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Chaotic Speed Synchronization Control of Multiple Induction Motors Using Stator Flux Regulation

Zhen Zhang\(^1\), K. T. Chau\(^1\), and Zheng Wang\(^2\)

\(^1\)Department of Electrical and Electronic Engineering, The University of Hong Kong, Hong Kong  
\(^2\)School of Electrical Engineering, Southeast University, Nanjing 210096, China

This paper presents and implements a new control approach to synchronize the chaotic speed of multiple induction motors. The direct torque control (DTC) using the stator flux regulation (SFR) method is the key for the chaotic speed control. Additionally, a nonlinear control approach, namely the adaptive time-delayed feedback control, is proposed to regulate the reference torque for the SFR-based DTC unit. By using the proposed control scheme, multiple induction motors can achieve a synchronized chaotic speed within a controllable boundary.

**Index Terms**—Chaos, direct torque control, flux regulation, induction motor, motor drive, synchronization.

I. INTRODUCTION

INSTEAD of stabilizing chaotic behavior in electric motors [1], it is becoming attractive to positively utilize the chaotic behavior for high-performance applications. Namely, the chaotic pulse width modulation (PWM) can be utilized to suppress the annoying acoustic noise of PWM inverter drives, while the chaotic motion can be utilized to improve the energy efficiency and homogeneity of liquid mixing or soil compaction. Differing from random motion, chaotic motion exhibits random-like behavior but bounded, which is completely deterministic without involving any random element or stochastic process. Hence, the chaotic behavior is well bounded while its controller is much simpler and easier for implementation than the random type [2], [3].

Recently, chaos in motor drives has been actively researched. A chaotic PWM scheme was proposed to reduce electromagnetic interference in AC motor drives [4]. A design-oriented approach was developed to chaoize a doubly salient permanent magnet motor for soil compaction [5]. A time-delay feedback control (TDFC) approach was implemented to chaoize a DC motor for liquid mixing [6]. A control-oriented approach was proposed for the chaoization of permanent magnet synchronous motor [7].

However, the aforementioned studies have been focused on the chaoization of a single motor. The chaotic speed synchronization of multiple motors is absent in literature, which is an important issue in some industrial applications such as the drug production line. Because of the parameter differences in motors and the high sensitivity of chaos to the initial conditions, it is difficult to synchronize the chaotic speed of multiple electric motors. The purpose of this paper is to propose an adaptive time-delayed feedback control (ATDFC) method to produce a torque reference for the SFR-based DTC unit [8], hence synchronizing the chaotic speed of multiple induction motors (IMs). The proposed controller can offer the definite advantages of fast torque response and robustness.

II. STATOR FLUX REGULATION

In the stator coordinates, the electromagnetic model of the IM can be expressed as [9]

\[
\begin{align*}
\psi_s &= u_s - R_s i_s, \\
\dot{\psi}_r &= j\omega\psi_r - R_r i_r,
\end{align*}
\]

(1)

where \(\psi_s\) is the stator flux, \(\psi_r\) is the rotor flux, \(i_s\) is the stator current, \(i_r\) is the rotor current, \(R_s\) is the stator resistance, \(R_r\) is the rotor resistance, \(\omega\) is the rotating speed and \(u_s\) is the stator voltage. The electromagnetic torque \(T\) can be expressed as

\[
T = \frac{L_{sr}}{\sigma L_s L_r} \psi_s |\psi_r| \sin \delta
\]

(2)

where \(L_s\) is the stator self-inductance, \(L_r\) is the rotor self-inductance, \(L_{sr}\) is the mutual inductance, \(\delta\) is the angle between the stator flux and rotor flux, and \(\sigma = 1 - L_{sr}^2/L_s L_r\). Consequently, \(T\) can be directly controlled by regulating \(\psi_s\) [10]. When the value of \(\psi_s\) is regulated to be constant, \(T\) can be controlled by \(\delta\).

As shown in Fig. 1, the key of the SFR-based DTC unit is to estimate \(\psi_s\) and to calculate \(T\) by using the transformed currents and voltages. Firstly, the measured voltage and currents are decomposed into the orthogonal stationary \(\alpha\) and \(\beta\) axes, and \(\alpha\) axis aligns with the phase-A armature winding of the IM. Then, \(\psi_s\) can be represented by

\[
\psi_s = \int (u_s - R_s i_s) dt
\]

(3)

while \(T\) is given by

\[
T = \frac{3 \pi}{2} \left( \frac{\psi_s}{2} \times \dot{i}_s \right)
\]

(4)

where \(u_s = [u_{s\alpha}, u_{s\beta}]^T\) are the voltages on the \(\alpha\) and \(\beta\) axes respectively, and \(\dot{i}_s = [\dot{i}_{s\alpha}, \dot{i}_{s\beta}]^T\) are the currents on the \(\alpha\) and...
Fig. 1. Direct torque control using stator flux regulation.

