

## EFFECTS OF INTER-PARTICLE FRICTION ON THE CRITICAL STATE BEHAVIOR OF GRANULAR MATERIALS: A NUMERICAL STUDY

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**Key words:** Granular materials, DEM, Inter-particle friction, Critical state.

**Abstract.** Critical state soil mechanics (CSSM) gives a theoretical framework for soil modelling. This paper investigates the effect of inter-particle friction on the critical state behavior of DEM assemblies with grading representative of a real soil. It is found that the angle of shearing resistance at the critical state increases with increasing inter-particle friction ( $\mu$ ) and in void ratio (e) vs logarithmic mean effective stress ( $\log(p')$ ) space the critical state locii have higher e values when  $\mu$  is higher. An atypical CSL in e-log $p'$  space that deviates from experimental observations and the classical CSSM behavior was observed when  $\mu = 0.5$ . Micro-scale analyses show that this can be attributed to the emergence of a higher number of floating particles due to the increasing self-stability of strong force chains with increasing inter-particle friction. This study recommends the use of inter-particle friction lower than 0.5

in DEM simulations of element testing in soil mechanics.

## 1 INTRODUCTION

The basic concept of critical state soil mechanics (CSSM) is that soils when sheared to large strain levels will deform continuously without a change of volume and stress state<sup>[1]</sup>. There is a unique relationship between the void ratio ( $e$ ) and the logarithm of the mean effective stress ( $\log(p')$ ) at the critical state which is defined as the critical state line or locus (CSL). The mechanical behavior of soils has been found to depend on the relative position of their current states ( $e$  and  $p'$ ) with respect to their critical states<sup>[2]</sup>. CSSM serves as a fundamental concept in soil modelling and has been widely applied in constitutive modelling. These CSSM concepts were developed based on macro-scale experimental observations and CSSM-based constitutive models are routinely used in continuum soil mechanics analysis. However, as soil is particulate system, its macro response depends on the particle-scale properties and interactions.

Sliding, rolling and buckling are the three major particle-rearrangement patterns that govern the macro-mechanical responses of granular materials. The resistance to sliding of particles against each other largely depends on the inter-particle friction. It has been observed in both physical experiments<sup>[3]</sup> and numerical studies using distinct element method (DEM)<sup>[4]</sup> that the inter-particle friction plays an important role in the behavior of granular materials, e.g., the dilatancy rate, peak and residual strength increase with increasing inter-particle friction<sup>[5,6,7]</sup>.

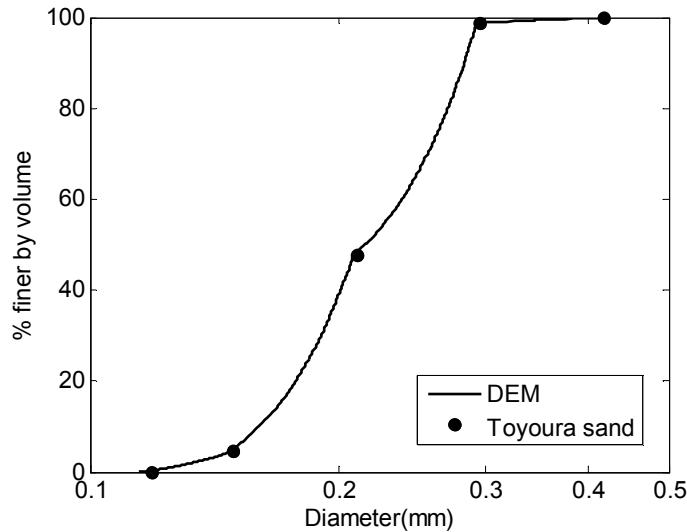
Previous studies of the influence of inter-particle friction coefficient ( $\mu$ ) on the critical state only focused on the angle of shearing resistance at the critical state<sup>[5,6]</sup>, while the role of inter-particle friction on the critical-state in terms of void ratio ( $e$ ) and mean effective stress ( $p'$ ) was neglected but should be of more interest as it governs the deformation path from initial state to critical state. Previous studies also considered only a single case for each inter-particle friction coefficient and thus could not define a CSL. This study extends the previous work of Thornton<sup>[5]</sup> and Yang et al.<sup>[6]</sup> to study the effect of inter-particle friction on the critical state behavior of a granular material with a realistic grading. In particular, the effect of inter-particle friction coefficient on the CSL in  $e$ - $\log(p')$  space is discussed. The micro-scale explanations for the effect of inter-particle friction on the macro response are explored by evaluating the mechanical redundancy and structural anisotropy of the particulate system.

