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Efficient All Top-\(k\): Computation—A Unified Solution for All Top-\(k\), Reverse Top-\(k\) and Top-\(m\) Influential Queries

Shen Ge, Leong Hou U, Nikos Mamoulis, and David W. Cheung

Abstract—Given a set of objects \(P\) and a set of ranking functions \(F\) over \(P\), an interesting problem is to compute the top ranked objects for all functions. Evaluation of multiple top-\(k\) queries finds application in systems, where there is a heavy workload of ranking queries (e.g., online search engines and product recommendation systems). The simple solution of evaluating the top-\(k\) queries one-by-one does not scale well; instead, the system can make use of the fact that similar queries share common results to accelerate search. This paper is the first, to our knowledge, thorough study of this problem. We propose methods that compute all top-\(k\) queries in batch. Our first solution applies the block indexed nested loops paradigm, while our second technique is a view-based algorithm. We propose appropriate optimization techniques for the two approaches and demonstrate experimentally that the second approach is consistently the best. Our approach facilitates evaluation of other complex queries that depend on the computation of multiple top-\(k\) queries, such as reverse top-\(k\) and top-\(m\) influential queries. We show that our batch processing technique for these complex queries outperform the state-of-the-art by orders of magnitude.

Index Terms—All top-\(k\) queries, view-based index

1 INTRODUCTION

Many real life applications support ranking of products according to user preference functions. For example, consider an online store (e.g., Amazon), which ranks blue-ray discs according to the preferences of customers. Preferences could be explicitly expressed by each user, or implicitly derived from user purchase records. Preferences are typically defined on some product features. For example, blue-ray discs could be ranked based on their movie cast and release date; recent movies having a good cast rank higher than others. To simplify illustration and analysis, in the rest of the paper, we assume that product features take values from a normalized numerical domain; e.g., the quality of casting takes a score from 0 (worst) to 1 (best). This way, the products can be modeled by multidimensional points; e.g., points \(p_1, p_2, p_3\), and \(p_4\) are used to represent four products respectively in Fig. 1. Modeling objects in such a multidimensional space is common for diverse types of queries, such as top-\(k\) queries [1], [2], [3], skyline queries [4], [5], and market analysis queries [6], [7].

Given a preference function \(f\), we can rank the products \(p \in P\) according to \(f(p)\). Fig. 1 shows three linear functions \(f_a, f_b,\) and \(f_c\) which create three object rankings as shown in the right part of the figure. Each function is of the form \(f(x_1) = x_1 + x_2 \cdot y\), such that \(0 \leq f(x_1)\). The functions are represented as vectors in the space that contains the points. The object ranking for a specific function \(f\) can be determined by the order of the points are met if we sweep a line perpendicular to the vector of \(f\) from point \((1,1)\) towards point \((0,0)\). Different customers may have completely different preferences. For instance, \(f_b\) represents the preferences of a customer, \(u_b\), who is concerned more about the quality of casting than the release date. Accordingly, \(p_2\) is the best product according to \(u_b\)’s preferences. Without loss of generality, we assume that all preference functions \(f\) are linear and the coefficients of them are normalized; the score \(f_p(p)\) of an object \(p\) is computed by the inner product \(\sum_{i=1}^{d}(f[i] \cdot p[i])\) of \(f\)’s weights vector with \(p\)’s feature vector.

Generally speaking, users are more interested in top ranked products. Given a constant \(k\), in addition with a ranking function \(f\), a top-\(k\) query [1], [2], [3] returns the \(k\) highest ranked objects according to \(f\). For example, consider the four products in Fig. 1. For \(k=3\) and user \(u_a\), whose preferences are captured by the linear function \(f_a = 0.5x + 0.5y\), the result of the top-\(k\) query is \(TOP^3(f_a) = \{p_1, p_2, p_3\}\).

Many applications have millions of users and numerous top-\(k\) queries may have to be evaluated simultaneously. Recommendation systems of online stores are such an application (i.e., recommendations to numerous users currently online). As another example, consider a second-hand cars company, which recommends cars to customers before the summer season; the company issues multiple top-\(k\) queries, one for each customer (depending on his/her individual preferences), simultaneously. The result can be computed by issuing an individual top-\(k\)-query for each user, \(TOP^k(f_i)\). This iterative approach becomes too expensive.

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when a large number of queries have to be evaluated over a large number of products. Thus, developing specialized techniques for processing multiple top-k queries is an important problem that has been overlooked in past research. We call this problem the all top-k query, \( \text{ATOP}_k \).

To the best of our knowledge, there is no efficient approach to compute multiple top-k queries simultaneously. In this paper, we study two batch processing techniques for this problem. The first is a batch indexed nested loops approach and the second is a view-based threshold algorithm. We also propose several novel optimization techniques for these methods.

Besides products recommendation, other tasks, such as product promotion analysis [8] and identifying the most influential products [9], can benefit from an efficient approach for computing multiple top-k queries simultaneously, as we discuss in Section 3. We demonstrate the utility of our result in these complex analysis tasks; when \( \text{ATOP}_k \) is used as a search module for reverse top-k [8] and top-m influential [9] queries, the evaluation cost of these queries greatly decrease.

The rest of the paper is organized as follows: we provide formal definitions and review preliminary concepts in Section 2. The applicability of \( \text{ATOP}_k \) in the evaluation of related queries is discussed in Section 3. An intuitive batch processing technique is introduced in Section 4. In Section 5, we present an alternative batch processing approach which extends the view-based threshold algorithm [10] and fully optimize it. Section 6 discusses how we can use our techniques to support related queries, including reverse top-k and top-m influential queries. In Section 7, we experimentally evaluate our methods using synthetic and real data. Section 8 discusses related work. Finally, Section 9 concludes the paper.

## 2 Preliminaries

This section includes all formal definitions and preliminary concepts, based on which we build our solutions. We begin by defining top-k and all top-k queries.

**Definition 1 (Top-k query, \( \text{TOP}_k(f) \)).** Given a set of products \( P \), a preference function \( f \), and a positive integer \( k \), the top-k query \( \text{TOP}_k(f) \) returns a subset of \( k \) products from \( P \), such that \( f(p_1) \geq f(p_2) \), \( \forall p_1 \in \text{TOP}_k(f), p_2 \in P \setminus \text{TOP}_k(f) \).

**Definition 2 (All top-k query, \( \text{ATOP}_k \)).** Given a set of products \( P \), a set of preference functions \( F \), and a positive integer \( k \), the all top-k query \( \text{ATOP}_k \) returns \( \text{TOP}_k(f) \) for every function \( f \in F \).

The reverse top-k query [8] is a derived concept. Given a product \( p_i \) and a set of user preferences, a reverse top-k query, \( \text{RTOP}_k(p_i) \), returns the users who have \( p_i \) in their top-k results (Definition 3). For example, for the data in Fig. 1, \( \text{RTOP}_2(p_2) \) returns functions \( f_9 \) and \( f_6 \) since \( p_2 \) is ranked 2nd and 1st by \( f_9 \) and \( f_6 \), respectively. Product promotion is an application of \( \text{RTOP}_k(p_i) \). Assume that a property agent is promoting a new building to customers via web advertisements. To minimize cost, the agent should advertise the building only to those customers who are potentially interested in it; in other words, product \( p_i \) should be advertised to users who would highly rank \( p_i \) based on their known preferences.

