnetic compatibility (EMC) and the mechanical performance for EV motor drives, and the aforementioned performance indexes are compared for the practical applicability.

I. INTRODUCTION

Because of the increasing consumption of traditional energies and the deterioration of our living environment, studies on EVs have been undertaken by various ways for improving the energy efficiency, the driving performance and the environment quality [1]-[10], such as the energy management, the EV architecture design, and the electric motor design and drive. Especially in the EV motor drives, the PWM scheme which is commonly used in AC motor drives has recently attracted wide attention [11]-[13]. Fig. 1 depicts the EV motor drive system, where the controller can produce the PWM pulses to drive the electric motor for the desired speed based on the feedback signal. Hence, the quality of PWM pulses is important to improve the performance of EV motor drives.

However, along with the switching frequency of PWM schemes is significantly raised for the reduction of their size and weight, the EMI is increased inevitably [14], which results in the degradation of EMC for electronic devices. It may deteriorate the EV motor drive system, even result in instability. Both the driving performance and the safety of EVs may be then challenged. Thus, a new PWM algorithm is necessary to be developed for the EV motor drive system, which can prevent aforementioned issues existing in the conventional PWM schemes.

In recent years, the development of the chaotic PWM algorithm has received considerable attention. A number of ideas about the chaoization of PWM signals have been tested in experimental prototypes. A CAFM-SPWM scheme was proposed and implemented by using Logistic map to chaoize a frequency-modulated signal which then modulates the carrier frequency [15]. A new chaotic SPWM scheme, namely the HCFM-SPWM method, was developed, which simultaneously chaoize both the carrier frequency and the pulse position [16]. In [17], a new chaotic SVPWM based on the induction motor drive, namely CAFM-SVPWM method, is proposed to improve the EMC for electric propulsion. In such way, the chaotic frequency modulator can be generated by using the Logistic map to generate chaotic sequence for chaoization of the amplitude of the standard frequency modulator. Additionally, a CFFM-SVPWM scheme is proposed and implemented, in order to reduce the EMI in electric motor drives [11].

In this paper, both the chaotic SPWM and the chaotic SVPWM algorithms are discussed. The SPWM pulses can be modulated chaotically by various methods, including CAFM-SPWM, CPPM-SPWM and HCFM-SPWM, while the SVPWM pulses can be chaoized by CAFM-SVPWM and CFFM-SVPWM methods. Each chaotic PWM algorithms has advantages and disadvantages, and even some chaotic PWM signals are not suitable for the EV drive system but for some certain subsystems of EVs. Thus, this paper draws a comparative analysis between the chaotic SPWM and the chaotic SVPWM based on different chaoization methods with respect to the aforementioned performance indexes, in order to make a practical applicability analysis for electric motor drives in EVs.
II. CHAOTIC SPWM DRIVE SYSTEM

Fig. 2 depicts the chaotic SPWM algorithm, which can be chaoized by three methods, such as CAFM-SPWM, CPPM-SPWM and HCFM-SPWM. Both the carrier frequency and the pulse position are chaoized by the chaotic sequence $\xi \in (0, 1)$ which is generated by the Logistic map. It can be expressed as $\xi_{n+1} = \alpha \xi_n (1 - \xi_n)$ where $\alpha \in [0.1, 3.9]$. The corresponding bifurcation diagram of the Logistic map is generated by plotting the extreme points of $\xi_n$ with respect to different values of $\alpha$ as shown in Fig. 3(a). It illustrates that $\alpha$ can be utilized to tune the spectral power distribution from no frequency modulation (FM), to periodic FM, to multi-periodic FM and finally to chaotic FM. Fig. 3(b) depicts the largest Lyapunov exponent, which denotes the rate of the attractor growth in the state space by the direction of maximum growth, is plotted with respect to different values of $\alpha$. Since the chaotic behavior occurs for $\alpha \in [3.57, 3.9]$, $\alpha = 3.9$ is selected for chaoization.

A. CAFM-SPWM

By referring to the standard frequency modulation (FM), the CAFM-SPWM can be generated by adding a sinusoidal frequency perturbation to the fixed carrier frequency and makes the real carrier frequency vary around the fixed frequency. Thus, the frequency of the carrier signal can be represented as:

$$f = f_c + \Delta f \sin(2\pi f_m t)$$

(1)

where $f$ is the real switching frequency, $f_c$ is the fixed switching frequency, $\Delta f$ is the deviation frequency and $f_m$ is the modulation frequency. Fig. 4 (a) depicts the generation algorithm of the CAFM-SPWM pulses.

