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A Numerical Design of Interactors for General Descriptor Systems

Delin Chu and Y. S. Hung

Abstract—In this technical note, we develop a numerically reliable method to design an interactor for a general descriptor system relative to a stability region $C_\sigma$ and an offending zero set $\Omega$. Our main result is based on a condensed form. This condensed form is independent of the offending zeros $\Omega$, and is computed using only orthogonal transformations and hence is numerically backward stable.

Index Terms—Descriptor systems, interactor, orthogonal transformations.

I. INTRODUCTION

Throughout this technical note, the following notation will be used:
- The rank of a constant matrix $A$ is denoted by $\text{rank}(A)$. The generic rank of a rational (transfer function) matrix $G(s)$ is defined to be $\text{rank}_g(G(s)) = r$ if $\text{rank}(G(s)) = r$ for almost all $s \in \mathbb{C}$. Obviously, for any constant matrix $A$, it holds that $\text{rank}_g(A) = \text{rank}(A)$.

It is well-known that in many control problems, system zeros occurring on the stability boundary and/or at infinity cause (numerical or computational) difficulties in the solution process. Such zeros will be referred to as offending zeros, which by definition are mutually exclusive with the stability region of the system. One way to overcome the difficulty associated with offending zeros is to use an interactor to cancel them and then work with the modified system that does not contain offending zeros. Such an approach has been used to solve the LQ regulation problem, the singular filtering problem, the inner-outer factorization problem, and the singular $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control problems [1], [3], [4], [8], [13], [18].

Consider a general descriptor system of the form

\[
\begin{align*}
\dot{E} \dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{align*}
\]

where $E, A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{l \times n}$, $D \in \mathbb{R}^{l \times m}$, $E$ is singular, and the pencil $A - sE$ is assumed regular so that existence and uniqueness of solution to (1) can be guaranteed. Clearly, the transfer function of the system (1) relating the output $y$ to the input $u$ is given by

\[
G(s) = \frac{-sE + A}{C}B.
\]

Definition 1: Given a stability region $C_\sigma$ and an offending zero set $\Omega \subset (\mathbb{C} \cup \{\infty\})/C_\sigma$. If $m$-by-$m$ rational matrix $U(s)$ satisfies that: (i) $U^{-1}(s)$ exists, is proper and stable relative to $C_\sigma$, and (ii) $G(s) = G(U(s))$ has no zeros in the set $\Omega$ and has the same poles as $G(s)$, then $U(s)$ is called an interactor of system (1) with respect to the stability region $C_\sigma$ and the offending zero set $\Omega$.

The design of an interactor for a standard linear time-invariant system with full normal column rank and $C_\sigma = C^{-}$ has been studied in [4], [7], [9], [12], [16], [19], [20]. In this technical note, we study the design of the interactor for descriptor systems of the form (1) which include proper systems without full normal column rank and non-proper systems. Our main result is based on a very technical condensed form for system matrices $E$, $A$, $B$, $C$, and $D$. This condensed form has a structure containing all necessary information for the design problem of the interactor studied in the present technical note. Such a structure is novel, and enables interactors to be designed for general descriptor systems.

Our results have two nice features compared with the existing ones in [4], [16], [19], [20] as follows.

1) In contrast to the existing results, based on a condensed form which can be computed using only orthogonal transformations and hence is numerically backward stable, we construct the interactor $U(s)$ for any stability region $C_\sigma$ and offending zero set $\Omega$ without computing any (generalized) eigenvalue/eigen-space of the system matrix of system (1) corresponding to the offending zeros and any Lyapunov-like equations. This feature is important because, for example, the eigen-spaces of the matrix pencils corresponding to infinite eigenvalues, pure imaginary eigenvalues and eigenvalues on the unit circle are sensitive to the perturbations and are in fact difficult to compute numerically

2) Some existing works have the shortcoming that the offending zeros cannot be guaranteed to be replaced by the new desired zeros. They might be replaced with additional singular structure which for applications is even worse than “bad” zeros. However, we parameterize the interactors $U(s)$ for any offending zeros $\Omega$ in terms of a tuning parameter $X$, by which the desired interactors $U(s)$ for any given offending zeros $\Omega$ can be obtained and the “bad” zeros can be replaced by the new desired zeros in the compensated rational matrix $G(s)U(s)$.

