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<tr>
<td>Author(s)</td>
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<tr>
<td>Citation</td>
<td>Proceedings of the IEEE, 2013, v. 101, p. 451-472</td>
</tr>
<tr>
<td>Issued Date</td>
<td>2013</td>
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<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10722/185860">http://hdl.handle.net/10722/185860</a></td>
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Skin-Effect Loss Models for Time- and Frequency-Domain PEEC Solver

This paper addresses the problem of modeling broadband skin-effect loss for conducting planes and 3-D shapes for the problems of signal, power, and noise integrity in electronic industry.

By Albert E. Ruehli, Life Fellow IEEE, Giulio Antonini, Senior Member IEEE, and Li Jun Jiang, Member IEEE

ABSTRACT | A challenging and interesting issue for the solution of large electromagnetic problems is the efficient, sufficiently accurate modeling of the broadband skin-effect loss for conducting planes and 3-D shapes. The inclusion of such models in an electromagnetic (EM) solver can be very costly in compute time and memory requirements. These issues are particularly important for the class of signal, power, and noise integrity (NI) problems. In this paper, we concentrate on partial element equivalent circuit (PEEC)-type methods which are suitable for the solution of this class of problems. Progress has been made recently in the design of skin-effect models. The difficult issues are broadband frequency-domain or time-domain problems. These models are considered in this paper. We present several solution methods, and we compare results obtained with these approaches.

KEYWORDS | Modified nodal analysis (MNA); noise integrity (NI); partial element equivalent circuit (PEEC); power integrity (PI); signal integrity (SI); transmission line (TL)

I. INTRODUCTION

A large class of problems has conventionally been solved using integral equation techniques. These include scattering and microwave problems, e.g., [1] and [2], and antenna structures, e.g., [3]. Another set of problems has become important recently, which includes electronic packaging, signal integrity (SI), power integrity (PI), and noise integrity (NI) problems [4]–[6]. NI problems like electromagnetic interference (EMI) noise are also very important. Frequency-domain applications include radio-frequency (RF)/microwave on-chip circuit issues like the modeling of circuits with the correct Q-factor which is sensitive to losses. Losses are also important for the modeling of antenna efficiency and gains in an integrated circuit (IC) environment. Time-domain solutions are important for many very large-scale integration (VLSI) SI/PI/NI and channel response problems. Our observation is that, in general, time-domain solutions are computationally less expensive than solutions in the frequency domain.

The solution of these large electromagnetic problems includes the modeling of the broadband skin-effect loss for the conducting planes and 3-D conductors. The computation with such models in an electromagnetic (EM) solver can be very costly in both compute time and memory requirements. We found that validation of the results with other models is very important. To our surprise, some approaches performed below expectations. It became clear during our research that a broadband model for problems with a significant 3-D current redistribution with frequency or time is challenging. The low-frequency inductive behavior is difficult to model while the prediction of resistive dependence seems to be easier to find. Of course, at very high frequencies, the currents are confined close to...
the surface, which makes the prediction of the current flow easier for many problems.

A. Transmission-Line-Based Models

In the last decade, skin-effect loss models for transmission lines (TLs) both in time and frequency domains have been a key research issue. Today, many skin-effect models exist for EM codes for 2-D TLs [7]–[11]. Such models are much easier to construct from a current flow point of view for several reasons. The transverse electromagnetic (TEM) mode allows the decoupling of the inductance and capacitance solution, which is computed based on 2-D solvers. This separation is also partially true for the partial element equivalent circuit (PEEC) solution [12]. Hence, the inductance/skin-effect loss problem is solved separately from the rest of the TL solution. The 2-D cross-sectional dependence of the current distribution is taken into account in more accurate 2-D models. However, we should note that these results are still based on a 1-D current flow direction, assuming infinitely long parallel conductors or practically very long conductors where the contribution of the end effects can be ignored. Also, a realistic TL problem usually involves only a small number of different cross sections for which we can precompute the skin-effect model data. This also has been done for on-chip skin-effect models [13], [14]. The simplifying aspect is that one direction current flow for TLs is only dependent on the distribution of the current in the 2-D cross-sectional dimension.

B. Causality, Hilbert Consistency, Stability, and Passivity

A different and important aspect is the fact that electromagnetic models should be causal and Hilbert consistent. This issue is well understood for TL models [7], [9], [15]–[18]. Other issues are stability and the stronger criteria of passivity. These criteria are hard to meet for 3-D electromagnetic solvers. This may lead to instabilities in the time domain while it can lead to inaccuracies in the frequency domain. Steady progress has been made toward solutions. Unfortunately, for PEEC models with delays or retardation, and other integral equation models, stability cannot be guaranteed. This issue has been the subject of another set of works. However, any part of the model which can be represented by nondelayed passive circuit elements is automatically Hilbert consistent in a circuit sense and passive. For this reason, we like to use circuit models as much as possible.

C. Three-Dimensional Current Flow Solution

The 3-D skin effect is much more difficult and time consuming to model in EM solvers due to the potential change in the current path with frequency or time. We use the conventional three orthogonal unit vectors to represent the arbitrary current flow directions. This is unlike the well-defined 1-D TL current flow along the direction of propagation. However, not all 3-D conductor problems are difficult to solve. The simplest case in this class is narrow frequency band modeling at very high frequencies where the skin depth is very small compared to the conductor thickness. In this case, the conductor thickness is not an issue, and conventional 1-D skin-effect models [19] can be used for this situation. In fact, these models are used widely, as has become evident to us from this work, even for a problem where the accuracy is insufficient. The appropriate use of these models needs a good understanding of the geometry.