Fig. 2. Selection of switching status for stator flux regulation.

\( \beta \) axes respectively. Then, the regulation of \( \psi_s \) is realized by selecting proper voltage space vectors.

According to the principle of switching voltage space vectors as shown in Fig. 2, the control signal of a 3-phase inverter can be determined by the location of \( \psi_s \) and the outputs of hysteresis loops for \( T \) and \( \psi_s \). The main advantages of this control are robustness and fast torque response. Thus, it is highly suitable for controlling fast-changing chaotic motion.

III. CHAOTIC SYNCHRONIZATION CONTROL

In this paper, the master-slave type is adopted for exemplification. Fig. 3 depicts the proposed chaotic speed synchronization control scheme. Based on the rotating speed difference \( \Delta \omega \) between the chaotic speed \( \omega_{\text{m}} \) of the master IM and the slave IM speed \( \omega_{\text{s}} \), the proposed ATDFC method can generate a torque reference \( \tau^* \). It is used for the SFR-based DTC unit to produce control signals for the inverter, hence driving the slave IM to track the chaotic speed of the master IM.

By applying the ATDFC method, the rotating speed difference of the master-slave IMs can be expressed as

\[
\dot{x}(t) = Ax(t) + Bu(t) \tag{5}
\]

where \( x(t) \) denotes the rotating speed difference of the master-slave IMs, \( A \) and \( H \) denote the coefficient matrices, and \( u(t) \) is the proposed controller which can be expressed as

\[
u(t) = K(t)[\Delta \omega(t) - \Delta \omega(t - \tau_c)] \tag{6}\]

where \( \tau_c \) denotes the delay time, and \( K(t) \) is the adaptive feedback gain. Hence, (5) can be written as

\[
\dot{x}(t) = Ax(t) + B\dot{K}(t)(x(t) - x(t - \tau_c)). \tag{7}\]

Consequently, the design criterion is to determine \( \dot{K}(t) \) such that the controlled system orbit can track the target

\[
\lim_{t \to \infty} \|x(t) - \bar{x}(t)\| = 0. \tag{8}\]

Taking \( \epsilon = x(t) - \bar{x}(t) \) and \( \Delta \dot{K}(t) = \dot{K}^* - \dot{K}(t) \), the corresponding error dynamical system can be obtained as

\[
\dot{\epsilon}(t) = Ae(t) + B(\dot{K}^* - \Delta \dot{K}(t))\dot{e}(t) - \bar{x}(t - \tau_c) - e(t - \tau_c). \tag{9}\]

In order to synchronize the chaotic speed, \( \pi[t] = 0 \) is required. Thus, it yields

\[
\dot{e}(t) = Ae(t) + B(\dot{K}^* - \Delta \dot{K}(t))\dot{e}(t) - \bar{x}(t - \tau_c) - e(t - \tau_c). \tag{10}\]

Then, the control objective is to force \( e(t) \to 0 \) as \( t \to \infty \). A Lyapunov function candidate is defined as

\[
V(c, \Delta \dot{K}) = e^TPe + \int_{\tau_c}^{t} e^TVe(t) + \frac{1}{\gamma} \Delta \dot{K}^T \Delta \dot{K} \tag{11}\]

where \( P \) and \( V \) are two positive definite matrices. The corresponding derivative is given by

\[
\dot{V} = e^T(t)Pe(t) + e^T(t)P\dot{e}(t) + e^T(t)Ve(t) - e^T(t - \tau_c)Ve(t - \tau_c) - \frac{2}{\gamma} \Delta \dot{K}^T(t)\Delta \dot{K}(t) \]

\[
- [V^{1/2}e(t - \tau_c) + V^{-1/2}K^*BP(e(t))][V^{1/2}e(t - \tau_c) + V^{-1/2}K^*BP(e(t)) + e^T(t)A^TP + PA + V + PBB^*V^{-1}K^*BP + K^*BP^2P + PBB^*][e(t) + 2\Delta \dot{K}^T(t) - \frac{1}{\gamma} \dot{K}(t) - \dot{K}(t)\dot{e}(t) - e^T(t)PBP(e(t) - e(t - \tau_c))] \tag{12}\]

Thus \( \dot{K}(t) \) can be chosen as

\[
\dot{K}(t) = -\gamma \int_{\tau_c}^{t} e^T(t)PBP(e(t) - e(t - \tau_c))dt \tag{13}\]
TABLE I  
PARAMETERS OF INDUCTION MOTORS