II. MAIN RESULT

The following result is a direct application of the theory on the generalized upper triangular form of an arbitrary matrix pencil [5], [21].

Theorem 2: Let $C_\sigma$ be the given stability region. Assume system (1) with $E, A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$, $C \in \mathbb{R}^{l \times n}$, and $D \in \mathbb{R}^{l \times m}$ is detectable relative to the stability region $C_\sigma$ and rank

\[
E := P^T \begin{bmatrix} E_1 & 0 & n_2 & n_3 \\ 0 & 0 & 0 & 0 \\ E_{21} & E_{22} & E_{23} \\ E_{31} & 0 & 0 & 0 \\ E_{41} & 0 & 0 & 0 \\ \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ \end{bmatrix}
\]

\[
A := P^T \begin{bmatrix} A_1 & 0 & 0 \\ A_{21} & A_{22} & A_{23} \\ A_{31} & 0 & 0 \\ A_{41} & A_{42} & A_{43} \\ \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ \end{bmatrix}
\]

\[
B := P^T \begin{bmatrix} B_1 & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \\ B_{41} & B_{42} & B_{43} \\ \end{bmatrix} \begin{bmatrix} n_1 & n_2 & n_3 \\ \end{bmatrix}
\]

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where

\[
\begin{align*}
\text{rank}(E_{11}) &= n_1, \quad \text{rank}(E_{22}) = n_2, \quad \text{rank}(A_{33}) = n_3 \\
\text{rank}(-sE_{11} + A_{11}) &= n_1, \quad \forall s \in \mathbb{C} \\
\text{rank}_s \left[ 
\begin{bmatrix}
-sE_{22} + A_{22} & -sE_{23} + A_{23} & B_{21} \\
0 & A_{33} & B_{31} \\
A_{42} & A_{43} & B_{41}
\end{bmatrix}
\right] &= n_2 + n_3 + m_1.
\end{align*}
\]  

(4) 

(5) 

(6)

\textbf{Remark 1:} We can assume that \(\text{rank} \left[ \begin{bmatrix} E \\
\mathbb{C} \end{bmatrix} \right] = n\) without loss of generality. Since if it is not the case, then we can achieve this by orthogonal transformations, as follows.

- Compute the QR factorization of \(C^T\) to get orthogonal matrix \(Q_1 \in \mathbb{R}^{n \times n}\) such that \(CQ_1 = \begin{bmatrix} 0 & C_3 \end{bmatrix}\) with \(\text{rank}(C_3) = \tau_3\). Denote

\[
E_1 = \begin{bmatrix} n - \tau_2 & \tau_2 \\
\mathcal{E}_1 & \mathcal{E}_3 \end{bmatrix}, \quad A_1 = \begin{bmatrix} A_1 & A_{41} \\
A_{42} & A_{43} \end{bmatrix}.
\]

(7)

Since the pencil \(A - sE\) is regular, so, \(\text{rank}_s(s\mathcal{E}_1 - A_1) = n - \tau_2\). Thus, we get orthogonal matrices \(P \in \mathbb{R}^{n \times n}\) and \(Q_2 \in \mathbb{R}^{(n-\tau_2) \times (n-\tau_2)}\) by computing the generalized upper triangular form \([5], [21]\) of the pencil \(A_1 - s\mathcal{E}_1\) such that

\[
P(s\mathcal{E}_1 - A_1)Q_2 = \begin{bmatrix} n - \tau_2 & \tau_2 \\
\mathcal{E}_{11} - A_{11} & s\mathcal{E}_{12} - A_{12} \\
0 & s\mathcal{E}_{22} - A_{22} \end{bmatrix} \begin{bmatrix} n - \tau_2 & \tau_2 \\
\tau_2 & \tau_3 \end{bmatrix}.
\]

(8)

where \(\text{rank}(\mathcal{E}_{22}) = \tau_2\), and \(\text{rank}(\mathcal{E}_{11} - A_{11}) = n - \tau_2 - \tau_3\) for all \(s \in \mathbb{C}\). Denote

\[
P(s\mathcal{E}_3 - A_3) = \begin{bmatrix} s\mathcal{E}_{13} - A_{13} \\
s\mathcal{E}_{23} - A_{23} \\
B_{12} \end{bmatrix}, \quad PBP = \begin{bmatrix} B_{11} & 0 & 0 \\
B_{21} & 0 & 0 \\
0 & 0 & 0 \end{bmatrix}.
\]