To limit the scope of this paper, we do not consider finite element solution techniques. We will concentrate on the solution of 3-D wide-band problems. Unfortunately, very few skin-effect models truly represent the 3-D current flow. However, several researchers have recently made advances in skin-effect models for 2-D and 3-D current flow. Among the integral-equation-based solutions is the 3-D volume filament model (3D-VFI) as it is used in several papers [20]–[22] and for nonorthogonal shapes in [12] and [23]. In the 3D-VFI model, the conductors are subdivided into filament cells in all three directions, for nonorthogonal conductors as well. The model is very accurate but it can be computationally expensive without the use of fast techniques. For this reason, other skin-effect models have been designed and new research like the work presented in this paper is in progress. With PEEC, compute time savings have been obtained with very large aspect ratio cell subdivision of the conductors. Hence, it is not surprising that, for our class of problems, we prefer rectangular or quadrilateral cell shapes rather than the popular triangular Rao–Wilton–Glisson (RWG) basis functions [24]. A distinction between the two integral equation approaches is the representation of conductors, PEEC with specific shaped cells, which can have very large aspect ratios, the RWG with a large number of easier to compute triangular shapes.

Approximate skin-effect models have been attempted for thick conductors, e.g., [25] and [26]. However, a general solution for these problems is very challenging. Differential equation models like the generalized surface impedance (GSI) [27] have a full cell-to-cell impedance coupling model inside the lossy conductor. It is evident that due to the large conductor spacings, a larger number of the impedance cells are coupled in all three directions internal to the conductor. An important aspect for these problems is the 3-D skin-effect loss representation for discontinuities like corners in conductors. Approximate VFI models were implemented in [22] and [28] for 3-D geometries to obtain a more compute-time-efficient solution. In this model, all the nodes for a cross section are shorted. However, such models have to be applied with care for general geometries.

D. General Purpose Solutions

General purpose electromagnetic solvers are needed to deal with a large variety of difficult problems. The PEEC method with a modified nodal analysis (MNA) solver provides separate inductive–resistive and capacitive
circuits, which leads to a well-conditioned low frequency and a proper dc solution [29], [30]. Other integral equation formulations [31], [32] have been devised to improve the low-frequency behavior for formulations. Recently, a new description has evolved for the integral equation formulations where the unknowns are included, such as the current charge integral equation (cqIE) for [31] and [32], since both current and charge are used as unknowns. Hence, an appropriate name for the electric field PEEC IE seems to be the potential current electric field integral equation (pcEFIE) for the electric field part since the unknowns are potentials (p) and current (c). The pcEFIE formulation has other advantages since potentials are very useful output quantities for many applications.

1) EXP Skin-Effect Loss Models: In this paper, we compare different models for the skin-effect representation. We call a class of skin-effect models the EXP type, for lack of a better name. They are based on the analytical 1-D exponential behavior for the penetration of the skin-effect current [19]. Several authors have developed different frequency-domain EXP models, probably in part unaware of each other’s research. Some of these works are [14], [33]–[35]. In fact, the first model presented in this paper for thin conductors is related to this type of model. We found that applying an EXP-based model to nonthin conductors was difficult. We have to recognize that the EXP model is locally oriented and that for thicker conductors the representation inside the conductors requires additional currents as is apparent from this work. However, we show that applying this type of model for very thin conductor problems yields efficient solutions.

2) GSI Models: A model which is suitable for larger cross sections is the so-called GSI model [27], [36], [37]. In this approach, a differential equation description is used, internal to the conductor, which relates the surface currents such that they can be used with the PEEC model. The current dependence in both transverse directions is included in the implementation [37]. However, the current is restricted to be in the longitudinal direction only, as in TL models. It is obvious that for the general 3-D current flow, this solution becomes more challenging. In this work, we extend the GSI model for true 3-D current flow. However, we also found that the approach is limited if we apply the internal–external inductance approximation.

3) General Aspects of Volume Models: A general property of the PEEC technique is that widely different cell sizes and cell aspect ratios are used for efficiency reasons [12]. However, this depends on the geometrical details identified by the meshers. This issue is also of importance for skin-effect models which need to be designed for the specific geometry. Hence, the information obtained from the meshers to identify the best solution is a key component for an efficient overall solution.

The first model considered in this work is designed for relatively thin conductor geometries such as printed circuit boards (PCBs) and thin wires [38]. Fortunately, for very thin conductors, the coupling to the immediate cells in the longitudinal direction inside the conductor is small, provided that the conductor thickness is small compared to the cell size. However, the coupling between the top and bottom surfaces is very strong at low frequencies. Then, a quasi-1-D model can be applied for this case such as the ones given in [8], [33], [39], and [40]. This can be called a simpler form of the GSI model [27], [36], [37] as will be apparent in Section IV-B. Even for thin conductors, the problems are more challenging to solve with sufficient accuracy than we anticipated. We use a true 3-D current flow solution for thick conductor GSI solutions. For the thin conductor case, the current flow is still mostly laminar along the cell surface. We should note that the thin portion of the geometry may represent only part of an overall structure, which may also involve larger conductors. For the general case, we should segment the conductors into different sections with different shapes or thicknesses. Hence, we also assume that the meshers is able to identify the type of a conductor segment like a thin conductor part.

