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Abstract—MapReduce has become the dominant programming model for processing massive amounts of data on cloud platforms. More and more enterprises are now utilizing hybrid clouds, consisting of private infrastructure owned by themselves and public clouds such as Amazon EC2, to process their spiky MapReduce workloads, which fully utilize their own on-premise resources while outsourcing the tasks only when needed. With disparate workloads of different MapReduce tasks, an efficient scheduling mechanism is in need to enable efficient utilization of the on-premise resources and to minimize the task outsourcing cost, while meeting the task completion time requirements as well. In this paper, a fine-grained model is described to characterize the scheduling of heterogeneous MapReduce workloads, and an online algorithm is proposed for joint task admission control into the private cloud, task outsourcing to the public cloud, and VM allocation to execute the admitted tasks on the private cloud, such that the time-averaged task outsourcing cost is minimized over the long run. The online algorithm features preemptive scheduling of the tasks, where a task executed partially on the on-premise infrastructure can be paused and scheduled to run later. It also achieves desirable properties such as meeting a pre-set task admission ratio and bounding the worst-case task completion time, as proven by our rigorous theoretical analysis.

I. INTRODUCTION

The MapReduce framework [1] and its open source versions such as Hadoop [2] have become the dominant programming models for data-intensive and computation-intensive applications in cloud data centers. In the MapReduce framework, a job may spawn many small Map and Reduce tasks that can be executed concurrently on multiple virtual machines, achieving significant fault tolerance and job completion time reduction. With the rapid increase of the number of jobs that are coded based on the MapReduce framework, efficient schedulers for such workloads on cloud computing resources are becoming increasingly important.

Although there have been a number of models and solutions for scheduling MapReduce workloads on cloud platforms [3][4], we point out the following important aspects that are not receiving sufficient attention from the existing work, as the motivation for our research:

(1) Temporally spiky workloads: Arrival of MapReduce jobs is not only non-uniform but even spiky. Instead of provisioning resources according to the peak-level workload, it is common for an enterprise to dispatch its workloads across a hybrid cloud, consisting of its own on-premise data center and a public cloud, as long as moderate job outsourcing cost can be achieved and job service delay can be guaranteed [5]. It is not trivial to design a scheduler that can strategically dispatch the workloads to guarantee long-term cost optimality, especially when the a priori knowledge about the job arrival pattern is unknown. Hajjat et al. [6] focus on one-time application deployment on hybrid clouds, without considering dynamic arrivals of workloads. Zhang et al. [5] propose an intelligent algorithm to factor workloads and dynamically determine the service placement across an on-premise server and a cloud, without diving into resource scheduling inside a private cloud.

(2) Heterogeneous workloads: Different MapReduce jobs may contain different numbers of Map and Reduce tasks, which can span several orders of magnitude [7], with highly heterogeneous task running times, which generally follow a long-tailed distribution [8]. Scheduling with the assumption that all jobs are the same or all tasks are the same, is not practical and results in low efficiency. We seek to model the MapReduce workloads at a fine-grained level, i.e., to characterize the lifespans and execution sequence of the tasks in each MapReduce job in details, in order to schedule the jobs efficiently onto the available cloud resources.

(3) Preemptive job execution: Preemption refers to pausing a running task, checkpointing its status and resuming it later on, making resource available for another task to run first [9]. As a task may span multiple time slots, preemption is a necessary mechanism to guarantee that more urgent jobs, e.g., production jobs, are not starved, while also allowing the cloud to be used for less urgent jobs when available, e.g., experimental and research jobs [7]. On the other hand, different amounts of workloads may remain after a task is preempted, which adds complexity to the optimization model for task scheduling. The Dynamic Priority Scheduler [10] uses preemption to achieve fairness among users, without guaranteeing the completion time of each job. Maguluri et al. [11] discuss preemptive algorithms for job scheduling to achieve optimal throughput in cloud computing clusters, without theoretically addressing the job service delays.

(4) Quality-of-service (QoS) guarantee: Two types of QoS are important for MapReduce task scheduling over a hybrid cloud. One is the admission rate of the workloads, defined as the ratio between the amount of workloads admitted into the system and the amount of workloads users submit. The other is the maximum allowed completion time of each job (referred to as the maximum tolerable job completion time), defined to be the duration from the time when the job is admitted into the
system to the time when it is completed in the private cloud or outsourced to the public cloud. For example, production jobs are typically associated with a tight completion time, while non-production jobs can tolerate more scheduling delay. The scheduler needs to strategically schedule the incoming jobs, in order to satisfy a pre-set workload admission ratio, while completing every admitted job by its completion deadline.