## 2 DEM SIMULATIONS

The simulations were run using the PFC3D DEM software<sup>[8]</sup>. The particle size distribution (PSD) used is shown on Figure 1 and is representative of the Japanese standard sand, Toyoura sand<sup>[9]</sup>. The simplified Hertz-Mindlin contact model was used with particle shear modulus ( $G$ ) and Poisson's ratio ( $\nu$ ) taken to be 29 GPa and 0.12 respectively (the properties of quartz given by Simmons and Brace<sup>[10]</sup>). Gravity was not simulated and the rigid walls were frictionless.

A non-contacting cloud composed of between 16,073 and 16,578 particles were first generated randomly within a cylindrical domain at half of their target sizes. The particles were then uniformly expanded to their final sizes and cycled to equilibrium. A servo control

mechanism was then introduced to adjust the positions of the rigid walls continuously until a stable isotropic stress state was attained. These isotropic assemblies were subjected to different loading paths (conventional drained compression (CDC), constant volume compression (CVC) or constant mean effective stress ( $p'$ ) compression (CPC)) until the critical state was reached. Two inter-particle friction coefficients ( $\mu = 0.25$  and  $0.5$ ) were used during shearing. The simulations carried out in this study are summarized in Table 1.



**Figure 1:** Comparison between the particle size distributions of DEM assemblies and Toyoura sand

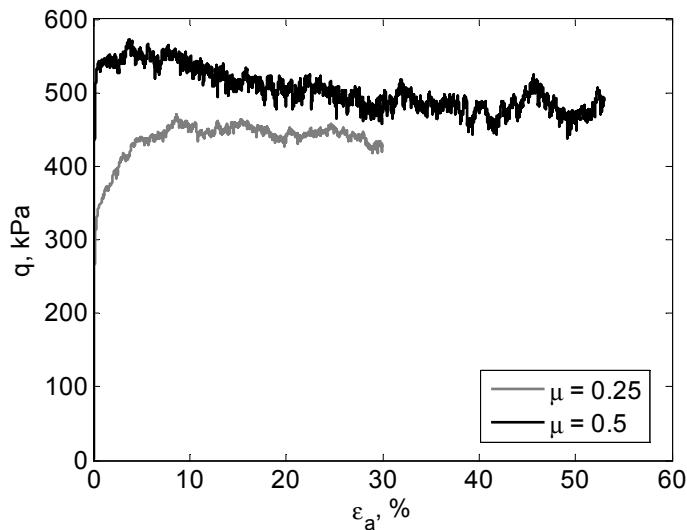
**Table 1:** Simulation summary

ID	Initial void ratio $e_0$	Initial confining pressure $p'_0$ (kPa)	Loading path	$\mu$
0.627-5000-CDC	0.627	5000	CDC	0.5
0.645-1000-CDC	0.645	1000	CDC	0.5
0.648-500-CDC	0.648	500	CDC	0.5
0.651-100-CDC	0.651	100	CDC	0.5
0.581-5000-CPC	0.581	5000	CPC	0.25
0.596-1000-CPC	0.596	1000	CPC	0.25
0.601-100-CVC	0.601	100	CVC	0.25
0.648-500-CDC	0.648	500	CDC	0.25
0.648-500-CVC	0.648	500	CVC	0.25