4) Surface Models: Our comparison would be incomplete without considering surface skin-effect models. These models avoid unknowns located inside the conductors. It is clear that the number of internal cells needed for a volume model has to exceed a minimum threshold for the surface approach to be more effective than the volume models, e.g., [41]. Surface techniques are based on the surface equivalence principle [42], [43]. The most popular class of surface formulations is based on the Poggio–Miller–Chang–Harrington–Wu–Tsai formulation (PMCHWT) [44]–[48] and similar models [49], [50]. The surface modeling of lossy conductor structures has recently received more attention [48], [51], [52]. In our comparison, we use the new simplified surface formulation called generalized impedance boundary condition (GIBC) [32], [53]. This approach is specifically tailored to the solution of the skin-effect problem using only two integral equations in comparison to four required for the PMCHWT.

E. Anomalous Skin-Effect Loss and Surface Roughness

For completeness, we want to consider two relatively new issues for our applications of skin-effect loss models for the problems at hand. Mainly, this is a result of the continuous miniaturization and higher speed of semiconductor circuits. The first issue is an on-chip problem. As the wire dimensions are reaching dimensions below 0.2 μm, the so-called anomalous skin effect is increasing the surface resistivity for a layer which is about 0.04 μm for copper [54]. This issue has also been considered in a recent paper where examples are given for the resistivity degradation
toward the edge [55]. It is clear that the implementation of the anomalous skin effect for very small conductors requires a volume like VFI or GSI conductor model to include the change in resistivity.

The second issue is surface conductor roughness. This issue is especially important for PCBs where roughness is used to enhance the adhesion of the conducting layers. Of course, this is in addition to the general roughness of the surfaces. Stochastic methods have been used frequently for surface scattering problems [56]; they also have been proposed for surface roughness applications [57]–[60]. For large problems, we need to resort to approaches such as [61]–[63], where the roughness is taken into account with a modification of the material properties. Hence, the techniques described in these papers can be applied also for the 3-D models in this paper.

F. Paper Outline

In Section II, we introduce the solution of the external region 1 with a pcEFIE or a cEFIE, which is the same for all the techniques considered in this paper. Then, narrow-band and wideband skin-effect models are considered in Sections III and IV. The first model considered is the 3D-VFI model, while Sections IV-B–IV-D present the thin and thick GSI models. Finally, in Section V, we consider the surface equivalence models. Section VI compares results obtained with the different skin-effect loss models.

II. ELECTRICAL PEEC INTEGRAL EQUATION MODEL

A. PEEC Circuit Equations

All models in this paper are presented in the context of an integral equation environment. As is shown schematically in Fig. 1, the problems to be solved consist of the outside region, which may be air or a dielectric. This is shown as region 1, while lossy conductors are represented as region 2. For the models presented in this paper, we use different approaches for the solution in the conducting region 2. In this section, we present the PEEC model derivation. As is done in a wide range of numerical discretized methods, we apply the method of residuals [64] to obtain a finite approximation of the integral equation. This consists of simplified trial functions to locally represent the current flow and charge distribution. Here, we use representations for the geometry in terms of circuit elements. It is interesting that they can be directly related to a Galerkin-type trial function due to fundamental symmetry of the concept of circuit elements like capacitance, resistance, and inductance. Of course, these concepts have been extended to nonrectangular coordinates. An integral or inner product is used to reformulate the integral equation into circuit equation.

The total tangential electric field at a space point \( r \) on the surface of a conductor is given by [12]

\[
E^{\text{inc}}(r, t) = \frac{J(r, t)}{\sigma} + \frac{\partial A(r, t)}{\partial t} + \nabla \Phi(r, t)
\]

where \( E^{\text{inc}} \) is the incident electric field, \( J \) is the current density in the conductor at the general coordinate point \( r \), and \( A \) and \( \Phi \) are vector and scalar potentials, respectively. Our inner product integration converts each electric field term (1) into the fundamental form \( \int E \cdot dl = V \) where \( V \) is a voltage or potential difference across the circuit element. This transforms the sum of the electric fields in (1) into the Kirchhoff voltage law (KVL).

The vector potential \( A \) in (1) for a single conductor at the field point \( r \) is given by

\[
A(r, t) = \mu \int_{V'} G(r, r') J(r', t_d) \, dv'
\]

where the volume integral \( V' \) extends over the current carrying conductor. The retardation time is given by \( t_d = t - |r - r'|/c \), which simply is the free space travel time between points \( r \) and \( r' \). Here, \( c \) is the speed of light. In the frequency domain, we can take the retardation inside the integral or outside the integral as \( e^{i(r-r')} \) as an approximation in the frequency domain. It is noted that, in the formulation derived here, both the retardation and Green’s functions are free-space quantities where

\[
G(r, r') := \frac{1}{4\pi |r - r'|}.
\]