(9)

Obviously, it holds that \(\text{rank} \left[ \begin{bmatrix} \mathcal{E}_{22} & \mathcal{E}_{23} \\
0 & C_3 \end{bmatrix} \right] = \tau_2 + \tau_3\) and

\[
G(s) = D + C(sE - A)^{-1}B = D + \begin{bmatrix} 0 & C_3 \end{bmatrix} \begin{bmatrix} s\mathcal{E}_{22} - A_{22} & s\mathcal{E}_{23} - A_{23} \end{bmatrix}^{-1}B_2.
\]

(10)

Moreover, \(\begin{bmatrix} A_{22} & A_{23} \\
\mathcal{E}_{22} & \mathcal{E}_{23} \\
0 & C_3 \end{bmatrix}\) is detectability relative to \(C_x\) provided \((A, E, C)\) is detectability relative to \(C_x\). Hence, we can rename

\[
A := [A_{22} & A_{23}], \quad E := [\mathcal{E}_{22} & \mathcal{E}_{23}], \quad B := B_2, \quad C := [0 & C_3].
\]

\textbf{Theorem 3:} Let \(C_x\) be the given stability region and \(\Omega \subset (C \cup \{\infty\})/C_x\) the given offending zero set. Assume system (1) with \(E, A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n}\) and \(D \in \mathbb{R}^{p \times m}\) is detectable relative to \(C_x\) and \(\text{rank} \left[ \begin{bmatrix} E \\
C \end{bmatrix} \right] = n\). Suppose that orthogonal matrices \(P, Q, W\) and the form (3) have been determined. Partition

\[
P = \begin{bmatrix} p_1 & n_2 & n_3 & m_1 \\
p_{11} & p_{12} & p_{13} & p_{14} \end{bmatrix}, \quad W = \begin{bmatrix} m_1 & m - m_1 \\
W_1 & W_2 \end{bmatrix}.
\]

(11)

(i) \((A_{12}, E_{22}, A_{22})\) is detectable relative to \(C_x\). Furthermore, there exists \(X \in \mathbb{R}^{n_2 \times m_1}\) such that the pencil \(A_{22} - XA_{42} - sE_{22}\) is stable relative to \(C_x\). Define

\[
U(s) = \begin{bmatrix} -sE_{22} + A_{22} & -sE_{23} + A_{23} & B_{21} \\
0 & A_{33} & B_{31} \\
A_{42} & A_{43} & B_{41} \\
0 & 0 & W_1 \end{bmatrix} = \begin{bmatrix} -X & 0 \\
0 & 0 \\
-1 & 0 \\
0 & 0 \end{bmatrix}.
\]

(12)

Then, \(U(s)\) is an interactor of system (1) relative to the stability region \(C_x\) and offending zero set \(\Omega\). Moreover

\[
\hat{G}(s) = G(s)U(s) = \begin{bmatrix} -sE + A \\
C \end{bmatrix}, \quad P_{24}X + P_{14}BW_2 \quad DW_2.
\]

(13)

its finite zeros are contained in the spectra of the pencil \(A_{22} - XA_{42} - sE_{22}\), and it has exactly \(n_1 + m_1\) infinite zeros.

(ii) If system (1) is stabilizable relative to \(C_x\), then \(\hat{G}(s)\) above is also stabilizable relative to \(C_x\).

\textbf{Proof:} Let \(Q = \begin{bmatrix} n_1 & n_2 & n_3 \\
Q_1 & Q_2 & Q_3 \end{bmatrix}\).