**Definition 3 (Reverse top-k query).** Given a product \( p \), a positive integer \( k \), a set of products \( P \) and a set of user preferences \( F \), the reverse top-k query \( \text{RTOP}_k(p) \) returns a subset of user preferences \( F \), such that \( \text{RTOP}_k(p) \subseteq F \), and \( f_i \in \text{RTOP}_k(p) \) if and only if \( \exists q \in \text{TOP}_k(f_i) \) such that \( f(q) \geq f(i) \).

The problem of finding the most influential products has been recently studied by Vlachou et al. [9]. The influence score \( I^k(p_i) \) of a product \( p_i \) (Definition 4) is defined by the number of customers who have \( p_i \) in their top-k preferences.

**Definition 4 (Influence score, \( I^k \)).** Given product data set \( P \), user preferences \( F \), and a positive integer \( k \), the influence score of a product \( p \) is defined as \( I^k(p) = |F'\subseteq F \) such that \( F' \leq F \). For example, \( \text{RTOP}_2(p_2) \) is an application of \( \text{RTOP}_k(p_2) \) is a derived concept. Given a product data set \( P \), a set of product preferences \( F \), and a positive integer \( k \), the \( \text{RTOP}_k(p) \) finds the \( k \) products that have the highest influence score in \( F \).

Accordingly, the top-m influential query [9], \( \text{ITOP}_m \), finds the \( m \) most influential products (Definition 5). Ranking is based on the influence scores \( I^k \). \( \text{ITOP}_m \) finds products of significant impact in the market. Identifying products of high influence in a large database (e.g., database of houses, second-hand cars, etc.) can help companies to assess the popularity of their current products and/or design new ones with features similar to the most popular products. For instance, the iPad is considered a good product because it is ranked highly by many customers in a survey [11]. Intuitively, the influence of a product in the market is the number of customers who consider it intriguing (i.e., rank it high in their preferences).

**Definition 5 (Top-m influential query).** Given a product data set \( P \), a set of users preferences \( F \), and a positive integer \( k \), the top-m influential query \( \text{ITOP}_m \) returns a subset of \( m \) products from \( P \) such that \( \text{ITOP}_m \subseteq P \) and \( |\text{ITOP}_m| = m \), \( I^k(p_i) \geq I^k(p_j), \forall p_i \in \text{ITOP}_m \). For example, in Fig. 1, let \( k = 3 \) and consider the three user preference functions \( F = \{f_1, f_2, f_3\} \). The four products \( \{p_1, p_2, p_3, p_4\} \) have influence scores \( \{3, 2, 3, 1\} \) respectively. The score of \( p_1 \) is only 1 because it appears in the top-3 set of only one function (\( f_1 \)). Thus, \( \text{RTOP}_3(p_1) \) returns \( \{p_1, p_4\} \).

In this paper, we study \( \text{ATOP}_k \) and show how it can be used as a module for efficient evaluation techniques for \( \text{RTOP}_k(p) \) and \( \text{ITOP}_m \). Table 1 summarizes the notation used throughout the paper. Our solution builds on methods for top-k queries using materialized ranking views [10]. A materialized ranking view is simply the result of a top-k...
query. Das et al. [10] proposed a Linear Programming adaptation of the Threshold Algorithm (LPTA), which extends the Threshold Algorithm (TA) [2] to apply on views. LPTA sequentially accesses the results of two or more materialized object rankings, based on different views, in order to compute the top-k objects of a new function. When an object \( p \) is accessed from view \( v_i \), a random access is performed at each of the other views to calculate the aggregate feature score of object \( p \). LPTA keeps track of the \( k \) objects with the highest scores seen so far. These \( k \) objects become the final top-\( k \) result if they have better scores than the maximum possible score for all unseen objects. The maximum possible score is computed by linear programming in [10]. We illustrate this process by an example in Section 5.

3 APPLICATIONS OF ATOP\(^k\) AS A MODULE

Besides its direct applications (e.g., in recommender systems), discussed in the Section 1, ATOP\(^k\) can also be used as a processing module of other queries. We note that the solution for reverse top-\( k \) problem proposed in [8] does not scale well, because every reverse top-\( k \) query is answered by issuing a set of essential top-\( k \) queries. If multiple reverse top-\( k \) queries are issued (e.g., multiple products are to be promoted at a holiday season), some of these top-\( k \) queries might even have to be executed multiple times. Also in [9], the object influence scores are calculated by reverse top-\( k \) queries, therefore the proposed solution does not scale well according to our discussion above.

In Fig. 2, we briefly summarize the relationship between the all top-\( k \) (ATOP\(^k\)) query that we study in this paper and RTOP\(^k\)(\( f \))/ITOP\(^m\)_\( k \). In [8], a reverse top-\( k \) query RTOP\(^k\)(\( f \)) is computed by a set of top-\( k \) queries; however, not all these queries need to be evaluated due to the use of pruning strategies. In addition, according to [9], the influence score of a product \( I^k(\( p \)) \) is equivalent to the size of the reverse top-\( k \) result. Given a set of products and a set of preference functions, the top-\( m \) influential query ITOP\(^m\)_\( k \) is evaluated using the influence scores of the products. Therefore, a large number of top-\( k \) queries are implicitly involved in a top-\( m \) influential query. Although pruning strategies and fine-tuned execution ordering are employed in the state-of-the-art solutions for RTOP\(^k\)(\( f \)) and ITOP\(^m\)_\( k \) queries in [8] and [9], respectively, neither solution optimizes the core ATOP\(^k\) module of these queries. In other words, an efficient evaluation technique for all top-\( k \) queries (ATOP\(^k\)) would greatly benefit the evaluation of RTOP\(^k\)(\( f \))/ITOP\(^m\)_\( k \) queries.

4 BATCH TOP-k PROCESSING

Top-\( k \) queries are extensively studied in the literature [1], [2], [3], [10]. The state-of-the-art techniques aim at minimizing the cost of a single top-\( k \) query with the use of thresholding and/or indexing structures. However, there is a lack of research on multiple top-\( k \) evaluation. Motivated by this, in this section, we propose a batch processing technique that indexes not only the objects but also the functions, to support all top-\( k \) computation.

This method can be considered as the counterpart of block indexed nested-loops in relational databases and spatial join queries in spatial databases [12]. Suppose that the objects are indexed by a multidimensional index, e.g., R*-tree [13], and the functions are also partitioned in groups. To group the functions, we can first order them according to their position on the Hilbert curve [14] that indexes the space of function coefficients. Then, we split the curve into subintervals, each defining a group, such that each group contains no more than a ratio \( \delta \) of the functions. Intuitively, a group contains a small number of similar functions that would share a number of results. Processing the functions in the group simultaneously would be faster than executing the queries individually, as some search cost would be shared among the functions in the group. In Section 7, we study the choice of \( \delta \) and evaluate alternative grouping strategies.