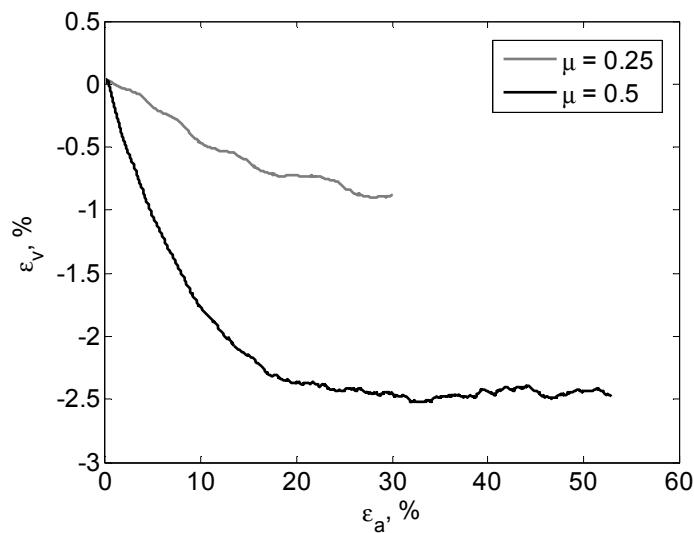
### 3 MACRO STRESS STRAIN BEHAVIOR

In this study, the conventional definition of the deviatoric stress ( $q$ ) and mean effective stress ( $p'$ ) in soil mechanics is adopted.  $q$  is defined as the difference between the major and minor principal stresses ( $q = \sigma'_1 - \sigma'_3$ ), while  $p'$  is the mean value of the three principal stresses ( $p' = (\sigma'_1 + \sigma'_2 + \sigma'_3)/3$ ). Compressive volumetric strains are taken to be positive.

Figure 2 illustrates the effect of inter-particle friction on the macro stress-strain response. The two samples had the same initial state ( $e_0 = 0.648$ ,  $p'_0 = 500$  kPa) and were sheared under conventional drained condition to large strain levels with inter-particle friction coefficients of 0.25 and 0.5. As shown in Figure 2 (a), the sample with  $\mu = 0.5$  is stiffer and yields a high peak as well as residual strength than the sample with  $\mu = 0.25$ . Figure 2 (b) shows that both samples contract initially before dilating until the volumetric strain becomes constant, i.e., the critical state is attained. The sample with  $\mu = 0.5$  is more dilative than the sample with  $\mu = 0.25$ .



(a) Evolution of deviatoric stress



(b) Evolution of volumetric strain

**Figure 2:** Effect of inter-particle friction on the stress-strain response

## 4 CRITICAL STATE OBSERVATIONS

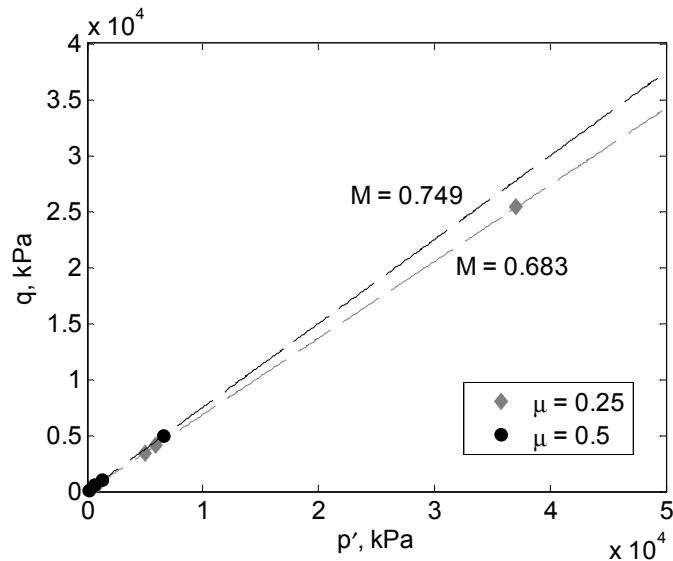
### 4.1 Macro-scale observations

The CSL in  $q-p'$  space is illustrated in Figure 3. The critical state locus in terms of  $q$  and  $p'$  can be represented by a unique linear relationship with slopes  $M$  equaling to 0.749 and 0.683 for  $\mu = 0.5$  and  $\mu = 0.25$  respectively, regardless of the initial state and loading conditions. The slope  $M$  is related to the angle of shearing resistance at the critical state by the following equation:

$$\varphi'_{cv} = \sin^{-1}\left(\frac{3M}{6 + M}\right) \quad (1)$$

Therefore the angles of shearing resistance are  $19.4^\circ$  and  $17.9^\circ$  for  $\mu = 0.5$  and  $\mu = 0.25$  respectively, i.e., the angle of shearing resistance at the critical state increases with increasing  $\mu$ . This is in line with the previous studies based on a single case <sup>[5,6]</sup>.

Figure 4 shows the critical state relationships in  $e-\log(p')$  space. The void ratio is taken as the void ratio within the homogeneous interior zone as suggested by Huang et al. <sup>[11]</sup>. The critical states of  $\mu = 0.5$  case are located above those of  $\mu = 0.25$  case, implying that samples with a higher  $\mu$  are less compressible or more dilative than samples with a lower  $\mu$ . For  $\mu = 0.5$  case, the critical state void ratio increases with increasing  $p'$ , while for the  $\mu = 0.25$  case, the critical state void ratio decreases with increasing  $p'$  and the critical state locus can be represented by a unique relationship. The increase of critical-state void ratio with increasing  $p'$  when  $\mu = 0.5$  contradicts both the laboratory observations and the basic concept of classic CSSM. The reason for this atypical phenomenon will be discussed in section 4.2.



**Figure 3:** Effect of inter-particle friction coefficient on the CSL in  $q-p'$  space

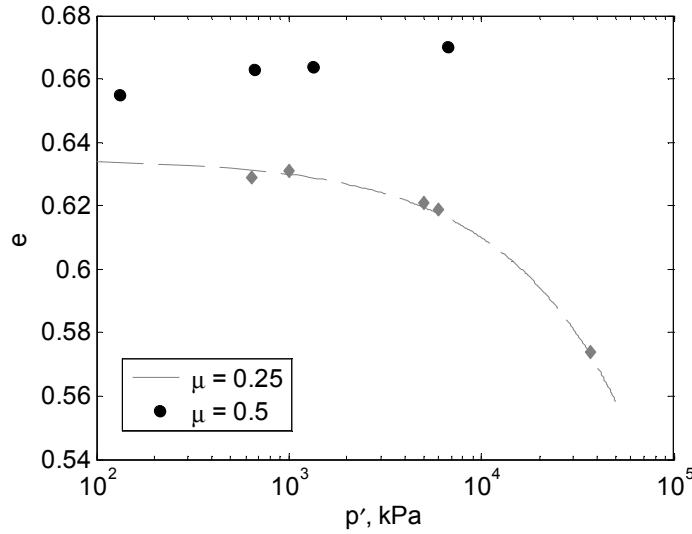
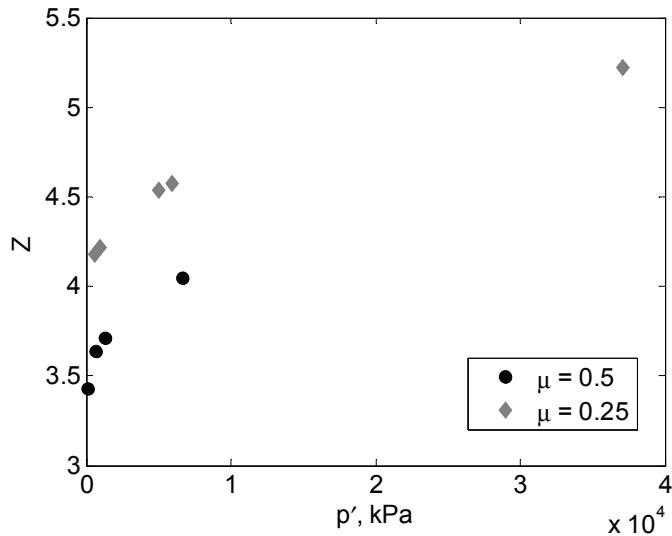