To the best of our knowledge, there are no existing studies that address all the above four aspects in their MapReduce scheduler design. Our contributions in this paper are highlighted as follows:

- We describe a detailed model to characterize Map and Reduce tasks with different workloads spanning one or more time slots, and the execution sequence of the tasks in a MapReduce job.
- We build an optimization framework for joint task admission control into the private cloud, task outsourcing to the public cloud, and VM allocation to execute the admitted tasks on the private cloud, such that the time-averaged outsourcing cost is minimized over the long run.
- We design an online algorithm based on the Lyapunov optimization framework, which features efficient task preemption, and is proven to meet a pre-set job admission ratio and bound the worst-case task completion time, based on our rigorous theoretical analysis.

In the remainder of this paper, we detail the system model and problem formulation in Sec. II, design algorithms for job-to-task arrival process conversion and online scheduling in Sec. III, rigorously prove the guarantee of worst-case task completion time and cost optimality of the algorithms in Sec. IV, and conclude the paper in Sec. V.

II. System Model and Problem Formulation

We consider a time-slotted system. In each time slot, users in the enterprise submit a number of MapReduce jobs to be processed by the hybrid cloud. Description of each job contains a Map/Reduce tasks in the job and expected completion time for the job, to be detailed in Sec. II-A. The system will first convert the completion time requirement of each MapReduce job to the completion time requirements of Map and Reduce tasks contained in the job, and then makes admission control decisions, i.e., either enqueueing the tasks into appropriate task queues to be potentially handled by the on-premise data center, dispatching tasks to the public cloud, or rejecting the job. For tasks in the task queues, the system further decides whether to schedule their execution on VMs in the on-premise data center, or outsource them to the public cloud, in order to minimize the outsourcing cost while guaranteeing all the QoS requirements. An illustration of the system is given in Fig. 1.

We address the following two problems: (1) Given the scheduling is carried out at the task level rather than the job level, how should the input job completion deadline requirement be converted to individual tasks’ completion deadlines for efficient scheduling? (2) How should the system schedule the tasks strategically to achieve minimized outsourcing cost while satisfying all the QoS requirements? We model these problems as follows. Key notation is summarized in Table I for ease of reference.

A. Job-to-task Completion Deadline Conversion

The system receives job submissions from users, in the sequence of $H_i$, $i = 1, 2, 3, \ldots$. Since we are going to restrict the discussion in this subsection within the scope of one job, we drop the subscript that distinguishes the jobs, for simplicity of the notation. Each job can be represented by a 6-tuple $< J, K, D', T_m, T_r, E >$, which is defined as follows:

- $J$ is the set of Map tasks. $K$ is the set of Reduce tasks.
- $D'$ is the user-specified maximum tolerable completion time for the job.
Each task can then be characterized by a Map task and any Reduce task with dependency in between. Let $m$ be the maximum tolerable completion time for the task, i.e., the maximum tolerable completion time for the task, $l$ is the required running time of the task. Here, $M$ is the set of all possible VM types, all possible tolerable completion times, and all possible running times, respectively. The latter two are in the units of time slots in our system. The tasks submitted in the same time slot $t$ and with the same 3-tuple $(m, d, l)$ produce a task arrival process with arrival rate $A_{mdl}(t)$ in $t$. We suppose the arrival rates are upper bounded by $A_{max}$.

B. Queueing Model and Control Decisions

Tasks that are submitted to the system are queued before they are scheduled to run. Especially, tasks with the same $(m, d, l)$ are enqueued in queue $Q_{mdl}$, $\forall m \in M$, $\forall d \in D$, $\forall l \in L$. Each element in a task queue represents one unit of task workload, i.e., the workload corresponding to one-time-slot execution of the task on a type-$m$ virtual machine. When a task is enqueued into $Q_{mdl}$, $l$ elements are appended to the tail of the queue; when a task is scheduled to run on its required VM for one time slot, an element belonging to that task departs from the queue; when a decision has been made for outsourcing a task in $Q_{mdl}$, all the remaining elements associated with that task are removed from the queue. We denote the backlog of task queue $Q_{mdl}$ at $t$ as $Q_{mdl}(t)$, which indicates the number of elements (i.e., the number of unit workload) in the queue.