Let \( F_g \) be a group of functions; the group maximum score \( s_{\text{max}}(\( p \)) \) of an object \( p \) computed by the functions of the group is

\[
F_g = \max_{g \in \text{functions}} \max_{e \in \text{entries}} f_g(e, \mathbf{p}[e])
\]

For a given \( F_g \), we traverse the nodes and objects in the R*-tree (e.g., Fig. 3a) in descending order of the group maximum score. We first load the root of the R*-tree, calculating \( s_{\text{max}} \) of all entries in it (i.e., for each minimum bounding rectangle (MBR)). The maximum possible score \( s_{\text{max}}(m) \) of an MBR \( m \) is the maximum score of any possible object inside \( m \). If higher values are preferred in each dimension, the corner point of an MBR with the largest values in all dimensions is the point with the maximum score. We put all accessed R*-tree entries and their maximum scores into a priority queue and access them in descending maximum score order. Each time an entry \( e \) is de-heaped, if \( e \) is a nonleaf entry (e.g., \( M \) in Fig. 3a), we calculate the maximum scores for all its children and insert them into the priority queue. If \( e \) is a leaf MBR (e.g., \( m_i \) in Fig. 3a), then all functions in \( F_g \) are computed.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
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<tbody>
<tr>
<td>( F )</td>
<td>the set of user preferences</td>
</tr>
<tr>
<td>( p )</td>
<td>the set of products</td>
</tr>
<tr>
<td>( f(p) )</td>
<td>score of product ( p ) by user preference ( f )</td>
</tr>
<tr>
<td>( p[i] )</td>
<td>the ( i )-th dimension value of ( p )</td>
</tr>
<tr>
<td>( f[i] )</td>
<td>the ( i )-th coordinate (weight) of ( f )</td>
</tr>
<tr>
<td>( \text{TOP}^k )</td>
<td>a top-( k ) query</td>
</tr>
<tr>
<td>( \text{ATOP}^k )</td>
<td>a reverse top-( k ) query</td>
</tr>
<tr>
<td>( I^k(( p )) )</td>
<td>top-( k ) influence score of product ( p )</td>
</tr>
<tr>
<td>( \text{ITOP}^m_( k ) )</td>
<td>a top-( m ) influential query</td>
</tr>
</tbody>
</table>

Table 1: Summary of Notations

Fig. 2. Relationship of different queries.

[1017]
against all the points in that leaf node and the candidate lists of those functions in $F_g$ are updated accordingly. As an optimization (see Lemma 1 below), we avoid processing an MBR $m$ for a function $f \in F$ if the upper bound $f(m)$ (computed using the best corner of $m$) is worse than the $k$ best scores of $f$ computed so far. We name this batch processing technique as Batch Indexed Nested Loops algorithm (BINL). We list the pseudocode for BINL in Algorithm 1.

**Algorithm 1. BINL Algorithm**

Algorithm BINL($R, F, k$)

$R$ is the R*-tree index of the set of objects $P$

1: partition $F$ into $g$ groups $\{F_1, \ldots, F_g\}$ by Hilbert curve  
2: for all $F_i \in \{F_1, \ldots, F_g\}$ do  
3: \hspace{1em} en-heap ($R$.root, 0) into $PQ$  
4: \hspace{1em} while $PQ$ is not empty do  
5: \hspace{2em} de-heap the top element $m$ from $PQ$  
6: \hspace{2em} if $m$ is a non-leaf MBR then  
7: \hspace{3em} for all $m_i \in m$ do  
8: \hspace{4em} compute the maximum possible score $s_{\text{max}}^f(m_i)$ to $m_i$  
9: \hspace{4em} en-heap ($m_i, s_{\text{max}}^f(m_i)$) into $PQ$  
10: \hspace{2em} else if $m$ is a leaf MBR then  
11: \hspace{3em} for all $f_i \in F_i$ do  
12: \hspace{4em} if $f_i(m)$ is better than $k$-th candidate of $f_i$ then  
13: \hspace{5em} evaluate $f_i$ for all objects in $m_i$  
14: \hspace{5em} update the candidate list of $f_i$

**Lemma 1 (MBR pruning).** An MBR $m$ needs not be evaluated by a function $f$ if $f(m)$ is no better than the $k$th score for the objects seen so far, where $f(m)$ is the maximum score of function $f$ for any point in $m$.

Fig. 3b illustrates an example for BINL. Assume that we are processing the group of functions $F_g = \{f_a, f_b\}$. The accessing order based on $s_{\text{max}}$ can be conceptually captured by the order a perpendicular plane to the dashed arrow in the figure crosses the MBRs. Suppose that $k = 2$ and we have already accessed four MBRs, $M, M_a, m_a$, and $M_b$; $p_2$ and $p_3$ have already been seen by $f_a$ and $f_b$ and we have $m_d, m_a, m_c$ in the priority queue. Next, we get $m_d$ from the priority queue, which is a leaf MBR, therefore its contents are evaluated using the functions in $F_g$. Note that only $f_b$ evaluates the objects in $m_d$ while $f_a$ prunes $m_d$ because $f_a(m_d) < f_a(p_2) < f_a(p_3)$.

**Discussion.** Techniques similar to BINL have been proposed before for All Nearest Neighbors Queries (ANNs) in spatial databases [12]. We note that BINL does not support early termination, because the group traversing order is generally different from the early termination order of every single user preference function in that group, which means that we have to traverse the R*-tree once for every group of functions.

**5 A View-Based Approach**

In this section, we investigate an alternative, more efficient approach than BINL. A well-accepted general paradigm for efficient query processing, for different data and query types, is to take advantage of materialized views with precomputed results [15]. As discussed in Section 2, LPTA [10] can be used to compute top-$k$ queries using views. Here, we demonstrate LPTA by an example in Fig. 4a. In this example, we use the same objects set from Fig. 1 and construct two views, $v_1$ and $v_2$. Assuming that $v_1$ and $v_2$ have been accessed two times, respectively, the regions being accessed are shaded in the figure. Note that the unseen region must be convex if all view functions are linear. Given a linear function, the maximum score of any objects in the convex unseen region must be smaller than or equal to the scores of the convex points (of the unseen region), which can be computed by linear programming.

After two sorted accesses from each view, only three objects, $p_2, p_3$, and $p_4$, are seen so far and the preference function $f_a$ keeps $p_3$ as the top-1 candidate. LPTA returns $p_3$ for $f_a$ since the current maximum possible score $s_{\text{max}}^f(f_a)$ (computed by linear programming) is already worse than the candidate’s score, $f_a(p_3)$.

To support batch processing, when an object $p$ is accessed from a view, we can evaluate its scores for multiple top-$k$ queries. For every top-$k$ query being evaluated, we update the current result set if necessary. A function is marked as stopped if its $k$th candidate score is no worse than the maximum possible score. Based on this idea, we can answer multiple top-$k$ queries by traversing each view once. We call this method Batched Linear Programming adaptation of the Threshold Algorithm (BLPTA). The pseudo code of BLPTA can be found in Algorithm 2.

**Algorithm 2. BLPTA Algorithm**

Algorithm BLPTA($V, P, F, k$)

1: for all $f \in F$ do
2: \hspace{1em} TOP$^k(f) = \emptyset$ and mark $f$ as running
3: \hspace{1em} while $F$ is not empty do
for all \( v \in V \) do  
1: access in round-robin fashion.
2: fetch next object \( p \) from \( v \) and update accessed regions
3: for all \( f \in F \) not marked as \( \text{stopped} \) do
4: if \( f(p) \) is better than \( k \)-th object \( \text{TOP}_k(f) \) then
5: remove \( k \)-th object and insert \( p \) into \( \text{TOP}_k(f) \)
6: compute maximum possible score \( s_{\text{max}}(f) \)
7: if \( k \)-th object in \( \text{TOP}_k(f) \) is better than \( s_{\text{max}}(f) \) then
8: mark \( f \) as \( \text{stopped} \) and remove \( f \) from \( F \).