The scalar potential is similarly

\[
\Phi(r, t) = \frac{1}{\varepsilon_0} \int_{V'} G(r, r') q(r', t_d) \, dv'.
\]
Before applying the inner product integration to (1), we compute the electric field at a point \( r \) located on the conductor surface in Fig. 1. We set the externally applied electric \( E^{\text{inc}} \) field as zero, unless we deal with a NI or external field EMI problem, and we substitute for \( A \) and \( \Phi \) from (2) and (4), respectively. The integral equation given by the tangential component of the electric field of (1) on the surface of the conductor is

\[
\mathbf{i} \cdot E^{\text{inc}} (r, t) = \mathbf{i} \cdot \left[ \frac{f(r, t)}{c_0} \right] + \mathbf{i} \cdot \left[ \mu \int \frac{G(r, r')}{\partial t} \frac{\partial f(r', t_d)}{\partial t} \, dv' \right] + \mathbf{i} \cdot \left[ \frac{\nabla}{c_0} \int G(r, r')q(r', t_d) \, dv' \right] - \mathbf{i} \cdot \mathcal{K}(r, r')\mathbf{M}(r')
\]  

(5)

where \( \mathbf{i} \) is a tangential unit vector on the conductor surfaces. The \( \mathcal{K} \) operator is used in Section V and is given in (13) and (14).

Equation (5) is a time-domain formulation which can easily be converted to the frequency domain by using the Laplace transform operator \( s = \partial/\partial t \), where the time retardation will transform to \( e^{-st} \), where \( t \) is the delay time. For electrically small structures, the delay may be ignored. In the last step to obtain the KVL loop, we apply the inner product integration. This leads to the PEEC equivalent circuit which corresponds to the loop given in Fig. 2. Of course, a realistic problem consists of multiple loops. More details are given in many papers, e.g., [12], [20], [65], and [66].

In the results presented in this paper, we do not include the capacitive contribution of the external model to the overall skin-effect loss model. For the volume filament model (VFI), this corresponds to the PEEC model inside the conductor where the capacitance part of the PEEC model does not exist since this would include resonances which would mask the results of interest. In the GSI models, we replace the series resistance in Fig. 2 with an impedance \( Z_s \) in Section IV-B. Hence, the impedance \( Z_s \) can include different loss models.

The final system of equations is given in a form MNA formulation [67]. This approach is very important for the problems at hand since it provides a solution which extends from \( dc \) to the frequency limit, which is determined by the largest cell size [29], [30]. It should be noted that this system is nonsingular at low frequencies. Also, for typical SI/PI/NI problems, additional connections are included to the PEEC model to other circuit elements, which can easily be included in the MNA matrix with additional circuit element stamps, which leads to a closed \( dc \) circuit loop.

The MNA equations were originally set up according to the scheme in [65]. In [68], unknowns were saved by eliminating auxiliary current. Now, we use formulations which are based on the matrix Kirchoff current law (KCL) [69]. This treats all PEEC model elements in a matrix stamp fashion for the known topology rather than element by element, as is done for conventional circuits.

As is done in [69], we set up the MNA system by splitting the matrix KCL into submatrices corresponding to the components as

\[
A_1 I_c + A_4 I_s + A_3 I_t + A_2 I_i = 0
\]  

(6)

where \( A_1 \) represents the connections to the potential input current sources. From Fig. 2, it is evident how we subdivide the matrix KCL according to the element types shown for a multitude of loops. Since all capacitor nodes are connected to ground, \( A_c \) is an identity matrix and \( A_s \) is minus an identity matrix. Hence, what we call the total capacitive current is

\[
I_T = I_c - I_s = -A_1 I_i - A_4 I_i
\]  

(7)

by using (6) where \( A_T \) represents the partial inductance connections. The PEEC circuit current controlled current source models are [12] given by

\[
\begin{bmatrix}
I_{i_1} \\
I_{i_2} \\
I_{i_3} \\
I_{i_4}
\end{bmatrix}
= 
\begin{bmatrix}
0 & p_{12}/p_{11} & p_{13}/p_{11} & p_{14}/p_{11} \\
p_{21}/p_{22} & 0 & p_{23}/p_{22} & p_{24}/p_{22} \\
p_{31}/p_{33} & p_{32}/p_{33} & 0 & p_{34}/p_{33} \\
p_{41}/p_{44} & p_{42}/p_{44} & p_{43}/p_{44} & 0
\end{bmatrix}
\begin{bmatrix}
I_{T_1} \\
I_{T_2} \\
I_{T_3} \\
I_{T_4}
\end{bmatrix}
\]

(8)

which can be written as \( I_i = WI_c \), where the total capacitive current is \( I_T = I_c - I_i \), and where the element retardation can be included in each term. We can write these two equations as \( WI_c = (I + W)I_i \), where we use \( I \) for the identity matrix to distinguish it from the current vector. If
we multiply (7) by \((\hat{I} + W)\), where we also use the
previous equation, then
\[
(\hat{I} + W)I_c - W I_c = -(\hat{I} + W)A I_c - (\hat{I} + W)A I_c
= I_c.
\] (9)

The branch relations for the capacitors are \(I_c = P_d^{-1}s \Phi_n\),
where \(P_d\) is a diagonal matrix with \(p_{mn}\) on the diagonal or
\(P_d = \text{diag}(p_{11}, p_{12}, \ldots, p_{nn})\). Also, for the inductive-
resistive branch, \(V_\ell = (sL_p + R)I_\ell\). Here, \(L_p\) is the matrix
of coupled partial inductances and \(R\) is a diagonal, or
nearly diagonal, matrix.