We formulate decision variables in our scheduling framework as follows:

1) Task admission or outsourcing upon arrival: In time slot $t$, $I_{mdl}(t)$ tasks among the $A_{mdl}(t)$ arrived tasks are admitted into queue $Q_{mdl}$, by appending $I_{mdl}(t)$ elements at the end of the queue. $O_{mdl}(t)$ newly arrived tasks are outsourced to run in the public cloud, and the rest $A_{mdl}(t) - I_{mdl}(t) - O_{mdl}(t)$ tasks are rejected.

2) Task outsourcing after admission: At time slot $t$, $D_{mdl}(t)$ units of workloads at the head of the queue $Q_{mdl}$ are outsourced to the public cloud. These $D_{mdl}(t)$ units of workloads may span a number of tasks at the head of the queue, which are partially completed or not scheduled for running at all yet. For outsourced workload, on-demand VMs from public cloud are rented to run them immediately without occurring any further delay.

3) Task scheduling on the private cloud: At the beginning of time slot $t$, $N_{mdl}(t)$ VMs of type $m$ are scheduled to run tasks of type $(m, d, l)$. $N_{mdl}(t^-)$ is the number of left-over type-$(m, d, l)$ task requests observed at the start of time slot $t$, i.e., tasks that were scheduled to run on type-$m$ VMs before time slot $t$, and have not yet been completed until the end of time slot $t - 1$. When $t = 1$, we set $N_{mdl}(t^-) = 0$.

If $N_{mdl}(t) \geq N_{mdl}(t^-)$, it denotes that all $N_{mdl}(t^-)$ left-over tasks continue running on their occupied VMs in time slot $t$, and $N_{mdl}(t) - N_{mdl}(t^-)$ type-$m$ VMs are newly allocated to run the next $N_{mdl}(t^-)$ waiting tasks in queue $Q_{mdl}$. If $N_{mdl}(t) < N_{mdl}(t^-)$, only $N_{mdl}(t)$ among the $N_{mdl}(t^-)$ left-over tasks can continue running, and the other $N_{mdl}(t^-) - N_{mdl}(t)$ left-over tasks are preempted. We maintain the first-come-first-served (FCFS) principle at the task level, i.e., between any two running tasks, the one arrived earlier at the queue has higher priority to continue running.

In time slot $t$, one unit of workload for each of the $N_{mdl}(t)$ head-of-the-queue tasks is removed from queue $Q_{mdl}$, i.e.,
\( N_{mdl}(t) \) elements are removed from \( Q_{mdl} \). We note that since each task is represented by multiple adjacent elements in the queue, the removed elements belong to different tasks, and may not be neighboring elements. Essentially, we maintain the FCFS principle at the task level only, while it is not necessarily FCFS at the level of elements. An illustration of this queueing model is given in Fig. 3. Devising a scheduling algorithm with a guarantee of worst-case completion time under such a queueing model is challenging.

![An illustration of the task queueing model: A—E represent task indices.](image)

The update of queue \( Q_{mdl} \) is as follows:

\[
Q_{mdl}(t + 1) = \max \{Q_{mdl}(t) - N_{mdl}(t), 0\} + I_{mdl}(t), \forall m \in M, d \in D, l \in L.
\]

We note that \( N_{mdl}(t) \) and \( D_{mdl}(t) \) are in the number of unit task workload, while the other quantities \( A_{mdl}(t) \), \( I_{mdl}(t) \), \( O_{mdl}(t) \) and \( N_{mdl}(t) \) are in the number of tasks.

### C. Quality-of-Service Constraints

The first QoS metric under consideration is the task admission ratio, which is the ratio between the number of processed tasks (including outsourced tasks and scheduled tasks) and the number of submitted tasks. The task admission ratio in the system should be no smaller than a given threshold \( \alpha \) (set by the enterprise), i.e.,

\[
\lim_{T \to \infty} \frac{\sum_{t=0}^{T} (I_{mdl}(t) + O_{mdl}(t))}{\sum_{t=0}^{T} A_{mdl}(t)} \geq \alpha.
\]

The other QoS requirement is to guarantee the worst-case completion time of tasks admitted into the queues, i.e., the tasks are processed within a specified maximum tolerable completion time. Instead of modeling this constraint explicitly, we treat it as a property of our proposed scheduling algorithm, which will be proved by rigorous analysis in Sec. IV.