At every iteration of BLPTA, we fetch the next object \( p \) from one of the views in a round-robin fashion and update the top-\( k \) candidates for each of the \( \text{running} \) functions. In Fig. 4b, the top-1 candidates of \( f_a, f_b, \) and \( f_c \) are \( p_1, p_2, \) and \( p_3 \), respectively, after 2 accesses from each of the views. The maximum possible scores, \( s_{\text{max}} \), of the functions are shown by three different lines in Fig. 4b. In this example, all functions are marked as \( \text{stopped} \) after the second access from each view since the corresponding \( s_{\text{max}} \) score is no better than the candidate score. Therefore, BLPTA exits the while-loop and returns the all top-\( k \) result.

BLPTA terminates early if all functions are marked as \( \text{stopped} \). However, this method is costly since 1) the maximum possible scores are computed by linear programming, 2) functions are not partitioned into groups, and 3) every object being accessed from views is unavoidably evaluated. In the remainder of this section, we discuss and resolve these three issues and propose an optimized version of the algorithm.

### 5.1 Avoiding Linear Programming

Given a set of precomputed views \( V \), BLPTA (and LPTA as well) can compute the top-\( k \) queries using a subset of \( V \) and the selection can be determined by the cost estimation technique suggested in [10]. However, the maximum possible score is still computed by linear programming. Considering the fact that this computation will be carried out for all \( \text{running} \) preference functions against all \( \text{accessed} \) objects, it easily becomes the bottleneck. Motivated by this, we first redesign our method to avoid linear programming computation. Instead of using a subset of precomputed views, we construct the views based on some constraints, such that the maximum possible score can be derived from the cross point of \( d \) \( \text{hyperplanes} \) (technique to be discussed shortly). We now introduce the constraints that we impose when constructing views (Definition 6).

**Definition 6 (d-Bounding views).** A preference function \( f \) is bounded by \( d \) views \{\( v_1, \ldots, v_d \)\} if and only if there exists a \( d \)-dimensional vector \( r \), such that \( \forall r_i, r_i \geq 0 \) and \( \sum_{i=1}^{d} r_i v_i = f \).

Intuitively, a preference function \( f \) being bounded by \( d \) views means that the direction of \( f \) is enclosed by the directions of \( d \) views. Fig. 5a demonstrates an example. Suppose that \( f_a = \frac{1}{2}x + \frac{1}{2}y \) and consider two views, \( v_1 = \frac{1}{2}x + \frac{1}{2}y \) and \( v_2 = \frac{1}{2}x - \frac{1}{2}y \), in the system. There exists a vector \( r = (\frac{1}{2}, \frac{3}{2}) \) that makes \( r_1 v_1 + r_2 v_2 = f_a \). Therefore, we say that views \( v_1 \) and \( v_2 \) are a set of \( d \)-bounding views for \( f_a \).

Besides, we define as the scanning hyperplane of a view \( v \), the hyperplane which is perpendicular to \( v \)'s vector and intersects the last object seen in \( v \). The dashed lines (orthogonal to the preferences vectors) in Fig. 5 illustrate scanning hyperplanes. Formally, if \( s \) is the last score seen in \( v \), the scanning hyperplane of \( v \) is defined by the set of points \( x \) which satisfy \( v[1]x[1] + \cdots + v[d]x[d] = s \).

By basic geometry, we can easily show that there is only one cross point \( \phi \) being intersected by \( d \) hyperplanes in the \( d \) dimensional space. We illustrate the cross point \( \phi \) in Fig. 5a. Assume that all user preferences in the system are bounded by \( d \) views. Theorem 1 shows that the cross point \( \phi \) is the point \( x \) that maximizes the score of any unseen objects (proofs of all theorems are in Appendix A, which can be found on the Computer Society Digital Library at http://doi.ieeecomputersociety.org/10.1109/TKE.2012.34).

**Theorem 1.** For a set of user preferences \( F \) being bounded by \( d \)-bounding views \{\( v_1, \ldots, v_d \)\}, \( f(\phi) \) is no worse than the score of any unseen objects, where \( \phi \) is the cross point of the scanning hyperplanes of the \( d \)-bounding views.

According to Theorem 1, \( f(\phi) \) can be viewed as the maximum possible score \( s_{\text{max}}(f) \) in BLPTA. Clearly, we can mark a function as \( \text{stopped} \) if \( f \) is bounded by the corresponding \( d \)-bounding views and the value of \( f(\phi) \) is not better than the \( k \)-th candidate score. The remaining problem is to calculate the cross point \( \phi \) of \( d \) scanning hyperplanes. For every view \( v_i \) and its last seen score \( s_i \), we have

\[
v_i[1]\phi[1] + \cdots + v_i[d]\phi[d] = s_i.
\]

Since we have \( d \) different equations in total, \( \phi \) can be found by solving a simple linear system, \( \phi = A^{-1}B \), where \( A \) is the set of \( d \) views and \( B \) is the set of last seen scores. Formally,

\[
\phi = \left( \begin{array}{c} v_1[1] & \cdots & v_1[d] \\ \vdots & \ddots & \vdots \\ v_d[1] & \cdots & v_d[d] \end{array} \right)^{-1} \left( \begin{array}{c} s_1 \\ \vdots \\ s_d \end{array} \right).
\]

**Discussion.** The views based computation can stop early if the preferences functions are bounded tightly by the views. For instance, we can mark \( f_a \) as \( \text{stopped} \) after accessing one object from each of views in Fig. 5a; while we need to access three objects in total from the views in Fig. 4a. However, finding the tightest \( d \)-bounding views is
Definition 7 (Simplex partitioning). Given a \( \Delta^{d-1} \) and a point \( p \) inside the simplex, \( \Delta^{d-1} \) can be partitioned into \( d \) isolated \( \Delta^{d-1} \) s being split from \( p \) towards the vertices of the simplex.

Theorem 2 shows that the function \( f_a \) passes through point \( p_a \), in the interior of \( \Delta^{d-1} \) if and only if \( f_a \) is bounded by \( \{v_1, \ldots, v_h\} \). In Fig. 7b, the corresponding \( d \)-bounding views of \( \Delta^1 \), \( \Delta^2 \), and \( \Delta^3 \) are \( \{v_1, v_3, v_4\} \), \( \{v_2, v_3, v_4\} \), and \( \{v_2, v_3, v_4\} \), which bound functions \( \{f_a, f_b, f_c\} \), and \( \{f_d\} \), respectively.

Theorem 2. A function (or a view) is bounded by a set of \( d \)-bounding views if and only if it passes through the interior of the \( (d - 1) \)-simplex defined by the \( d \)-bounding views.

For the sake of generating tighter boundings, we can recursively partition the simplex. On the other hand, this might create a large amount of views. Therefore, there is a tradeoff between achieved tightness and the number of views, which should be considered in the process.