B. CPPM-SPWM

Fig. 4 (b) depicts the generation of the CPPM-SPWM algorithm. The chaoization of pulse positions is also carried out by the Logistic map. This chaotic bit train functions to modulate the pulse position signal $q$ resulted from a comparison between the carrier signal $S_c$ and the reference signal $S_{ref}$ as:

$$q = \begin{cases} 0, & S_c \geq S_{ref} \\ 1, & S_c < S_{ref} \end{cases}$$

(2)

Consequently, the desired CPPM-SPWM signal is generated by applying the AND operator to $q$ and chaotic bits.

C. HCFM-SPWM

As shown in Fig. 5, the HCFM-SPWM scheme is a combination of the modulation of the carrier frequency by a chaotic sequence and the variation of pulse positions by chaotic bits in each switching cycle. The carrier frequency is chaoized by using the chaotic sequence generated by the Logistic map. Then, the HCFM-SPWM can be computed by using resulted pulses and a train of chaotic bits which is used to chaoize the pulse positions.

III. CHAOTIC SVPWM DRIVE SYSTEM

The chaotic SVPWM scheme can be applied in the EV introduction motor drive system. The dynamic of the
induction motor drive resulting from rotor field orientation is expressed as:

$$\frac{di_d}{dt} = \frac{R_L^2}{L_i}i_{mr} + \frac{R_L}{L_i}i_d + \frac{\omega_r}{L_i}i_q + \frac{u_d}{L_i}$$

(3)

$$\frac{di_q}{dt} = -\frac{R_L^2}{L_i}i_{mr} - \frac{R_L}{L_i}i_d - \frac{\omega_r}{L_i}i_q + \frac{u_q}{L_i}$$

(4)

$$\frac{\omega_r}{dt} = \frac{R_s}{L_s}i_d + \frac{R_s}{L_s}i_q$$

(5)

$$\omega_r = \omega_{mr} + \frac{R_s}{L_s}i_q$$

(6)

where $i_d$ and $i_q$ are the stator current components along the direction of rotor flux and along the orthogonal direction of rotor flux respectively, $i_{mr}$, $\omega_r$, and $\omega_{mr}$ are the magnetizing current in the rotor, the speed of rotor flux and the rotor speed respectively, $u_d$ and $u_q$ are the stator voltage components along the $d$-axis and the $q$-axis, respectively, $R_L$ and $R_s$ are the winding resistances of the stator and rotor respectively, and $L_s$, $L_r$ and $L_m$ are the stator inductance, the rotor inductance and the mutual inductance respectively.

Fig. 6 depicts the control block diagram of the chaotic SVPWM scheme in EV motor drive system, which has two closed-loop controllers, such as the inner-loop current controller and the outer-loop speed controller. Both the current controller and the speed controller utilize the PI control method. The sampling rate of the current controller varies with the switching frequency of the chaotic SVPWM inverter while the sampling rate of the speed controller is kept to be a constant value. Due to the variable sampling rate, the PI parameters in the current controller is updated during each sampling interval, namely $K_p=K_p$ and $K_i=K_i$. $K_p$ and $K_i$ are the discrete proportional parameter and integral parameter respectively, and $K_p$ and $K_i$ are the continuous proportional parameter and integral parameter respectively.

Fig. 7 shows the diagram of the inverter output voltage space vectors, which has eight switching states of the three upper transistors as: $S_0=(0, 0, 0)$, $S_1=(1, 0, 0)$, $S_2=(1, 1, 0)$, $S_3=(0, 1, 0)$, $S_4=(0, 0, 1)$, $S_5=(1, 0, 1)$ and $S_6=(1, 1, 1)$. In each switching period, the sequence of the switching states is: $S_0(T_0)$, $S_1(T_1)$, $S_2(T_2)$, $S_3(T_3)$, $S_4(T_4)$, $S_5(T_5)$, $S_6(T_6)$, and $S_0(T_7)$, where $S_4$ and $S_6$ are the active switching states. The active switching states correspond to the two adjacent basic voltage vectors $\vec{V}_1$ and $\vec{V}_2$ between where $\vec{V}_1$ locates. The switching time $T_1$, $T_2$ and $T_0$ are computed to be:

$$T_1 = \frac{2\sqrt{3}V_1}{\sqrt{3}V_1} \sin(60^\circ - \theta)$$

(7)

$$T_2 = \frac{2\sqrt{3}V_1}{\sqrt{3}V_1} \sin \theta$$

(8)

$$T_0 = T_r - T_1 - T_2$$

(9)

where $\theta$ is the phase angle of $\vec{V}_1$ and $T_r$ is the switching period.