### D. Outsourcing Cost Minimization

Since the on-premise infrastructure is constructed and always maintained by the enterprise regardless of the workloads, our scheduling algorithm focuses on the minimization of the task outsourcing cost, which is the payment to the public cloud, over the long run. Let \( H(m) \) denote the charge by the public cloud for processing a unit of task workload on a type-\( m \) VM. The overall outsourcing cost in time slot \( t \) is

\[
C(t) = \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} (I_{mdl}(t) + D_{mdl}(t))H(m).
\]

Our objective is to minimize the time-averaged outsourcing cost in the long term as follows:

\[
\min \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} C(t)
\]

subject to:

\[
\lim_{T \to \infty} \sum_{t=0}^{T} (I_{mdl}(t) + O_{mdl}(t)) \geq \alpha,
\]

\[
\forall m \in M, d \in D, l \in L,
\]

\[
\sum_{d \in D} \sum_{l \in L} N_{mdl}(t) \leq N_{mdl}^{\text{tot}}, \forall m \in M, \forall t \in [0, T],
\]

\[
I_{mdl}(t) + O_{mdl}(t) \leq A_{mdl}(t),
\]

\[
\forall m \in M, d \in D, l \in L, t \in [0, T],
\]

\[
D_{mdl}(t) \leq D_{mdl}^{\max}, \forall m \in M, d \in D, l \in L, t \in [0, T].
\]

\[
O_{mdl}(t) \leq Z^{+} + 0, \forall m \in M, d \in D, l \in L, t \in [0, T],
\]

\[
O_{mdl}(t) + D_{mdl}(t) \leq Z^{+} + 0, \forall m \in M, d \in D, l \in L, t \in [0, T].
\]

\[
N_{mdl}(t) \leq Z^{+} + 0, \forall m \in M, d \in D, l \in L, t \in [0, T].
\]

Here, \( N_{mdl}^{\text{tot}} \) is the total number of type-\( m \) VMs, and \( D_{mdl}^{\max} \) is the maximum number of units of outsourced workloads. (6) is to specify the admission ratio requirement. (7) shows that the number of scheduled tasks for each type of VM should not exceed the total number of VMs of that type in the private cloud.

### III. THE ONLINE SCHEDULING ALGORITHM

We next detail the algorithm for converting job submission sequence into task submission sequences, and then design the dynamic scheduling algorithm based on the Lyapunov optimization framework.

#### A. Converting Job Submission Sequence to Task Submission Processes

For each submitted job, we need to decide the submission time and maximum tolerable completion time for each of the Map and Reduce tasks contained in the job, in order to generate the task arrival process \( A_{mdl}(t) \).

Although there exist many feasible solutions satisfying the constraints in (1), we seek to derive a simple and efficient solution method, which probably achieves performance optimality together with our scheduling algorithm, as follows.

For each Reduce task \( k \in K \), find the Map task that has the maximum required running time among all the Map tasks that task \( k \) depends on, i.e., select

\[
j' = \arg \max_{j \in J} \{ e(j, k)T_j^{\text{map}} \}.
\]

Hence, the slack time to allow timely scheduling of Map task \( j' \) and Reduce task \( k \) is \( D'_k - T_{j'}^{\text{map}} - T_k^{\text{reduce}} \). We allocate one half of this slack time to the scheduling of Reduce task \( k \), and set its maximum tolerable completion time as

\[
D'_k = T_k + \frac{D'_k - T_{j'}^{\text{map}} - T_k^{\text{reduce}}}{2}.
\]

Suppose this MapReduce job is submitted at \( t_0 \). The submission time of Reduce task \( k \) is set to \( t_0 + D'_k \).

After deciding the submission time and maximum tolerable completion time of each of the Reduce tasks using the above method, we then decide the submission time and tolerable time of each of the Map tasks. For Map task \( j \in J \), the maximum tolerable completion time is computed as the job’s tolerable
complemet time minus the largest tolerable completion time among those of all the Reduce tasks depending on Map task $j$, i.e., $D_{m}^{\nu} = D' - D_{\nu}^{\prime}$, where $k' = \arg\max_{k \in K} (j, k)D_{k}$. The submission time of each Map task is set to the same as that of the job, i.e., $t_0$.