Accordingly, we propose an algorithm that recursively partitions the initial simplex. After each partitioning, we assign each function to the subsimplex where its projection falls. We use a parameter \( \lambda \) to control the number of views being created during this process. We do not further split a simplex if the number of functions being bounded by it is less than a ratio \( \lambda \) of the total.

The partitioning procedure is described by Algorithm 3. We first construct the simplex \( \Delta^{d-1} \) based on the \( d \)-bounding views \( V \) (e.g., \( v_1, v_2, \) and \( v_3 \) in Fig. 7) and assign the entire set of preferences functions to \( \Delta^{d-1} \). \( F \) denotes the associated preference function set \( F \) of the simplex \( \Delta^{d-1} \). Lines 3-12 describe an iterative process that recursively partitions the simplex. Given a point inside a simplex (e.g., the average point of all vertices, \( v_{avg} \)), we partition the simplex \( \Delta^{d-1} \) into \( d \) sub-simplices using Definition 7 (line 5). Every bounding function \( f_d \) of \( \Delta^{d-1} \) is assigned to one of the \( d \) sub-simplices. Clearly, the simplex is not tight enough if it bounds many functions. Therefore, we further partition a subsimplex if the number of bounding functions is larger than a threshold (controlled by parameter \( \lambda \)).

Algorithm 3. \( d \)-bounding views partitioning

Algorithm partitioning(\( V, F, \lambda \))

1: construct \( \Delta^{d-1} \) for \( V \) and set \( \Delta^{d-1} = F \)
2: push \( \Delta^{d-1} \) into a queue \( Q \)
3: while \( Q \) is not empty do
4: \( \Delta^{d-1} := Q.pop() \)
5: partition \( \Delta^{d-1} \) into \( \{\Delta_1^{d-1}, \ldots, \Delta_n^{d-1}\} \) using \( v_{avg} := AVG(V) \in V \)
6: for all \( f \in \Delta^{d-1} \) do
7: assign \( f \) to \( \Delta^{d-1} \) if \( f \) is in the interior of \( \Delta^{d-1} \)
8: for all \( \Delta_i^{d-1} \in \{\Delta_1^{d-1}, \ldots, \Delta_n^{d-1}\} \) do
9: if \( size(\Delta_i^{d-1}, F) \geq \lambda \cdot size(F) \) then
10: push \( \Delta_i^{d-1} \) into \( Q \) to further partition \( \Delta_i^{d-1} \)
11: else
12: \( \Delta_G := \Delta_G \cup \{\Delta^{d-1}\} \)
13: return \( \Delta_G \)

5.3 Simplex Execution Order

Even though the simplices (generated by Algorithm 3) can be evaluated independently at any order, the memory...
usage can be controlled better if the execution order is well designed. According to our partitioning approach, each simplex contains $d$ views and each view is used by multiple simplices. A view can be removed from memory after all relevant simplices have been evaluated. To minimize the total memory usage, we should define an execution order such that the maximum number of views kept in memory is minimized. Finding the optimal order is a combinatorial problem, therefore we adopt a greedy approach, where the next simplex is decided by the views kept in memory. Intuitively, a view $v_{\text{min}}$ should be cleaned up first if $v_{\text{min}}$ is used by the fewest simplices among all views in memory. In other words, we first evaluate all simplices that use $v_{\text{min}}$ in order to remove $v_{\text{min}}$ from memory as early as possible. The effectiveness of this approach is demonstrated in our experiments.

5.4 Accessing Multiple Objects from Views
Recall that whenever a leaf MBR $m$ is accessed by BINL, every function $f_g$ in $F_g$ first examines whether $m$ can be pruned by the candidate set of $f_g$, according to Lemma 1 (see Section 4). However, the objects being accessed from views are unavoidably evaluated by the functions in BLPTA. For the sake of batch pruning, we fetch a set of objects from a view instead of one object at each access. In order to have stable performance at different data distributions, we stop fetching objects from a view if the volume of the accessed objects’ MBR is larger than a threshold $\omega$. In addition, we apply the same pruning idea as BINL, i.e., not every object is necessarily evaluated by the functions, improving pruning effectiveness.

5.5 Putting All Together
We are now ready to present our ETA algorithm (Efficient adaptation of the Threshold Algorithm), which integrates all techniques been discussed. ETA first partitions the functions into groups; each of group is bounded by a corresponding set of $d$-bounding views (see Section 5.2). Given the execution order of the groups (see Section 5.3), we evaluate the functions in batch using the corresponding $d$-bounding views. At every iteration, for each group, we access the views in a round-robin fashion. At each access, we fetch multiple objects from the views, until the MBR $m$ of them has a larger volume than $\omega$ (see Section 5.4). Subsequently, we update the cross point $\phi$ of $d$ scanning hyperplanes (see Section 5.1).

In the next step, we examine whether the objects belonging to $m$ should be examined by a function using the MBR pruning technique (see Lemma 1 in Section 4). Moreover, the result of a function $f$ is confirmed by the condition whether $f(\phi)$ is no better than the candidate set of $f$, and $f$ is marked as stopped in this case (see Section 5.1). The all top-$k$ results of a group are found as soon as all functions in the group are marked as stopped. Algorithm 4 is a detailed pseudocode for ETA.

Algorithm 4. ETA Algorithm
Algorithm ETA($V, P, F, k, \omega, \lambda$)
1: for all $f \in F$ do
2: $TOP^k(f) \leftarrow \emptyset$ and mark $f$ as running
3: $\Delta_G := \text{partitioning}(V, F, \lambda)$ $\triangleright$ Section 5.2
4: while $\Delta_G$ is not empty do

Fig. 8. Cost analysis.
5: $\Delta_i^{d-1} := \text{select}(\Delta_G)$; $\Delta_G := \Delta_G - \{\Delta_i^{d-1}\}$ $\triangleright$ Section 5.3
6: while $\Delta_i^{d-1} \cdot F$ is not empty do
7: for all $v \in \Delta_i^{d-1} \cdot V$ do
8: fetch objects until their MBR volume $V_m \geq \omega$ $\triangleright$ Section 5.4
9: compute cross point $\phi$ using $d$-scan hyperplanes $\triangleright$ Section 5.1
10: for all $f \in \Delta_i^{d-1} \cdot F$ do
11: if $f(m)$ is better than $k$-th score in $TOP^k(f)$ then
12: for all $p \in m$ do
13: if $f(p)$ is better than $k$-th score in $TOP^k(f)$ then
14: remove $k$-th object and insert $p$ into $TOP^k(f)$
15: if $f(\phi)$ is no better than $k$-th score in $TOP^k(f)$ then
16: mark $f$ as stopped and remove $f$ from $\Delta_i^{d-1} \cdot F$

In our implementation for ETA, we assume that the set of objects is indexed by a multidimensional access method and that the views are not precomputed and materialized. A view is computed on-demand using an off-the-shelf top-$k$ computation algorithm (e.g., BRS [3]). In order to reduce memory consumption of computed view rank lists, the memory held for a view is released after the view is no longer needed.