Finally, the branch to nodal voltages is related to the
potential as [69], [70]
\[
V_\ell = A_i^T \Phi_n.
\] (10)

Note that in a PEEC circuit we have the real ground
node which also is the potential at infinity \(\Phi = 0\). Entering the
above equations into the MNA matrix, observing that \(A_i\) and
\(A_j\) are simply diagonal matrices, we can write for the
cEFIE model
\[
\begin{bmatrix}
    sP_d^{-1}(\hat{I} + W)A_i \\
    -(sL_p + R)
\end{bmatrix}
\begin{bmatrix}
    \Phi_n \\
    I_\ell
\end{bmatrix}
= \begin{bmatrix}
    (\hat{I} + W)A I_c \\
    0
\end{bmatrix}.
\] (11)

Hence, we can see that using the matrix KCL, we can
stamp multiple elements of the same type in a matrix
sense.

In principle, we could also reduce it by eliminating the
potential \(\Phi\) in (11) to get a conventional current-only
cEFIE model to
\[
\begin{bmatrix}
    sL_p + R \\
    1 + A_i^T P_d(\hat{I} + W)A_i
\end{bmatrix}
I_\ell
= \begin{bmatrix}
    1 + A_i^T P_d(\hat{I} + W)A I_c
\end{bmatrix}.
\] (12)

However, the low-frequency \(1/s = 1/(2\pi f)\) behavior of the
current-only formulation is clearly apparent. Hence, we
use (11) for the SI/PI/NI class of problems to avoid the
singularity.

We need an additional term in the EFIE, including
magnetic currents \(M\) for the surface-equivalence-
principle-based techniques in Section V. This can be
done both for the pcEFIE and the cEFIE formulation. The
magnetic-current-dependent part is given by
\[
\mathcal{K}(M) = \frac{1}{\epsilon_n} \nabla \times F_n
\] (13)

where the electric vector potential \(F_n\) is
\[
F(r, t) = \epsilon_n \int G(r, r')M(r', t_d) \, ds'.
\] (14)

Hence, \(K\) will be utilized in Section V.

The generalized impedance boundary condition
(GIBC) model requires the magnetic coupling, which is
included in the pcmEFIE model in the equivalent circuit in
Fig. 3 as dependent voltage source \(V_{e1}\).

### III. NARROWBAND HIGH-FREQUENCY
SKIN-EFFECT MODEL

Narrowband or single-frequency high-frequency skin-
effect models are the most simple to construct. We can
use a 1-D EXP model for current penetration or skin depth
\(\delta\) given by [19] where
\[
\delta = 1/\sqrt{\pi \mu \sigma f}
\] (15)

where \(f\) is the frequency, \(\sigma\) is the conductivity of the
conductor, and \(\mu\) is the permeability, which is usually \(\mu_0\).
It is evident from this that at high frequencies, the skin
depth restricts the current to a well-defined path close to
the surface. We assume that a conductor or a ground plane
is, in this case, thicker than the skin depth \(\delta\). Hence, we
can assume that the two sides of the plane are isolated
from each other. As shown in Fig. 5, we assume that we cut
out a surface cell of a size \(\Delta x\) and \(\Delta y\), where at a given
angular frequency \(\omega\), the equivalent current penetration
thickness is \(\delta\). This results in an equivalent impedance of
the cell of \(Z_i = R_i + j \omega L_i\), with
\[
R_i = \frac{\Delta x}{\sigma \delta \Delta y}, \quad L_i = \frac{\Delta x}{\omega \sigma \delta \Delta y}.
\] (16)
Further, due to the small diffusion distance $\delta$ into the conductor, we also can assume that the lateral loss coupling between two neighboring conductor surface cells is small. As a consequence, all the skin-effect loss couplings to other cells can be ignored.

To couple the two models, we simply place the above impedance $Z_s = R_s + j\omega L_s$ into the PEEC circuit in Fig. 2, where the current in the model is $I = I_x$, the current through the model. For very high frequencies, we do not have to take the couplings to other cells into account, which we consider in the three models considered in Section IV.

**IV. WIDEBAND SKIN-EFFECT MODELS**

At very low frequencies, the conductors are transparent to the magnetic field, and the conductors essentially reduce to resistive circuit models. At the highest frequencies, the conductor model reduces to decoupled top and bottom surfaces as is discussed in Section III. Often, the top and bottom layers are assumed to be totally independent. Hence, a wideband skin-effect conductor model has to be very general to cover the entire frequency range with both situations.

**A. 3D-VFI Skin-Effect Model**

The first wideband skin-effect model we consider is the 3D-VFI model. In this approach, the current can be represented by components in all three directions to properly represent the general current flow case. We will not give a detailed derivation of the model since numerous papers are available, which are cited in Section I.

The VFI model requires the subdivision of the thickness into cells, as shown in Fig. 4. This makes possible the distribution of the current among the layers. Further, the horizontal $x$-$y$ cells for a flat conductor are shown in Fig. 5. Finally, the vertical connections in Fig. 6 lead to the vertical 3-D part of the model. Hence, all three directions of currents internal to the conductors are properly represented in this conventional PEEC mesh.

The PEEC equivalent circuit corresponding to a conductor node is shown in Fig. 7. It is clear that the partial inductive circuit model is inductively connected to the partial inductances in Fig. 2 beside all the internal nonorthogonal cells. This leads to large coupled sections in the inductance part of the circuit matrix.