Figure 4: Effect of inter-particle friction coefficient on the CSL in  $e$ - $\log p'$  space

#### 4.2 Micro-scale analysis

The coordination number ( $Z$ ) is defined as the average number of contacts per particle and is given by  $2N_c/N_p$ , where  $N_c$  and  $N_p$  are the number of contacts and particles within the sample, respectively.  $Z$  is a micro-scale indicator of packing density. A higher  $Z$  indicates a tighter packing. The relationship between  $Z$  and  $p'$  at critical state is shown in Figure 5. It is clear that  $Z$  increases with increasing  $p'$  in both high and low  $\mu$  cases, indicating that particles are more closely packed under higher stresses. Given the same stress level,  $Z$  is higher in the  $\mu = 0.25$  case than in the  $\mu = 0.5$  case. This has two implications: the sample with low inter-particle friction is more compressible than the comparable sample with a high inter-particle friction; and the particles are more stable when the inter-particle friction increases and thus fewer lateral supports are needed. The former one is in accordance with the macro observations as described in section 4.1. To explain the latter implication, we firstly consider the extreme case where  $\mu = 0$ . In this frictionless situation ( $\mu = 0$ ), for a 3D system a single particle has only three translational degrees of freedom (the rotational degree of freedom can be ignored as there is no driven force in the tangential directions to cause the rotational movement). Therefore at least three constraints are needed to maintain the kinematic and mechanical stability of a single particle. In the frictionless situation, a single contact can only provide a restriction to the relative movement which is along the contact normal direction, i.e., there is only one constraint direction. Therefore at least three contacts are needed for a single particle to be stable. In this case, the minimum coordination number in frictionless case is 6 ( $2N_c/N_p = 2 \cdot 3/1=6$ ). When the inter-particle friction is increased and  $\mu > 0$  the relative sliding movement between particles at the contact is resisted and thus the number of contacts for a single particle to maintain stability decreases.



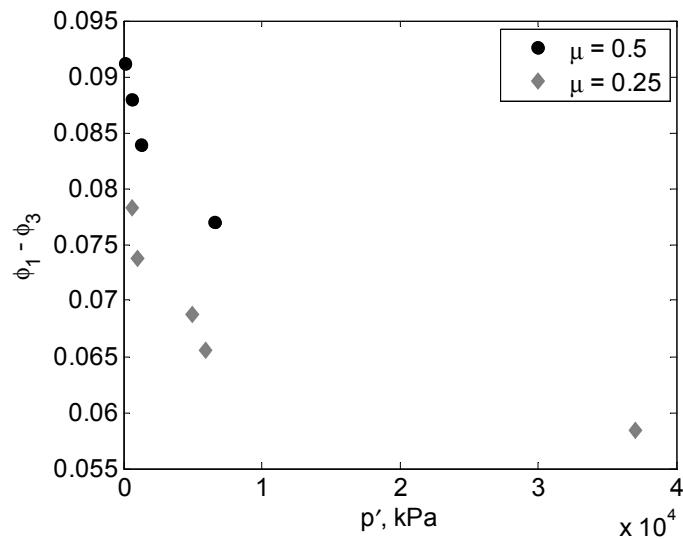
**Figure 5:** Relationship between Z and  $p'$  at critical state

The enhancement of self-stability by increasing  $\mu$  can also be speculated from the structural anisotropy at the critical state. The fabric anisotropy can be quantified using the second-order fabric tensor proposed by Satake <sup>[12]</sup> as follows:

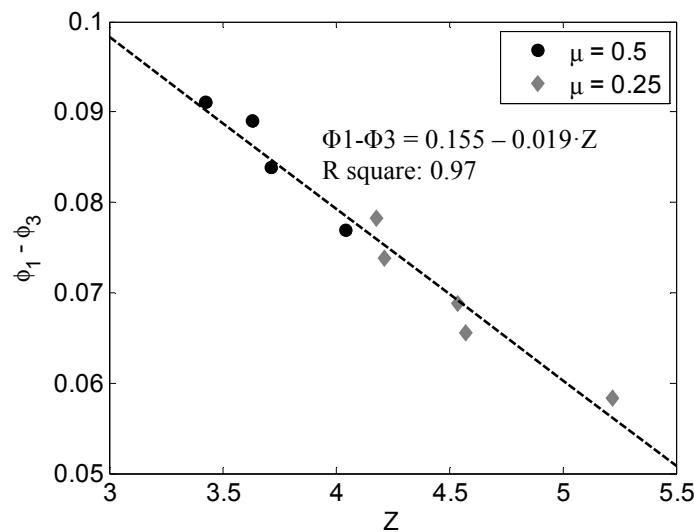
$$\Phi_{ij} = \sum_{k=1}^{N_c} n_i^k n_j^k \quad (2)$$

where  $n_i^k$  denotes the unit contact normal. The deviatoric fabric  $\Phi_d$  is the difference between the maximum eigenvalue ( $\Phi_1$ ) and the minimum eigenvalue ( $\Phi_3$ ) of the fabric tensor and it has been widely used to quantify the structural anisotropy of granular assemblies <sup>[5,7]</sup>. Figure 6 illustrates the relationship between  $\Phi_d$  and  $p'$  at the critical state. Obviously, the structural anisotropy decreases with increasing mean effective stress for both  $\mu = 0.5$  case and  $\mu = 0.25$  case. The sample with a high  $\mu$  is more anisotropic than the sample with a low  $\mu$  corresponding to the same  $p'$ , suggesting that the strong contact force network in high  $\mu$  case is more stable and thus less reliant on the propping by an orthogonal network. Thus, samples with a higher  $\mu$  can sustain a higher magnitude of deviatoric loading than samples with a lower  $\mu$  as shown in Figure 2 (a).

The relationship between the deviatoric fabric and coordination number at critical state is illustrated in Figure 7. The structural anisotropy decreases with increasing coordination number in both low and high  $\mu$  case. It is worth noticing that the data collapse to a linear relationship with a high confidence ( $R^2 \approx 0.97$ ) irrespective of the  $\mu$  values and loading conditions. A similar observation has been made by Kruyt <sup>[13]</sup> who proposed that this relationship has a geometrical origin. This relationship (intercept and slope) may depend on the particle size distribution as the relative size between contacting particles may affect the arrangement of particles and the orientation of contact normals as well.