5.5.1 Cost Simulation Analysis
We observe that the benefit of tightening the views (i.e., minimizing $\lambda$) drops proportionally to the size of simplices in ETA. To demonstrate this, we propose a model that simulates the accessing cost for different view settings (i.e., represented by their angles) using 2D data. In Fig. 8a, we illustrate two different views ($v_1$ and $v_2$) where their angles to the top horizontal line are $\theta_1$ and $\theta_2$, respectively. For the sake of analysis, we assume that the objects are uniformly distributed in the domain area. Based on this assumption, the scanned area $a(m)$ of accessing a specific number of objects (i.e., $m$ objects) is the same for any view/ function (i.e., $a_1(m) = a_2(m)$).

Given the scanned area $a(m)$, the cross point $\phi$ of the scanning hyperplanes (computed by $a(m)$), and a bounded user preference function $\theta$, we can calculate the minimum accessed distance $D_{\text{min}}(a(m), \theta, \phi)$ of the user preference in the unseen region by LPTA. Besides, given the angle $\theta$ and
the scanned area $k/N$ of a view/function, we can compute the accessed distance, $D(k/N, \theta)$, by simple geometry.

To determine the cost for a specific user function $f$ (i.e., represented by $\theta$), we need to count the number of accessed objects $m$ from each view such that the top-$k$ score is not worse than maximum possible score (i.e., minimum accessed distance). This can be modeled by $D_{\min}(a(m), \theta, \phi) \leq D(k/N, \theta)$, where $m$ can be calculated given the values of $k, \theta,$ and $\phi$.

Fig. 8b shows our simulation result as a function of $\lambda$ where the value of $\theta$ and $\phi$ can be derived by $\lambda$ and $k$ is set to 20. The relative cost decreases as $\lambda$ decreases; however, when $\lambda$ is smaller than 0.02, the benefit of further partitioning of the simplices is not significant. On the other hand, we have to compute more views if we decompose more simplices. It is clear that we should stop our simplex partitioning at some point by considering the tradeoff between these two factors. In this work, we set $\lambda$ to 0.02 based on both numerical and experimental analyses (see Section 7).

6 EFFICIENT REVERSE TOP-K AND TOP-M INFLUENTIAL COMPUTATION

In this section, we show how we can use our ATOP$^k$ algorithms to facilitate the evaluation of reverse top-$k$ RTOP$^k(p)$ and top-$m$ influential ITOP$^m_k$ queries. First, we briefly review the state-of-the-art solutions to these problems from [8] and [9].

6.1 State-of-the-Art RTOP$^k$ Solution

Given a set of objects $P$ and a set of preference functions $F$, the reverse top-$k$ query of an object $p \in P$ returns the subset of $F$ that contains $p$ in their top-$k$ result. A naïve method computes a reverse top-$k$ query by evaluating the preference functions one by one. Vlachou et al. [8] proposed evaluating the functions in a given order. Intuitively, the top-$k$ results are similar (or exactly the same) if two functions, $f_i$ and $f_j$, are very close. In other words, if $f_i$ does not have $p$ in its top-$k$ result, then most probably it is not in $f_j$’s top-$k$ either. Therefore, we can skip the evaluation of $f_j$ if $f_i(p) < \max_{p \in TOP^k(f_j)} f_j(p)$ since $p$ is ranked worse than at least $k$ other objects. This method is termed Reverse top-$k$ Threshold Algorithm (RTA) in [8]. However, this process might evaluate all functions, in the worst case.

We demonstrate the reverse top-$k$ computation in Fig. 9a. Given the execution order based on cosine similarity (i.e., $f_a, f_b, f_c$) and $k = 3$, we want to answer RTOP$^k(p_3)$. According to the given order, we first evaluate $f_a$ where the top-$k$ result is $\{p_3, p_1, p_4\}$ and find that $f_a$ is not in the reverse top-$k$ set of $p_3$. Before we evaluate next function $f_b$, we first apply $f_a$ on $f_a$’s top-$k$ set and compute a threshold $\theta = \max \{f_a(p_3), f_a(p_1), f_a(p_4)\}$. In this example, $f_a(p_3) < \theta$, which indicates that $f_a$ is not the reverse top-$k$ of $p_3$ and needs not be evaluated. On the other hand, $f_b(p_3) \geq \theta$, therefore $f_b$ has to be evaluated.

1. Closeness can be measured by a cosine function.

6.2 State-of-the-Art ITOP$^m_k$ Solution

Given a set of objects $P$, a set of functions $F$, and $k$, the top-$m$ influential query returns the $m$ objects that have the highest influence scores, defined by the size of RTOP$^k(p)$. A straightforward solution is to evaluate a reverse top-$k$ query for each object. Note that each reverse top-$k$ query is evaluated by multiple top-$k$ queries. The cost becomes too high if $F$ and $P$ are large. In [9], a technique that estimates the maximum possible influence score $U(q)$ of an object $q$ is proposed. This can be computed by

$$U(q) = |\cap_{q \in CDS(q)} RTOP^k(p_i)|,$$

where $CDS(q)$ is the constrained dynamic skyline of $q$ (see Definition 8).

Definition 8 (Constrained dynamic skyline set). Given a set of objects $P$ and an object $q$, we denote as $P_c \subseteq P$ the set of all objects $p_r$ such that $\forall_{r=1}^m : q[j] \leq p_r[j]$. An object $p_r \in P_c$ belongs to the constrained dynamic skyline set $CDS(q)$ of object q, if it is not dynamically dominated with respect to q by any other point $p' \in P_c$.

$CDS(q)$ finds a set of dynamic skyline objects in the region being constrained by $q$; this region is bounded from $q$ towards the best point $(1, \ldots, 1)$. In the example of Fig. 9b, suppose $k$ is set to 3, $CDS(p_5)$ contains $p_1, p_2$ and $U(p_5) = 2$ ($=|\{f_a, f_b\} \cap \{f_a, f_b, f_c\}|$).

Assuming that $P$ is indexed by a multidimensional access method, we can traverse the objects $p_i \in P$ in decreasing order of $U(p_i)$. Similar to other branch-and-bound (BB) processing techniques (e.g., [3]), the first $m$ deheaped objects are the result of the query. This BB algorithm is the best approach in [9] and it is much faster than the straightforward solution. However, BB essentially executes a large amount of top-$k$ queries indirectly, since every reverse top-$k$ query is evaluated by a set of top-$k$ queries.

6.3 Using All Top-$k$ Computation

In this section, we study how we can use ATOP$^k$ to evaluate RTOP$^k$ and ITOP$^m_k$. We also discuss why our approach is superior to the state-of-the-art solutions. RTOP$^k$ using ATOP$^k$. After having computed an ATOP$^k$, we have the top-$k$ results of all functions. For the objects and functions of Fig. 9, the ATOP$^k$ results are shown in Fig. 10a. By “inverting” this table, as shown in Fig. 10b we can obtain the reverse top-$k$ sets of all objects. Thus, any RTOP$^k$ query can be answered easily by fetching a row in the inverted table. The space requirement is only $O(|F| \cdot k)$. 