The switching frequency of SVPWM can be chaoized by using the CAFM-SVPWM and CFFM-SVPWM methods. The chaotic SVPWM scheme can not only retain the advantages of the traditional SVPWM but also reduce the conducted EMI and avoid mechanical resonance. Fig. 8 shows the flowchart of generation of switching period for CAFM-SVPWM. It shows that the switching frequency of SVPWM is updated at the end of each switching interval and the amplitude is modified at the end of each frequency-modulating period in the CAFM-SVPWM method. In addition, Fig. 9 depicts the flowchart of generation of switching periods using the CFFM-SVPWM. To avoid too large value of $I_i(U_m)$, the minimum threshold of $\zeta_i$ is forced to be 0.1. So, if $\zeta_i \geq 0.1$, the modulating frequency will be updated at the end of each interval $\zeta_i(U_m)$. Otherwise, the modulating frequency will be updated at the end of interval $1/(0.1U_m)$. The sampling rate of the variable switching period $T_v$ will be updated at the end of each interval $T_v$.

For both the CAFM-SVPWM and the CFFM-SVPWM, the symmetrical regular sampling is used, and the sampling rate is varied simultaneously with the switching rate for good dynamic performance of the induction motor drive.
IV. PERFORMANCE INDEXES

The performance indexes used in the comparative analysis are the following: the electromagnetic interference (EMI) and the mechanical resonance (MR).

A. EMI

The total conducted EMI is caused by the common-mode (CM) conducted emissions which are due to capacitive coupling of switching inverter output voltages to earth at the output side of the drive, and the differential-mode (DM) emissions which are related to the differential mode current between output phases. It has been verified that the high-voltage slew rate \((\text{dv/dt})\) of the PWM inverter output voltages are mainly responsible for the conducted EMI in the EV motor drives. Fig. 2 shows the configuration of conducted in the induction motor drive fed from a voltage source PWM inverter. The CM current flows between the input phases and the earth of the system and its excitation source is the high-frequency potential difference which exists between the earth point of the motor frame and the inverter DC-link midpoint. The excitation source of the CM conducted emission has been proven as [18]:

\[
V_{CM} = (V_A + V_B + V_C)/3 + V_d/2
\]  

The DM current flows between different phases, and the corresponding excitation source between any two phases \(i\) and \(j\) can be expressed in terms of corresponding inverter output voltages as \(V_i - V_j\) \((i, j = A, B, C)\) [19]. Thus, the spectrum of the inverter output voltages should be improved in order to reduce the EMI in EV induction motor drives. Since the conducted EMI with a frequency exceeding 9 kHz is stringently limited by the VDE standards, the maximum power spectral density (PSD) of \(V_{CM}\) and \(V_{DM}\) within 9-150 kHz is used as the first indicator.

B. MR

The mechanical vibration of the induction motor drive is mainly due to the magnetic force created on the stator [20]. The corresponding magnetic force density \(D(t, \phi)\) is produced by the PWM inverter output voltages as:

\[
D(t, \phi) = \frac{B(t, \phi)}{2\mu_0} \quad \text{(11)}
\]

\[
B(t, \phi) = \text{Re}(\tilde{B}e^{-j\phi}) \quad \text{(12)}
\]

\[
\tilde{B} = \frac{\psi_n}{w l} \quad \text{(13)}
\]

\[
\psi_n = \frac{\tilde{V}_1 - \sigma_j}{j \omega_1} + \frac{\tilde{V}_h (1 + \sigma_j)}{j \omega_1 (\sigma_j^2 + \sigma_h^2)} \quad \text{(14)}
\]

\[
\tilde{V} = \frac{2}{3} (V_A e^{j\pi} + V_B e^{-j\pi}) \quad \text{(15)}
\]

where \(\tilde{V}_1\) and \(\tilde{V}_h\) are respectively the fundamental component and the harmonic component of the inverter rotating output voltage vector; \(\sigma_j\) and \(\sigma_h\) are respectively the leakage factor of the stator and the rotor; \(\omega_1\) and \(\omega_m\) are respectively the fundamental component and the harmonic component of the angular speed of the inverter output voltage; \(w\), \(d\) and \(l\) are respectively the number of turns of the stator winding, the inner diameter and the active length of the stator; \(\psi_n\), \(\tilde{B}\) and \(B(t, \phi)\) are respectively the rotating magnetic flux density at time \(t\) and position \(\phi\). It shows that the magnetic force density \(D(t, \phi)\) is governed by the inverter output voltages \(V_A\), \(V_B\) and \(V_C\). As discussed in [21] and [20], there exist distinct spectral peaks of \(D(t, \phi)\) when the motor drive adopts the conventional PWM algorithm. If the frequencies of the peaks overlap with the natural frequency of the electric motor, the mechanical resonance will be induced. Thus, a proper PWM algorithm should be designed to not only decrease the peaky EMI but also avoid the mechanical resonance. In this paper, it shows that the power spectrum is mainly from \((f_s - \alpha j)\) to \((f_s + \alpha j)\). In order to evaluate the content of acoustic noises, the power of \(V_{CM}\) and \(V_{DM}\) within 2.99-3.01 kHz is used as the third performance index for the following comparative analysis.