An exception is the model for on-chip conductors where for the small conductors the skin effect is only moderate, which results in only a few partial inductances to represent the skin effect [71]. It is clear that without speedup techniques, only medium-sized problems can be solved on a computer with moderate memory. Different approximations have been made to include the partial inductance couplings [72]–[74]. In the approach in [22] and [28], the cross-section elements in (7) are shorted. This approach works well for shapes with a small diameter compared to the length. We found that the accuracy of the 3D-VFI model is good even for a modest number of cells.
For this reason, we use it as the standard solution for all the test problems in this paper.

### B. Approximate GSI Thin Conductor Skin-Effect Model

In Sections IV-B–IV-D, we consider the class of GSI models described in Section I. In these models, the mutual inductive coupling between the inside of a conductor and the outside of the surface are ignored. These models are related to the inductance part to the internal–external inductance concepts, which have been used for many years [75]. The internal inductance and loss model is based on a differential equation formulation. In this section, we treat the case where the conductors are thin, while thick conductors are considered in Section IV-D.

In this case, the external inductive model consists of a zero thickness partial inductance, which is connected in series to the internal-differential-equation-based model, the same way as in the surface model in Section II. This represents the inductance in Fig. 2 for the conductor surfaces [27]. This avoids the inductive couplings from the inside of the VFI skin-effect model. Hence, the approach can be less costly than the VFI model.

The solution of the current diffusion problem inside the conductor is fundamentally different from the external PEEC propagation model in Section II. For the thin conductor problem, the skin-effect model must represent the EXP current penetration on opposite surfaces. Hence, the mesh cells on the sides must be lined up. This corresponds to top and bottom cells in Fig. 6. However, due to the small assumed conductor thickness, the coupling to other cells including the neighboring cells is ignored. This leads to a very sparse internal impedance coupling matrix \( Z \). For broadband applications, the skin depth changes widely over the frequencies of interest. Hence, the impact of the skin effect on the impedance is large for thin conductor areas.

The next task is to provide an electromagnetic and finally a circuit model for the diffusion of current inside the thin conductor. Especially for the time domain, we want to keep the order or the number of poles of this skin-effect model as low as possible. We derive a simplified model for the thin conductor. The fundamental idea is based on the derivation of an appropriate equivalent circuit for the EXP function based on [39]. The skin-effect model in this work was designed for 1-D current dependence for a single surface. However, we require a model for the two sides of the thin conductor. The original model was designed for a TL 2-D model, but the basic approach also works in the context of partial inductances.

We approximate the model by breaking the thickness into layers, as shown in Fig. 4. However, as we consider below, we do not use uniform layers. This leads to an equivalent circuit model in the z-direction as is indicated by the dashed lines. This is the dominant skin-effect direction for the thin layer structures. The Maxwell equations for the interior of the conductor are

\[
\nabla \times E = -\mu \frac{\partial H}{\partial t} \quad (17)
\]

and

\[
\nabla \times H = J = \sigma E. \quad (18)
\]

We observe that for the conductor a simplified solution is that the following fields are nonzero: \( E = xE_x \) and \( H = yH_y \). Therefore, this reduces (17) to

\[
\frac{\partial E_x}{\partial z} = -\mu \frac{\partial H_y}{\partial t}. \quad (19)
\]

Further, (18) will also reduce to a simple form

\[
\frac{\partial H_y}{\partial z} = -\sigma E_x \quad (20)
\]

since in the derivation we discretize the plane thickness in the z-direction only shown in Fig. 4. Of course, we are also using the usual subdivision of the conductor into PEEC cells for the currents in the x- and y-directions shown in Fig. 5.

We start with \( z_1 = 0 \), and we label the thickness of the layers as \( k = 1, \ldots, N_k \), where \( N_k \) is the number of layers. Hence, the top layer ends at \( z_N_k = d \). Also, the thickness of the layers is given by \( \Delta z = z_{k+1} - z_k \).

To derive an equivalent circuit, we start with (17), and we multiply it by the cell length \( \Delta x \). The voltage drop along the layer \( k \) in the x-direction is given by \( \Delta x E_{x,k} = V_{x,k} \), which leads to

\[
V_{x,k+1} - V_{x,k} = -\mu \Delta x \frac{\partial H_{y,k}}{\partial t}. \quad (21)
\]
To convert this into an equivalent circuit, we define the differential inductance of layer \( k \) in the conventional way as
\[
L_k = \frac{\Phi_k}{I_{y,k}} = \frac{B_{y,k} \Delta x \Delta z_k}{I_{y,k}}
\]  
which results in \( B_{y,k} = \mu H_{y,k} \) and \( H_{y,k} = I_{y,k}/\Delta y \) to
\[
L_k = \frac{\mu \Delta x \Delta z_k}{\Delta y} .
\]  
(23)

With this, we can simplify (21) to
\[
V_{x,k+1} - V_{x,k} = -L_k \frac{\partial I_{y,k}}{\partial t} .
\]  
(24)

which is the circuit equation for an inductance element.