**Figure 6:** Relationship between the deviatoric fabric and  $p'$  at critical state

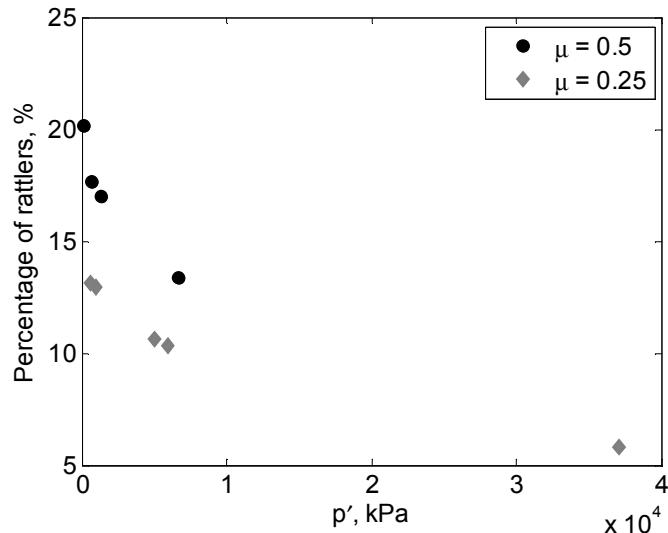


**Figure 7:** Relationship between structural anisotropy and coordination number at critical state

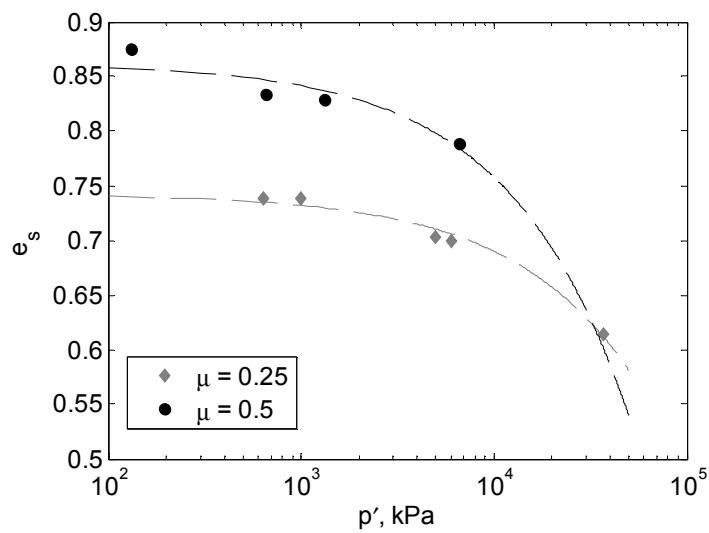
The number of rattlers (particles with < two contacts) is a direct indicator of the mechanical redundancy of a particulate system. Figure 8 plots the proportion of rattlers against  $p'$  at critical state. The proportion of rattlers is high at low stress levels and decreases rapidly with increasing  $p'$ . There are more rattlers in the  $\mu = 0.5$  case than in the  $\mu = 0.25$  case, suggesting that fewer particles are needed to maintain a stable force transmission network with increasing  $\mu$ , i.e., the particulate system has a higher degree of mechanical redundancy when  $\mu$  increases. These rattlers make no contribution to the overall force transmission. Therefore, following Thevanayagam and Mohan [14], the rattlers can be considered as part of the void space and an inter-granular void ratio ( $e_s$ ) can be defined as,

$$e_s = \frac{V_{void} + V_R}{V_s - V_R} \quad (3)$$

Where  $V_{void}$  is the real void volume,  $V_R$  is the volume of rattlers and  $V_s$  is the total volume of solids. Figure 9 plots the inter-granular void ratio ( $e_s$ ) against the mean effective stress ( $p'$ ). It is interesting to see that the atypical relationship between  $e$  and  $\log(p')$  vanishes and both the CSL for  $\mu = 0.5$  and  $\mu = 0.25$  case show a consistent trend with  $e_s$  decreasing with increasing  $p'$ .



**Figure 8:** Relationship between the percentage of rattlers and  $p'$  at critical state



**Figure 9:** CSL in terms of inter-granular void ratio and  $p'$

## 5 CONCLUSIONS

This study has investigated the effect of inter-particle friction coefficient ( $\mu$ ) on the critical state behavior of a granular material with a realistic grading. Macro-scale analysis exhibits that the sample with  $\mu = 0.5$  is stiffer and more dilative than the sample with  $\mu=0.25$ . The angle of shearing resistance at the critical state increases with increasing  $\mu$  and the locus of critical state points in e-log( $p'$ ) space for  $\mu = 0.5$  is above that for  $\mu = 0.25$ . While the major features of the critical state behavior of sands can be captured by DEM simulations with a low  $\mu$  (i.e. 0.25), the void ratio increases with increasing mean effective stress when  $\mu$  is high (i.e. 0.5) which is contradictory to the classic CSSM concept. Micro-scale analysis exhibits that the coordination number decreases while the structural anisotropy increases with increasing  $\mu$  and there is linear relationship between the structural anisotropy and coordination number at critical state which is independent of  $\mu$ . The use of high  $\mu$  leads to a high degree of mechanical redundancy which may be the origin of the weird macro response. Hence, in DEM simulations of element testing, the inter-particle friction coefficient should be lower than 0.5.

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