<table>
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<tr>
<th>Parameter</th>
<th>Master motor</th>
<th>Slave motor</th>
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<tr>
<td>Rated power (kW)</td>
<td>1.5</td>
<td>1</td>
</tr>
<tr>
<td>Rated voltage (V)</td>
<td>220</td>
<td>220</td>
</tr>
<tr>
<td>Rated speed (rpm)</td>
<td>1430</td>
<td>1430</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Stator resistance (Ω)</td>
<td>3.3</td>
<td>0.760</td>
</tr>
<tr>
<td>Stator inductance (mH)</td>
<td>43.9</td>
<td>22.4</td>
</tr>
<tr>
<td>Rotor resistance (Ω)</td>
<td>3</td>
<td>0.675</td>
</tr>
<tr>
<td>Rotor inductance (mH)</td>
<td>43.9</td>
<td>22.3</td>
</tr>
<tr>
<td>Mutual inductance (mH)</td>
<td>278</td>
<td>217.6</td>
</tr>
<tr>
<td>Rotating inertia (kgm²)</td>
<td>6.65×10⁻⁴</td>
<td>11.1×10⁻³</td>
</tr>
<tr>
<td>Viscous coefficient (N/m.rad/s)</td>
<td>5.5×10⁻⁶</td>
<td>7.355×10⁻⁴</td>
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</table>

Since $P$ and $V$ are two positive definite and symmetric constant matrices, the Riccati polynomial matrix is expressed as

$$
A^TP + PA + PB\tilde{K}\ast T V^{-1} \tilde{K}\ast B^TP + PB\tilde{K}\ast T + V
$$

which is either zero or semi-negative definite ($= 0$, $< 0$). Hence the control objective can be achieved: $\|e(t)\| \to 0$ as $t \to \infty$, namely the system orbits can track the expected state. That means multiple IMs can achieve the synchronized chaotic speed by choosing appropriate parameters of the ATDFC method. Differing from the conventional TDFC method mentioned in [5], the feedback gain of the ATDFC method can be tuned by using an adaptive law. Hence, the robustness of the chaotic speed synchronization control system is also improved.

IV. VERIFICATION RESULTS

The computational simulation is carried out by MATLAB Simulink. In order to verify the feasibility of the proposed chaotic speed synchronization control scheme, two IMs with different parameters are adopted for exemplification. The 3-phase 4-pole 1.5 kW 220 V IM is chosen as the master motor, while the 3-phase 4-pole 1 kW 220 V IM is chosen as the slave one. The key parameters are listed in Table I.

First, to examine the performance of the SFR-based DTC scheme, the master motor is chosen for verification. Since the proposed chaotic speed synchronization control is designed for liquid mixing, the operating speed range is below 300 rpm and the speed reference is selected as 191 rpm (20 rad/s). Fig. 4(a) and (b) show the simulated result of $T^*$ and $T$ respectively when $\psi_s^* = 0.157$ Wb, $\Delta T = 0.005$ Nm, $\Delta \psi = 0$ Wb, and $V_d = 70$ V. The corresponding simulated waveforms of $\psi_s^*$ and $\psi_s$ are shown in Fig. 5(a) and (b) respectively. It indicates that the output torque of the IM can track the torque reference quickly, while the stator flux can be regulated to follow the constant reference value. Thus, the proposed controller can offer a fast torque response for the chaotic speed control.

Second, the performance of the ATDFC method is examined by choosing $\tau_c = 0.325$ s, $T_a = 10^4 \mu s$, and $P = 1.3$. The parameter $\gamma$ should be positive, whereas its value does not significantly affect the control performance. So, the selection principle is to use the trial-and-error approach in such a way that the control performance is acceptable when choosing $\gamma = 1.218$. As shown in Fig. 6, the IM can exhibit chaotic motion with different boundaries by choosing the hysteretic flux boundary of $\Delta \psi = 0$ Wb and $\Delta \psi = 0.15$ Wb respectively. In addition, the amplitude of chaotic speed changes in accordance with the flux boundary, while the chaotic speed error between the master and slave motors varies synchronously. The simulated results in Figs. 7 and 8 confirm that the error of chaotic speeds between the master IM and slave IM converges to zero when the ATDFC method takes effect after $t = 7.3$ s. Additionally, the chaotic speed synchronization controller can be effective for different chaotic motion.
states, which verifies the improvement of the robustness performance.

It should be noted that the DTC is particularly attractive for torque control, but not for speed control. Thus, the DTC is inherently not the best choice for the proposed chaotic speed synchronization control. In addition, the master-slave structure may result in a speed-tracking error when the load torque varies.

V. CONCLUSION

In this paper, a chaotic speed synchronization control scheme has been proposed and implemented for multiple IMs, which is essential for application to production line processes. This key is to develop the SFR-based DTC which can inherently offer a fast torque response. Meanwhile, the ATDFC method is proposed to generate the torque reference for the SFR-based DTC unit. The verification results confirm that the proposed controller can offer a fast torque response and high robustness for the chaotic speed control. Also, it reveals that the amplitude of chaotic speed changes according to the flux boundary while the chaotic speed error between the master and slave IMs varies synchronously.

In the future, other control schemes will be investigated to supersede the DTC, aiming to have a more precise speed control.

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