V. COMPARATIVE ANALYSIS

The computational simulation of the aforementioned chaotic PWM algorithms is carried out by Simulink. By using the periodogram method, the power spectra of \(V_{CM}\) and \(V_{DM}\) based on the conventional PWM, CPPM-SPWM, CAFM-SPWM, HCFM-SPWM, CAFM-SVPWM and CFFM-SVPWM are computed from zero to 150 kHz under the same
Fig. 10. Simulated power spectrum: (a) $V_{CM}$ with conventional SPWM; (b) $V_{CM}$ with conventional SPWM; (c) $V_{CM}$ with CPPM-SPWM; (d) $V_{CM}$ with CPPM-SPWM; (e) $V_{CM}$ with CAFM-SPWM; (f) $V_{CM}$ with CAFM-SPWM; (g) $V_{CM}$ with HCFM-SPWM; (h) $V_{CM}$ with HCFM-SPWM.

Fig. 11. Simulated power spectrum: (a) phase voltage with CAFM-SVPWM; (b) line voltage with CAFM-SVPWM; (c) phase voltage with CFFM-SVPWM; (d) line voltage with CFFM-SVPWM.

$f_{sw}$ and $Af$. Fig. 10 and Fig. 11 depict the corresponding power spectra from zero to 20 kHz for the chaotic SPWM and the chaotic SVPWM respectively.

1) Chaotic SPWM algorithm

**EMI** - By comparing the maximum PSD of their $V_{CM}$ spectra in 9-150 kHz, namely 10.03 dBm/Hz for the conventional SPWM, 6.04 dBm/Hz for the CPPM-SPWM, 2.22 dBm/Hz for the CAFM-SPWM and -11.88 dBm/Hz for the HCFM-SPWM, it illustrates that the HCFM-SPWM has a remarkable improvement over other PWM schemes. Similarly, from their $V_{CM}$ spectra, namely 14.78 dBm for the conventional SPWM, -17.14 dBm for the CPPM-SPWM, -1.42 dBm for the CAFM-SPWM and -17.57 dBm for the HCFM-SPWM, it confirms that both the CPPM-SPWM and HCFM-SPWM have significant improvement over the others while the HCFM-SPWM is better than the CAFM-SPWM.

**MR** - By comparing the spectral power of their $V_{CM}$ spectra in 2.99-3.01 kHz, namely 10.03 dBm for the conventional SPWM, -17.14 dBm for the CPPM-SPWM, 2.22 dBm for the CAFM-SPWM and -19.55 dBm for the HCFM-SPWM, it illustrates that both the CPPM-SPWM and the HCFM-SPWM have a remarkable improvement over other PWM schemes. Then, from their $V_{CM}$ spectra, namely 10.03 dBm for the conventional SPWM, -11.72 dBm for the CPPM-SPWM, -1.42 dBm for the CAFM-SPWM and -17.57 dBm for the HCFM-SPWM, it confirms that both the CPPM-SPWM and HCFM-SPWM have significant improvement over the others while the HCFM-SPWM is better than the CPPM-SPWM.

2) Chaotic SVPWM algorithm

**EMI** - From the maximum PSD of the phase voltage spectra in 9-150 kHz, namely 4.28 dBm/Hz for the CAFM-SVPWM and 6.03 dBm/Hz for the CFFM-SVPWM, it illustrates that the CAFM-SVPWM produce a little better
In terms of the control algorithm, the chaotic SPWM algorithm has a lower computation complexity than the chaotic SVPWM algorithm. Additionally, the chaotic SPWM is designed for the open-loop electric motor control, while the chaotic SVPWM is developed for the closed-loop control based on the feedback current and rotating speed and can be used for the precise motor speed control. Thus, the chaotic SPWM can be utilized in some certain subsystems of EVs, such as the wiper system, and the chaotic SVPWM can be used in the precious motor drive, such as the EV drive system.

VI. CONCLUSION

In this paper, a comparative analysis of chaotic PWM algorithms is drawn for the practical applicability in EV motor drives. It shows that the HCFM-SPWM can offer the better performance in terms of the reduction of the EMI and the mechanical resonance than other chaotic PWM algorithms. However, the chaotic SPWM scheme is developed for the open-loop electric motor drive and cannot be utilized in the precise speed control. Thus, the chaotic SPWM is not commonly used in the EV drive system, but some certain subsystems. Additionally, the chaotic SVPWM algorithm is designed for the closed-loop electric motor drive system based on the feedback current and rotating speed. It shows that the CAFM-SVPWM can produce a little better performance for the EMI and mechanical resonance issues, but not a remarkable improvement. Therefore, both the CAFM-SVPWM and CFFM-SVPWM algorithms are suitable for EV drive system.

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