Similarly, starting from (20), we again replace the magnetic field with \( H_{y,k} = I_{y,k}/\Delta y \) and \( E_x = V_x/\Delta x \) to obtain
\[
\frac{1}{\Delta y} \frac{\partial I_{y,k}}{\partial z} = -\frac{V_{x,k}}{\Delta x} .
\]  
(25)

This leads to the local resistance of the block as
\[
\frac{1}{R_k} = \frac{\sigma \Delta z_k \Delta y}{\Delta x} .
\]  
(26)

If we numerically approximate the derivative \( \partial I_{y,k}/\partial z = (I_{y,k} - I_{y,k+1})/\Delta z \), we can finally find that
\[
I_{y,k} - I_{y,k+1} = \frac{V_{x,k}}{R_k} .
\]  
(27)

It is a small step to show that (21) and (27) correspond to the equivalent circuit in Fig. 8. Note that we assigned half the differential inductance to each vertical branch to get a symmetric circuit. Also on purpose, we make sure that the surface impedance between the terminals \( A \) and \( C \) is
\[
\lim_{\omega \to \infty} \text{Im}(Z) = 0
\]  
(28)

such that the circuit will become resistive for very high frequencies. This eliminates unwanted resonances at very high frequencies where the model becomes invalid due to the discretization. This circuit skin-effect part of the model is automatically Hilbert consistent since we represent it with the passive nondelay equivalent circuit in Fig. 8 [69].

1) Physics-Based Macromodel: Reduced-order models (ROMs) are usually constructed using approaches like VectorFit or other analytical model order reduced (MOR) approaches. Here, we use a direct, so-called physics-based MOR, PM–MOR, which is accomplished directly based on a physical simplification of a model.

In the 1-D model in Fig. 8, we have reduced the thickness of the cells toward the current carrying surfaces. This results in a drastic reduction of the number of layers required and in the number of eigenvalues or poles. We call the obtained model with the nonuniform meshing in the \( z \)-direction PM–GSI and also the thin GSI model since it applies only to thin conductors. We note that in [76] the cells are also reduced in thickness toward the surfaces for a VFI model, which results in a less expensive model. In fact, it is essential for this model.

As a test example for the PM–GSI model, we start out with a 1-D model based on the derivation in Section IV-B with the equivalent circuit (Fig. 8). The thickness of the first layer is chosen to be \( d_1 = \delta/2 \), where \( \delta \) is the skin depth for the highest frequency in the spectrum. The cell thickness is increased by a factor \( \alpha \) starting from the surface layer with the dominant current flow. Hence, for \( L \) layers, the thickness \( d \) of the conductor and the thickness multiplying factor for the thickness are related by the finite geometric series with the sum
\[
d = d_1 \frac{1 - \alpha^{(N_l+1)}}{1 - \alpha}
\]  
(29)

where \( d_1 \) is again the thickness of the first layer.
We would like to limit the number of layers \( N_l = 4, \ldots, 12 \) to keep the number of poles as low as possible in the MOR representation. To test our 1-D model, we compare it with the 1D-EXP model in Fig. 9 for the inductance and in Fig. 10 for the resistance. The results show very good agreement for a cell which is 30 \( \mu \)m thick with a seven-layer PM–GSI model. We are well aware that at low frequencies, a simple 1D-EXP model does not give a correct result as is apparent from both figures. However, the equivalent circuit yields the correct answer. The reduction in inductance for high frequencies in the PM–GSI model is due to the high-frequency limit introduced.

For the thin conductors, it is essential to include the top to bottom conductor coupling since the internal coupling between the top and the layer is very strong. However, the coupling to the neighboring cells is relatively weak. We added an additional port to the model shown in Fig. 8 to compute the trans-impedance between the ports. For the C to A port, we have \( Z_{11} \), and for the S to U, we have \( Z_{12} \), as shown in Fig. 8. We give an example for the evaluation of the real and inductive part of \( Z_{12} \). In the example, with a layer thickness of 2 \( \mu \)m, we have four increasing layers and four symmetrically decreasing layers toward the other surface. In the results in Fig. 11, the imaginary part is divided by \( \omega \) to get the inductive coupling. Hence, we have the desired two-port model for the cells on the opposite side of the conductors.

2) Frequency-Domain Solver for Physics-Based Macromodel: The evaluation of the ladder circuit impedances in the frequency domain, as shown in Fig. 8, can be solved efficiently due to the simple analytical solution for ladder circuits, which are available [77]. Hence, this can speed up the computation of the impedances which can be directly stamped into the MNA matrix in the frequency domain. The MNA matrix for the PEEC circuit with the impedance is similar to (11)

\[
\begin{bmatrix}
  s P^{-1}\hat{d} & (\hat{I} + W) A_i \\
  A_i^T & -(s L_p + Z_{S}(s))
\end{bmatrix}
\begin{bmatrix}
  \Phi_p \\
  I_i
\end{bmatrix} =
\begin{bmatrix}
  (\hat{I} + W) A_i I_i \\
  0
\end{bmatrix} \tag{30}
\]

where \( L_p \) is the zero thickness surface partial inductance matrix, accounting for the magnetic field coupling occurring among surface currents, \( Z_s \) is the model impedance, \( P_i \) and \( W \) are the coefficients of potential matrix describing the electric field coupling among surface charges, \( A_i \) is
the connectivity matrix, and $I_i(s)$ represents potential current sources at the nodes.