(i) The detectability of \((A_{12}, E_{22}, A_{22})\) relative to \(C_x\) follows directly from the detectability of \((C, E, A)\) relative to \(C_x\) and the condensed form (3). This implies that there exists matrix \(X \in \mathbb{R}^{n_2 \times m_1}\) such that the pencil \(A_{22} - XA_{42} - sE_{22}\) is stable relative to \(C_x\). From the condensed form (3), we have

\[
\begin{bmatrix} E \\
0 \end{bmatrix}, \quad Q = P = \begin{bmatrix} E_{11} & 0 & 0 \\
E_{21} & E_{22} & E_{23} \\
E_{31} & 0 & 0 \\
E_{41} & 0 & 0 \end{bmatrix}.
\]

(14)

\[
\begin{bmatrix} A \\
0 \end{bmatrix}, \quad Q = P = \begin{bmatrix} A_{11} & 0 & 0 \\
A_{21} & A_{22} & A_{23} \\
A_{31} & 0 & A_{33} \\
A_{41} & A_{42} & A_{43} \end{bmatrix}.
\]

(15)

\[
\begin{bmatrix} B \\
D \end{bmatrix}, \quad W = P = \begin{bmatrix} B_{21} & B_{22} \\
B_{31} & B_{32} \\
B_{41} & B_{42} \end{bmatrix}.
\]

(16)

Note that \(E_{22}\) is nonsingular, we have from (10) that

\[
\begin{align*}
E_{Q_2} &= P_{12}E_{22}, \quad E_{Q_3} = P_{13}E_{23}, \quad P_{22} = 0 \\
C_{Q_2} &= P_{24}A_{42}, \quad C_{Q_3} = P_{23}A_{33} + P_{24}A_{43} \quad \text{(11)}
\end{align*}
\]
Now, we obtain using the property (6) that

$$
U_1(s) = \begin{bmatrix}
U_{11}(s) & U_{12}(s) \\
0 & A_{23}
\end{bmatrix}
\begin{bmatrix}
B_{21} - XB_{41} & 0 \\
0 & B_{31}
\end{bmatrix}
\begin{bmatrix}
A_{42} & A_{43} & B_{41} \\
0 & 0 & 0 & I_{m-n+1}
\end{bmatrix}
W^T
$$

(12)

where $U_{11}(s) = -sE_{22} + A_{22} - XA_{42}, U_{12}(s) = -sE_{23} + A_{23} - XA_{43}, E_{22}$ and $A_{43}$ are nonsingular, the pencil $A_{22} - XA_{42} - sE_{22}$ is stable relative to $C_\sigma$, thus, $U_1(s)$ is proper and stable relative to $C_\sigma$. Furthermore, we obtain by using (10) and (11), and some simple and basic row and column transformations that

$$
\mathcal{G}(s)U(s) = G(s)U(s)
$$

(13)

where $\tilde{E}_{22}(s) = -sE_{22} + A_{22}, \tilde{E}_{23}(s) = -sE_{23} + A_{23}$. Obviously, $\mathcal{G}(s)$ has the same poles as $G(s)$. Furthermore, (10) and (11) yield that

$$
P^T
\begin{bmatrix}
-sE + A & P_{12}X + P_{14} & BW_2 \\
C & 0 & 0 \\
0 & BW_1 & 0
\end{bmatrix}
\begin{bmatrix}
I_{m_1} & I_{m_2} & I_{m_3} \\
0 & I_{m_4} & A_{42}A_{43}I_{m_5}
\end{bmatrix}
Q
\begin{bmatrix}
U_{11} & 0 & 0 \\
0 & 0 & 0 \\
U_{12} & X & B_{22}
\end{bmatrix}
= -sE_{11} + A_{11} & 0 & 0 & 0 \\
-sE_{21} + A_{21} & U_{11}(s) & U_{12}(s) & X \\
-sE_{31} + A_{31} & 0 & 0 & B_{22}
\end{bmatrix}
$$

$$
\begin{bmatrix}
P_{12}X + P_{14} & BW_2 \\
C & P_{24} & BW_2
\end{bmatrix}
\begin{bmatrix}
L_{11} & 0 \\
0 & L_{m_2} & L_{m_3} \\
A_{42}A_{43}I_{m_5} & 0 & 0
\end{bmatrix}
Q
\begin{bmatrix}
-sE + A & 0 & 0 \\
\tilde{E}_{22}(s) & \tilde{E}_{23}(s) & B_{21} \\
0 & 0 & A_{43} & B_{31} & 0
\end{bmatrix}
= -sE_{11} + A_{11} & 0 & 0 & 0 \\
-sE_{21} + A_{21} & U_{11}(s) & U_{12}(s) & X \\
-sE_{31} + A_{31} & 0 & 0 & B_{22}
\end{bmatrix}
$$