The above method guarantees that a Reduce task will not be enqueued until all of its dependent Map tasks have been completed. As this conversion method is deterministic, when the arrival process of the jobs is ergodic, the task arrival processes into the system are also ergodic.

B. Solving the Online Scheduling Problem

We apply the Lyapunov Optimization framework [13][14] to design an online algorithm to solve problem (5).

To guarantee the satisfaction of admission ratio constraints in (6), we associate a virtual queue $Q_{mdl}$ with each task queue $Q_{mdl}(\forall m \in M, d \in D, l \in L)$, with initial backlog $K_{mdl}(0) = 0$ and queue update

$$K_{mdl}(t+1) = \max\{K_{mdl}(t) + A_{mdl}(t) - l I_{mdl}(t) - l O_{mdl}(t), 0\}.$$  

To bound the worst-case completion time for each task, we apply the $\epsilon$-persistent service technique [14] to build a virtual queue $Z_{mdl}$ associated with each task queue $Q_{mdl}$, $\forall m \in M, d \in D, l \in L$, with initial backlog $Z_{mdl}(0) = 0$:

$$Z_{mdl}(t + 1) = \max\{Z_{mdl}(t) + 1_{\{Q_{mdl}(t) > 0\}/\epsilon_{mdl} - N_{mdl}(t) - D_{mdl}(t) + 1_{\{Q_{mdl}(t) = 0\} \cdot N_{mdl}^{\text{tot}}, 0}\}.$$  

Here $\epsilon_{mdl}$ is a pre-defined constant whose value is related to the desired worst-case completion time $d$ of tasks in queue $Q_{mdl}$. We will reveal their relationship in Sec. IV-B.

Let $\Theta(t) = (Q(t), Z(t), K(t))$ denote a vector consisting of all queues in our system, where $Q(t) = \{Q_{mdl}(t), \forall m \in M, d \in D, l \in L\}$, $Z(t) = \{Z_{mdl}(t), \forall m \in M, d \in D, l \in L\}$, and $K(t) = \{K_{mdl}(t), \forall m \in M, d \in D, l \in L\}$. Define the Lyapunov function as

$$L(\Theta(t)) = \frac{1}{2} \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} [Q_{mdl}(t)^2 + Z_{mdl}(t)^2 + K_{mdl}(t)^2].$$

The one-slot conditional drift-plus-penalty is

$$\Delta(\Theta(t)) + VC(t) \\
\leq B_1 + Z_{mdl}(t) + 1_{Q_{mdl}(t) > 0} (N_{mdl}^{\text{tot}} - 1_{Q_{mdl}(t) = 0} \cdot N_{mdl}^{\text{tot}}) \\
+ \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} [I_{mdl}(t)Q_{mdl}(t) - K_{mdl}(t)] \\
+ \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} [O_{mdl}(t) + V_{H}(m) - K_{mdl}(t)] \\
+ \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} [D_{mdl}(t) + V_{H}(m) - Q_{mdl}(t)] \\
- 1_{Q_{mdl}(t) > 0} (Z_{mdl}(t) - N_{mdl}(t)) + \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} N_{mdl}(t)(Q_{mdl}(t) + 1_{Q_{mdl}(t) > 0} Z_{mdl}(t)),$$

where $\Theta(t)$ is a controlling constant the purpose of which will be detailed in Sec. IV, and $B_1$ is a constant as follows:

$$B_1 = \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} (D_{mdl}^{\text{max}})^2 + 2 \sum_{m \in M} N_{mdl}^{\text{tot}} \sum_{d \in D} \sum_{l \in L} D_{mdl}^{\text{max}} \\
+ |D| \cdot |L| \cdot (N_{mdl}^{\text{tot}})^2 + \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} \max\{(1 - \alpha)^2, \alpha^2\} (A_{mdl}^{\text{max}})^2 \\
+ \sum_{m \in M} \sum_{d \in D} \sum_{l \in L} \max\{\epsilon_{mdl}^2, (N_{mdl}^{\text{tot}} + D_{mdl}^{\text{max}})^2\}.$$  

Based on the Lyapunov optimization framework [13], we derive a dynamic algorithm that observes queues in $\Theta(t)$ in each time slot $t$ and makes control decisions on $I_{mdl}(t)$, $O_{mdl}(t)$, $D_{mdl}(t)$, $N_{mdl}(t)$, $\forall m \in M, d \in D, l \in L$, that minimize the RHS (Right-Hand-Side) of (19), such that an upper bound for the time-averaged outsourcing cost is minimized. Except the constant terms, RHS of (19) can be decomposed into three parts, which we seek to minimize respectively in each time slot as follows.