![Figure 9. Examples of other queries.](image-url)
**Algorithm 5.** Top-\(m\) influential query using ATOP\(k\):

1. for \(\forall p \in P\), \(I^k(p) \leftarrow 0\) \> Initialize influence scores
2. \(ATOP^k \leftarrow \) run all top-\(k\) computation
3. for all \(f \in F\) do
4. for all \(p \in ATOP^k[f]\) do \> \(ATOP^k[f] \equiv TOP^k(f)\)
5. \(I^k(p) \leftarrow I^k(p) + 1\)
6. return the top-\(m\) objects \(p\) with respect to \(I^k(p)\)

**Discussion.** In [8] and [9], many top-\(k\) queries are evaluated if \(P\) and \(F\) are large, while in [9] multiple reverse top-\(k\) queries are executed and some of them may even share the same top-\(k\) queries, which are evaluated multiple times in this case. For a fair comparison, we implemented an optimized version of BB, named Optimized Branch-and-Bound algorithm (OBB), which caches the results of previously issued top-\(k\) queries and reuses them if necessary. Still, as we show in Section 7, OBB is much slower than our "ITOP\(^m\) using ATOP\(^k\)" approach.

### 7 Experimental Evaluation

According to the methodology in [4], we generated three types of data sets, independent (IND), correlated (COR), anticorrelated (ANT). In IND data sets, the feature values are generated uniformly and independently. COR data sets contain objects whose values are correlated in all dimensions. ANT data sets contain objects whose values are generated uniformly and independently.

**Table 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(</td>
<td>P</td>
</tr>
<tr>
<td>(</td>
<td>F</td>
</tr>
<tr>
<td>Dimensionality (d)</td>
<td>2, 3, 4, 5, 6</td>
</tr>
<tr>
<td>Data distribution for (P)</td>
<td>IND, ANT, COR, CLU</td>
</tr>
<tr>
<td>Data distribution for (F)</td>
<td>IND, CLU</td>
</tr>
<tr>
<td>(k)</td>
<td>2, 5, 10, 20, 40, 80</td>
</tr>
<tr>
<td>(m)</td>
<td>2, 5, 10, 20, 40, 80</td>
</tr>
<tr>
<td>BINL grouping ratio, (\delta)</td>
<td>0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1</td>
</tr>
<tr>
<td>ETA splitting ratio, (\lambda)</td>
<td>0.001, 0.002, 0.005, 0.01, 0.02, 0.05, 0.1, 0.2, 0.5, 1</td>
</tr>
<tr>
<td>ETA volume of accessed objects’ MBR, (\omega)</td>
<td>(10^{-7}, 10^{-6}, 10^{-5}, 10^{-4}, 10^{-3}, 10^{-2}, 10^{-1})</td>
</tr>
</tbody>
</table>
ETA has two parameters, $\lambda$ and $\omega$, and its cost is affected by both of them. We investigated how various values of these parameters affect the cost. Here, we plot the cost of ETA as a function of one parameter ($\lambda$ or $\omega$) while setting the other to the default value. Based on the result, we choose $\lambda = 0.02$ and $\omega = 10^{-4}$ that show robust performance at any dimensionality.

Scalability experiments. In this set of experiments, we demonstrate the superiority of our all top-$k$ methods, BINL (Section 4) and ETA (Section 5.5) compared to the naïve approach and a simple skyline-based solution (Skyband). The naïve approach evaluates the top-$k$ queries one-by-one using BRS [3]. Skyband first collects the objects in the $k$-skyband (using BBS [5]) and then evaluates the top-$k$ queries one-by-one over it.

Fig. 12a shows the response times of the four methods as a function of dimensionality $d$, after setting all other parameters to their default values. Cost grows exponentially with $d$ for all methods. ETA is at least 8, 2.5, and 1.5 times faster than Naïve, Skyband, and BINL, respectively, in all experiments. The skyline-based approach does not scale well with dimensionality due to the increasing number of objects in the skyband. For large values of $d$, the gap between BINL and ETA becomes smaller, because the MBRs that group multiple accessed objects in ETA becomes too large, reducing the effect of the MBR pruning technique.

Fig. 12b compares performance as a function of $k$. ETA is at least 8.6, 3.48, 2.1 times faster than Naïve, Skyband, and BINL, respectively. All methods are sensitive to $k$ since the problem becomes harder as $k$ increases.

The response times for different numbers of products $|P|$ are shown in Fig. 12c. The cost is not very sensitive to $|P|$ since the products are indexed and we only need to access a small fraction of the data.

Fig. 12d shows the response time of all methods for different numbers of functions $|F|$. The response time increases linearly with $|F|$, since there are more top-$k$ queries being evaluated. Our ETA still performs the best, followed by BINL, Skyband, and Naïve.

Data distribution. As shown in Fig. 13a, ETA is at least one order of magnitude faster than Naïve and 2.6 times faster than BINL for different data distributions of $P$ and independently distributed $F$. ANT distributed objects are the hardest case since top-$k$ computation becomes hard in this case. Interestingly, the gap between ETA and the other methods widens in this case. One of the reasons is that our $d$-bounding views partitioning technique provides better grouping than the Hilbert curve grouping. We also evaluated our methods for the CLU $F$ where we generate the functions coefficients in clusters. As shown in Fig. 13b, ETA is again the best method which is at least one order of magnitude faster than Naïve and 2.6 times faster than BINL. We conclude that ETA is the best method for all distribution combinations.

Fig. 13c plots the response time of all methods on the NBA real data set. We instantiated $P$ from this data set (12,278 records) and set other parameters to their default values. Again, ETA is consistently better than Naïve and BINL for all values of $k$. Summing up, ETA is the best solution for ATOP queries, typically being one order of magnitude faster than Naïve solution and 2-3 times faster than BINL.
In Fig. 13d, we demonstrate the response time of all methods using another real data set, Household. We instantiated \( P \) from the Household data set (including 3.6M records). We divided Household into four data sets with 516K, 514K, 1.25M, and 1.35M records from years 2003, 2004, 2005, and 2006, respectively. The feature values in Household are discrete, so there are some tuples having the same feature values in all dimensions; in this case the objects are grouped to a single capacitated object. The number of different discrete objects are 242K, 250K, 520K, and 542K, respectively, in the four years, while there are 1.55M different ones in total. We demonstrate the response time of all three methods of the data in these four years as well as the whole data in Fig. 13d. ETA again performs best in all cases, being at least 36 and 4 times faster than Naïve and BINL, respectively. Even though the cardinality of the entire data set (ALL) is several times larger than that of the yearly data sets, the response time does not increase much, being consistent with the trends in Fig. 12c.

**I/O cost and peak memory usage.** Figs. 14a and 14b show the I/O cost and peak memory usage\(^2\) of all three methods as a function of dimensionality \(d\), after setting all other parameters to their default values. The I/O costs of all three methods (Naïve, BINL, and ETA) grow exponentially with the dimensionality. This result is consistent with the corresponding response time experiment (Fig. 12a); ETA accesses several times to two order of magnitude fewer pages than other two methods. However, ETA may use more memory than Naïve and BINL since each view keeps some data structures for incremental top-k computation; still the required memory is not excessive. Also, if the execution order is randomly selected (ETA-NoOrder), then the execution consumes 3.89 times more memory than ETA at \(d = 6\).

**Reverse top-k and top-m influential computation.** We now demonstrate the use of \( ATOP^k \) queries in the computation of reverse top-k and top-m influential queries. For these two problems, we compare the state-of-the-art solutions [8], [9] to the \( ATOP^k \)-based alternatives that we introduced in Section 6.3.