Hence, we obtain from the property (5), nonsingularity of $A_{43}$, the stability of the pencil $A_{22} - XA_{42} - sE_{22}$ relative to $C_\sigma$ and relation $\Omega \subseteq (C \cap \{ \infty \})/C_\sigma$ that $\mathcal{G}(s)$ has no zeros of $\Omega$. Therefore, $U(s)$ is the desired interactor. Moreover, the finite zeros of $\mathcal{G}(s)$ are contained in the spectra of the pencil $A_{22} - XA_{42} - sE_{22}$ and $\mathcal{G}(s)$ has exactly $n_3 + m_1$ infinite zeros.

(ii) Furthermore, if system (1) is stabilizable relative to $C_\sigma$, i.e.,

$$
\text{rank} \begin{bmatrix} sE + A & B \end{bmatrix} = n \text{ for all } s \in C/C_\sigma, \text{ note that the pencil } A_{22} - XA_{42} - sE_{22} \text{ is stable relative to } C_\sigma, A_{43} \text{ is nonsingular, we have using (10) that for any } s \in C/C_\sigma,
$$

$$
\text{rank} \begin{bmatrix} -sE + A & 0 & 0 & 0 \\
0 & \tilde{E}_{22}(s) & \tilde{E}_{23}(s) & B_{21} \\
0 & 0 & A_{43} & B_{31} & 0
\end{bmatrix}
= n + n_2 + n_3 + m_1.
$$

Therefore, rank $[-sE + A P_{12}X + P_{14} BW_2] = n$ for any $s \in C/C_\sigma$, i.e., $\mathcal{G}(s)$ is also stabilizable relative to $C_\sigma$. □

Theorem 3 has provided the following algorithm to design an interactor for descriptor system (1).

**Algorithm 1:** (Construction of an interactor for descriptor system (1))

- **Input:** Stability region $C_\sigma$, offending zero set $\Omega \subseteq (C \cup \{ \infty \})/C_\sigma$, and descriptor system (1) which is detectable relative to $C_\sigma$.
- **Output:** An interactor $U(s)$.

**Step 1:**

Step 1. Compute the condensed form (3).

**Step 2:**

Step 2. Compute matrix $X \in \mathbb{R}^{n_2 \times m_1}$ by the Schur method for pole assignment in [22] such that the pencil $A_{22} - XA_{42} - sE_{22}$ is stable relative to $C_\sigma$.

**Step 3:**

Step 3. Compute $U(s)$ and $\mathcal{G}(s)$ by (8) and (9).

### III. NUMERICAL PROPERTIES OF ALGORITHM 1 AND A NUMERICAL EXAMPLE

Let $\bar{X}$ denote the computed $X$ using finite precision arithmetic, as opposed to exact arithmetic, and $\epsilon$ denote the machine precision. For the condensed form (3), since matrices $P, Q$ and $W$ are all orthogonal, we have [6] that

$$
\|\bar{P}\bar{P}^T - I\|_2 \approx \epsilon, \|\bar{Q}\bar{Q}^T - I\|_2 \approx \epsilon, \|\bar{W}\bar{W}^T - I\|_2 \approx \epsilon
$$

$$
\|\bar{P}^T \begin{bmatrix} E \\ C \end{bmatrix} - \bar{Q} - \bar{A}\|_2 \approx \|E\|_2 \epsilon,
\|\bar{P}^T \begin{bmatrix} A \\ C \end{bmatrix} - \bar{Q} - \bar{A}\|_2 \approx \|A\|_2 \epsilon
$$

Therefore, the computation of the form (3) is numerically backward stable. In addition, $X$ is computed by the Schur method for pole assignment in [22]. The Schur method in [22] is numerically reliable in the sense that it is based exclusively on numerically stable procedures like column and row compressions using QR factorizations [22]. Hence, all steps of Algorithm 1 can be performed by using numerically reliable procedures, and thus Algorithm 1 can be implemented by a numerically reliable way.