1) Admission control: We solve the following optimization problem to derive $I_{mdl}(t)$ and $O_{mdl}(t)$, for each $m \in M, d \in D, l \in L$:

$$\min I_{mdl}(t)[Q_{mdl}(t) - K_{mdl}(t)] + O_{mdl}(t)[V_{H}(m) - K_{mdl}(t)]$$

Subject to: constraints (8)(11)(12).

2) is a linear optimization problem. We observe the coefficients of the two variables $I_{mdl}(t)$ and $O_{mdl}(t)$, i.e., $Q_{mdl}(t) - K_{mdl}(t)$ and $V_{H}(m) - K_{mdl}(t)$, in (21), and derive the following solutions:

**Case 1:** If $Q_{mdl}(t) - K_{mdl}(t) \geq 0$ and $V_{H}(m) - K_{mdl}(t) < 0$, the solution is $I_{mdl}(t) = 0$ and $O_{mdl}(t) = A_{mdl}(t)$;

**Case 2:** If $Q_{mdl}(t) - K_{mdl}(t) < 0$ and $V_{H}(m) - K_{mdl}(t) \geq 0$, the solution is $I_{mdl}(t) = A_{mdl}(t)$ and $O_{mdl}(t) = 0$;

**Case 3:** If $Q_{mdl}(t) - K_{mdl}(t) \geq 0$ and $V_{H}(m) - K_{mdl}(t) \geq 0$, the solution is $I_{mdl}(t) = 0$ and $O_{mdl}(t) = 0$;

**Case 4:** If $Q_{mdl}(t) - K_{mdl}(t) < 0$ and $V_{H}(m) - K_{mdl}(t) < 0$, the solution is $I_{mdl}(t) = A_{mdl}(t)$ and $O_{mdl}(t) = 0$; otherwise, the solution is $I_{mdl}(t) = 0$ and $O_{mdl}(t) = A_{mdl}(t)$.

2) In-queue task outsourcing: We solve the following linear minimization problem to derive $D_{mdl}(t)$, for each $m \in M, d \in D, l \in L$:

$$\min D_{mdl}(t) [V_{H}(m) - Q_{mdl}(t)] - 1_{Q_{mdl}(t) > 0} Z_{mdl}(t)$$

Subject to: constraints (9)(10).

The solution to problem (22) is

$$D_{mdl}(t) = \begin{cases} 0 & \text{if } V_{H}(m) \geq Q_{mdl}(t) + 1_{Q_{mdl}(t) > 0} Z_{mdl}(t) \\
\max_{d \in D} & \text{otherwise} \\
D_{mdl}^{\text{max}} & \end{cases}$$

3) VM allocation for in-queue tasks: We derive $N_{mdl}(t)$, for each $m \in M$, by solving the following maximization problem:

$$\max_{d \in D} \sum_{l \in L} N_{mdl}(t)(Q_{mdl}(t) + 1_{Q_{mdl}(t) > 0} Z_{mdl}(t))$$

Subject to: constraints (13)(7).

As $Q_{mdl}(t) + 1_{Q_{mdl}(t) > 0} Z_{mdl}(t)$ is always non-negative, to maximize (23), we simply need to find $\{d', l\} = \arg\max_{d \in D, l \in L} Q_{mdl}(t) + 1_{Q_{mdl}(t) > 0} Z_{mdl}(t)$, in (24) and then set $N_{mdl}^{*} = N_{mdl}^{\text{tot}}$ and $N_{mdl} = 0, \forall d \neq d', l \neq l'$.

The above dynamic algorithm can be implemented by a controller module in the enterprise as follows: At the beginning
of each time slot \( t \), the controller receives submitted jobs and converts them into task arrivals at the current and future time slots. Then the controller solves the three optimization problems (21)(22)(23) to decide the optimal values of control variables on task admission, outsourcing and VM allocation, and schedules the tasks accordingly. Especially for in-queue task outsourcing and VM allocation, the controller first outsources tasks that are covered by \( D_{\text{mdt}} \) elements at the head of each queue \( Q_{\text{mdt}} \), and then schedules the next \( N_{\text{mdt}}(t) \) in-queue tasks to run on VMs. At the end of the time slot, the controller updates the status of all queues.