For reverse top-k queries, we plot the response time of \( ATOP^k \)-based reverse top-k search using ETA versus the average response time of \( RTOP^k \) processing using RTA [8]. The queries are selected randomly from \( k \)-skyband objects in order to avoid meaningless results (i.e., no user function considers the query in their top-k result). As shown in Fig. 15a, RTA is only 2.6 to 13.2 times faster than ETA when dimensionality \(d\) varies from 2 to 6. However, ETA computes the all top-k result which can be used to answer any reverse top-k result (see Fig. 10b). In other words, if we are to execute more than 13\( RTOP^k \) queries in \( d = 6\), ETA should be preferred to RTA, because the total response time will be better in this case. Thus, RTA is not appropriate in settings where multiple reverse top-k queries are to be executed. Comparing the two queries for different values of \(k\) (Fig. 15b) leads to similar conclusions.

For top-m influential queries, we compare our \( ITOP_m \) using \( ATOP^k \) (ITOP-ATOP) approach (see Section 6.3) to the state-of-the-art solution BB and its optimized version OBB (as discussed in Section 6.3). Fig. 16a shows the response time for these methods as a function of \( k \). As \( k \) increases, OBB becomes much better than original BB since OBB caches the results of previous top-k computations. However, OBB is still 17 times slower than our ITOP-ATOP approach, which performs an all top-k query and uses its result to evaluate the \( ITOP_m \) query. Fig. 16b shows how the cost is affected by \(m\). The response times of BB and OBB are linearly increasing with \(m\), because BB and OBB unavoidably compute more maximum possible influence scores when \(m\) becomes larger and this introduces additional reverse top-k queries. However, our approach is completely insensitive to \(m\) since we have already collected all necessary data for \( ITOP_m \) by an \( ATOP^k \) computation.

Fig. 17 shows some additional experiments on \( ITOP_k \) queries (varying dimensionality and data distribution). Our ITOP-ATOP method consistently beat other methods by 1 to 2 orders of magnitude for various values of \(d\). In addition, for different distributions of \(P\) for IND \(F\), our method greatly outperforms BB and OBB, especially in the ANT case where BB and OBB take 2,827 and 607 seconds, respectively, while ITOP-ATOP runs in only 0.64 seconds.

In summary, running an all top-k query using our best method ETA is a much better alternative that repetitive executions of RTA if multiple reverse queries are to be evaluated. In addition, evaluating an all top-k query using ETA and using its result to evaluate an \( ITOP_m \)
query is 1-2 orders of magnitude faster than the state-of-the-art method proposed in [9], even if this method is optimized to reuse cached results of top-$k$ queries.

8 RELATED WORK

8.1 Top-$k$ Queries

Top-$k$ queries [1], [2], [3] provide a convenient way for users to find important objects according to their preferences. In [2], a threshold algorithm has been proposed to combine object ranks from different sorted lists with a help of an aggregate function $f$. TA scans the lists sequentially, in a round-robin fashion, and computes the aggregate score of each encountered object, while maintaining the top-$k$ set. As soon as the aggregate scores of the remaining objects cannot exceed the top-$k$ scores found so far, TA terminates. Due to its popularity, many variants of TA have been proposed (e.g., [1]). Onion [20] and PREFER [21], [22] are two top-$k$ methods which rely on pre-processing techniques. Onion [20] pre-computes convex hull layers of the data and processes linear top-$k$ queries by scanning objects incrementally, from exterior layers to interior layers. Onion stops when it is guaranteed that the remaining layers cannot contain any other results. The high complexity of convex hull computations ($O(n^d/2)$ in $d$-dimensional space) makes Onion too expensive to be used in practice. Also Onion cannot be used when data set is frequently updated because recomputations of convex hulls are needed in this case. PREFER [21], [22] first generates materialized views; a top-$k$ query is answered by scanning the views with most similar preferences to the query. An algorithm to determine the best views to be materialized when top-$k$ queries are pipelined is proposed. However, as demonstrated in [22], we need to materialized many views before we can ensure satisfactory performance. In addition, similar to Onion, PREFER is only suitable for static data. The performance of Onion can be improved with the use of indexing [23]; however, building robust indexes is quite expensive.

BRS [3] is a branch-and-bound approach for answering top-$k$ queries over a set of objects that are indexed by an R*-tree. BRS uses a heap to maintain candidate entries, traversing the R*-tree in a top-down manner. At every iteration, BRS fetches the best entry from the heap. If the entry is a leaf entry of the R*-tree, then it is output as the next result in the ranking; the algorithm stops if we have enough results. If the entry is in an intermediate node, then the corresponding node is accessed and for each of its entries $e$ a max score is computed and $e$ is inserted into the heap. As shown in [3], BRS is an I/O optimal algorithm, meaning that it accesses only the tree nodes which may contain the top-$k$ results. Since max score is a general concept, this algorithm can be applied to both monotone and non-monotone preference functions.

Recently, a group recommendation problem has been studied in [24]. Given a group of people, a consensus relevance score function is used to model the interests and preferences of all group members. The score of an object is defined as a linear combination of group relevance and group disagreement. Using the monotonicity of relevance and disagreement, a TA-like algorithm is designed for top-$k$ processing. This paper shares the same intuition with our paper to recommend products to a group of users. However, we focus on providing different recommendations to different users based on their individual preferences, while the goal in [24] is to provide a consensus recommendation of all users. Another technical difference is that our methods are designed for computing multiple top-$k$ queries simultaneously for a large number ($\sim 10K$) of users, while the group size in [24] is very small ($\lt 10$). The proposed solution in [24] is obviously inapplicable to our problem.

8.2 Other Related Queries

There is plenty of work on skyline evaluation (e.g., [4], [5], [25]). The concept of skyline is based on the dominance relationship. The objective is to find the objects that are not dominated by others. The skyline operator was first proposed in [4]. Papadias et al. [5] proposed an incremental skyline algorithm that access a minimal number of nodes from an R*-tree that indexes the data. An object-based space partitioning method that provides efficient skyline computation in high dimensional spaces was proposed in [25].

Several novel types of queries have been proposed recently to assist the analysis tasks of product manufacturers. Li et al. [26] was the first paper to use the concept of dominance for business analysis from a microeconomic perspective. A data cube model (DADA) is proposed to summarize the dominance relationships between objects in all combinations of dimensions. The space is modeled by the grid (i.e., matrix) of dimensional value combinations (assuming that features have small integer domains) and each cell summarizes the dominance of products in it. In [6], the problem of creating competitive products have been studied. In [7], Wu et al. aim at finding the best subspace for a query object where it is highly ranked. Miah et al. [27] studied an optimization problem that selects a subset of attributes of a product $t$ such that $t$’s shortened version still maximizes $t$’s visibility to potential customers.

9 CONCLUSION

In this paper, we studied the problem of batch evaluation of numerous top-$k$ queries. To our knowledge, this is the first thorough study for this problem. We proposed two batch processing techniques; the first is a batch indexed nested loops approach and the second is a view-based threshold algorithm with a set of optimization techniques, including $d$-bounding views, simplex partitioning, and batch objects accessing. We demonstrated that ATOP queries can be used to boost the performance of reverse top-$k$ and top-$m$ influential queries. In the future, we plan to study alternative techniques for
queries that employ parallel processing. Moreover, we intend to study additional queries that can make use of \( ATOP^k \) as a module.

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**REFERENCES**


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