Note that the computational complexities for computing the generalized upper triangular form of a pencil $A - sC \in \mathbb{R}^{r \times r}$ and the QR factorization of a matrix $A \in \mathbb{R}^{r \times r}$ are $O(r^3 + k^3)$ [5], [6], [21], so, the computational complexity for computing the condensed form (3) is $O(n^3 + p^3 + m^3)$. In addition, the computational complexities for computing matrix $X \in \mathbb{R}^{n_2 \times m_1}$ by the Schur method in [22] such that the pencil $A_{22} - XA_{42} - sE_{22}$ is stable relative to $C_\sigma$ is $O(n_2^3 + m_1^3)$ which is at most $O(n^3 + m^3)$. Thus, the computational complexity of Algorithm 1 is $O(n^3 + p^3 + m^3)$.

We now present an example to illustrate Algorithm 1.

1) **Example 1:** Let $(C_\sigma, \Omega) = (C^- , C/C^-)$, and

$$
E = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \ A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}
$$

$$
B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \ C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
$$

and $D = \begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & -2 \\ 0 & 0 & 0 \end{bmatrix}$. 


Then system (1) is stabilizable and detectable relative to \( C_x \), it has a finite zero at \( s = 1 \), and

\[
G(s) = \begin{bmatrix}
1 & 2 & -2 \\
0 & -1 & -2 \\
0 & 0 & 0
\end{bmatrix} + s \begin{bmatrix}
1 & 3 & 0 \\
1 & 4 & 2 \\
0 & -1 & -2
\end{bmatrix} + s^2 \begin{bmatrix}
1 & 4 & 2 \\
0 & 0 & 0 \\
0 & 1 & 4
\end{bmatrix}.
\]

Obviously, \( G(s) \) is non-proper and is not of full normal column rank. By Algorithm 1, we get

\[
U(s) = \begin{bmatrix}
-sE_v + A_U \\
& & C_U \\
& & & D_v
\end{bmatrix}
\]

and

\[
\mathcal{G}(s) = G(s)U(s) = \begin{bmatrix}
x_1 & x_2 - 1.06066017177982 & -2 \\
x_1 & x_2 + 0.35355396039327 & -2 \\
0 & 0 & 0
\end{bmatrix} + s \begin{bmatrix}
1 & x_2 + 0.35355396039327 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix} + s^2 \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

where

\[
A_U(:, 1:3) = \begin{bmatrix}
0 & 1 & 0 \\
0 & -1.14142135623731 & -1.141421356237309 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
A_U(:, 4:5) = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

\[
E_v = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}, \quad B_v = \begin{bmatrix}
-x_1 & -p_2 \\
0 & 0 \\
0 & 0 \\
-1 & 0 \\
-1 & 0
\end{bmatrix}
\]

\[
C_v = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}, \quad D_v = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix}
\]

where \( x_1 > 0 \) and \( x_2 \) is arbitrary. It is easy to know that \( \mathcal{G}(s) \) has a zero at \( s = -x_1 \in \mathbb{C}^+ \). This example illustrates that by choosing \( x_1 \) as required, we can get new desired zeros of \( \mathcal{G}(s) \).

### IV. Conclusion

In this technical note, we have presented a numerically reliable method for computing an interactor \( U(s) \) relative to any given stability region and offending zeros for a general descriptor system (1). Our main result, Theorem 3, is based on the condensed form (3) which is independent of the given offending zeros \( \Omega \) and does not involve the computation of any (generalized) eigenvalues/eigenspaces of the system matrix of system (1), it is computed by only orthogonal transformations and hence is stable. We have parameterized the interactor \( U(s) \) in terms of the parameter \( X \) in (8), by means of which the interactor \( U(s) \) can be tuned and the new desired zeros of the compensated matrix \( G(s)U(s) \) can be chosen in \( C_x \).

How to apply the results obtain in this technical note to solve the \( H_\infty \) control problem for descriptor systems is an interesting future research issue. Offending boundary zeros of descriptor systems may contribute to controller order reduction [17]. Therefore, it seems that canceling these zeros using an interactor may yield a complicated controller since one has to combine the interactor for compensated system and the inverse of the interactor to obtain a controller for the original control problem. Hopefully, these possible drawbacks can be overcome since the compensated system \( G(s)U(s) \) and the inverse of the interactor \( U(s) \) are explicitly given in (9) and (12), respectively. This is an interesting issue for our future research.

### REFERENCES