IV. PERFORMANCE ANALYSIS

The proof of all lemmas and theorems in this section can be found in our technical report [15].

A. Strong Stability of Queues

Lemma 1: (Strong Stability of Queues) Our algorithm guarantees in all time slots,

\[
K_{\text{mdt}}(t) \leq K^\text{max}_{\text{mdt}} = VH(m) + \epsilon l A_{\text{max}},
\]

\[
Q_{\text{mdt}}(t) \leq Q^\text{max}_{\text{mdt}} = VH(m) + (1 + \alpha) l A_{\text{max}},
\]

\[
Z_{\text{mdt}}(t) \leq Z^\text{max}_{\text{mdt}} = VH(m) + \epsilon_{\text{mdt}}.
\]

As Lemma 1 shows the strong stability of queue (17), inequality (6) is always satisfied, i.e., the pre-set admission ratio is guaranteed.

B. Guarantee of Worst-Case Task Completion Time

Theorem 1: (Guarantee of Worst-Case Completion Time) The worst-case completion times of all tasks admitted into queue \( Q_{\text{mdt}} \) are upper bounded by the constant

\[
U_{\text{mdt}} = \left( \frac{(1 + l) Q^\text{max}_{\text{mdt}} + Z^\text{max}_{\text{mdt}}}{\epsilon_{\text{mdt}}} \right),
\]

where \( Q^\text{max}_{\text{mdt}} \) and \( Z^\text{max}_{\text{mdt}} \) are upper bounds of \( Q_{\text{mdt}}(t) \) and \( Z_{\text{mdt}}(t) \) defined in (26) and (27).

This shows that we can set \( \epsilon_{\text{mdt}} \) as

\[
\epsilon_{\text{mdt}} = \frac{(1 + l) Q^\text{max}_{\text{mdt}} + Z^\text{max}_{\text{mdt}}}{d},
\]

such that the completion times of tasks in \( Q_{\text{mdt}} \) are no larger than the maximum tolerable completion time of \( d \).

C. Optimality of Time-averaged Outsourcing Cost

For simplification of notation, we use \( \overline{X} \) to represent \( \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T} E(X(t)) \).

Lemma 2: (Existence of Optimal Stationary, Randomized Policy) For any ergodic job arrival process, there exists a stationary randomized control policy \( \pi \) that chooses \( I_{\text{mdt}}(t) \in [0, A_{\text{max}}] \), \( O_{\text{mdt}}(t) \in [0, A_{\text{max}}] \), \( N_{\text{mdt}}(t) \in [0, N_{\text{mdt}}] \), that solve problem (5) augmented with the following constraints:

\[
\epsilon_{\text{mdt}} \leq \overline{X}_{\text{mdt}} + D_{\text{mdt}}, \forall m \in \mathcal{M}, d \in \mathcal{D}, l \in \mathcal{L},
\]

with optimal time-averaged outsourcing cost \( \overline{C}^* \), only if the new problem is feasible.

Theorem 2: (Optimality of Outsourcing Cost) The time-averaged outsourcing cost achieved by our dynamic algorithm (denoted as \( \overline{C}^* \) hereafter) is within a constant gap from the cost of any stationary randomized control policy that solve problem (5) augmented with constraint (29), i.e.,

\[
\overline{C}^* \leq \overline{C} + \frac{B_1}{V} + |\mathcal{D}| \sum_{l \in \mathcal{L}} \sum_{m \in \mathcal{M}} H(m),
\]

where \( B_1 \) is the constant defined in (20).

The theorem shows that the performance of our dynamic algorithm can approach the optimal cost of the augmented problem within a constant gap, which decreases with the increase of \( V \). Meanwhile, from (26) we see worst-case backlogs grow linearly in \( V \). Therefore, \( V \) can be adjusted to achieve a desired tradeoff between the outsourcing cost and the completion times of tasks in the task queues.

V. CONCLUSION

This paper proposes a fine-grained model to characterize the scheduling of heterogeneous MapReduce workloads over a hybrid cloud, and an online algorithm for joint task admission control, task outsourcing and VM allocation. Based on the Lyapunov optimization framework, we show that the online algorithm can achieve close-to-minimal time-averaged task outsourcing cost over the long run, with guarantee of task admission ratio and worst-case task completion time. As ongoing work, we are implementing the algorithm in practical systems to further evaluate its performance.

REFERENCES