C. Approximate Thin Wire Skin-Effect Loss Model

The model in Section IV-B is efficient for thin-sheet-type geometries. Another similar class of problems are thin wires as they often occur in IC interconnect wires and wire bonds. Wire PEEC models have been considered before [84]. An example of a wire segment is shown in Fig. 12. Starting out with the same equations (17) and (18), a cylindrical skin-effect model can be derived, provided that the proximity effect is small enough. This is the case if the center-to-center distance between the wires is $> 2d$.

The current is strictly in the $z$-direction with a circular symmetric current density $J = J_z \hat{z}$. Then, the internal field is centered around the dashed line in Fig. 12, hence, the magnetic field in the $\phi$-direction in a cylindrical coordinate system where $H = H_\phi \hat{\phi}$. This considerably simplifies the vector operations.

If we take (18) and integrate it over the cross section, we get

$$\int_s (\nabla \times H) \cdot \hat{n} \, ds = \oint_t H \cdot d\ell = \int_s J \cdot \hat{n} \, ds$$

(31)

which reduces

$$I_2(r, t) = 2\pi \alpha H_\phi(r, t)$$

(32)

for $0 \leq r \leq a$ in our specific case.

If we take (17) and use $J = \sigma E$ to get

$$\nabla \times J = -\mu \sigma \frac{\partial H}{\partial t}$$

(33)

where with the current $J$ in the $z$-direction and $H$ in the $\phi$-direction, we get

$$\frac{\partial I_z}{\partial t} = -\mu \sigma \frac{H_\phi(r)}{\partial t}.$$ 

(34)

From this and (32), we finally get a first-order differential equation

$$\frac{\partial I_z}{\partial t} = -\frac{\mu \sigma}{2\pi r} \frac{\partial I_2(r)}{\partial t}.$$ 

(35)

The goal of this work is to come up with an efficient circuit model for the internal cylindrical conductor from the outside layers to the inside ones where the vertical current is impeded to go to the inside layers by the inductances. We can transform (35) by multiplying both sides by the section length $\ell$ and dividing by $\sigma$ and by approximating the spacial derivative for the first section as

$$\frac{\Delta z I_1 - I_2}{r_1 - r_2} = -\frac{\mu \Delta z}{2\pi r_2} \frac{\partial I_1}{\partial t}.$$ 

(36)

Finally, by multiplying by $r_1 - r_2$ and by replacing $I_1 = (I_p - I_1)/a_1$ and $I_2 = (I_1 - I_2)/a_1$, we get

$$\frac{\Delta z I_p - I_1}{\sigma a_1} - \frac{\Delta z I_1 - I_2}{\sigma a_1} = -\frac{\mu \Delta z (r_1 - r_2)}{2\pi r_2} \frac{\partial I_1}{\partial t}.$$ 

(37)

where $I_p$ is the current from the external partial inductance $\Delta z = |z_e - z_i|$. We define

$$L_2 = \frac{\mu |z_e - z_i|(r_1 - r_2)}{2\pi r_2}$$

(38)

as the differential diffusion inductance. Also, on the left-hand side of the equation, we recognize the resistors of the form $R_i = |Z_e - Z_i|/(\sigma a_1)$, where the area of the first section is $a_1 = \pi (r_1^2 - r_2^2)$, etc. Of course, for the first section, $r_1 = a$. The currents $I_1, I_2, \ldots$ pertain to the appropriate layers. With this, we can recognize the loop form of the circuit equation corresponding to (37) to be

$$R_1 I_p - R_1 I_1 + L_2 \frac{\partial I_1}{\partial t} + R_2 I_1 - R_2 I_2 = 0.$$ 

(39)

Since all the other loops are of the same form, we can construct the equivalent circuit in Fig. 13. Note that the
partial inductance of the outer zero thickness shell (43) has been added to the model. We emphasize that, since this model is a self-term, it can be applied to any orientation for a cylindrical conductor in the global coordinate system.

The formula for a partial inductance is given by

\[ L_{pm} = \frac{1}{a_m a_l} \frac{\mu}{4\pi} \int_{v_m} \int_{v_l} \mathbf{t}_m \cdot \mathbf{t}_l \frac{dV_m}{dV_l} \] (40)

where \( \mathbf{t} = \hat{z} \) are the unit vectors tangential to the currents and where \( R_{m,\ell} \) is the distance metric between the source and observation points. For a round bar, the partial inductance can be approximated as

\[ L_{p11} = \frac{\mu_0}{2\pi} \left[ \ln \left( \frac{\ell}{a} + \sqrt{\left( \frac{\ell}{a} \right)^2 + 1} \right) - \sqrt{1 + \frac{(a^2)}{\ell^2}} + \frac{a}{\ell} \right] \] (41)

where \( a \) is the wire radius and \( \ell \) is the length. We originally assumed that this is an approximation to the low-frequency inductance for a cylindrical conductor. However, the value we obtained from it seems to be closer to the value for a tube, at least for the aspect ratios we are considering in this paper. This approximate equation seems to be an average value for the partial inductance.

For this model, an important limiting partial inductance is given by the high-frequency limit. This infinite frequency limit is the partial inductance of a zero thickness cylindrical tube with a radius \( a \).

The partial inductance (40) for the cylinder reduces to

\[ L_{p_{11}} = \frac{\mu}{4\pi} \int_{\phi=0}^{\pi} \int_{z=0}^{\ell} \frac{dz \, dz' \, d\phi}{\sqrt{4a^2 \sin^2 \left( \frac{\phi}{2} \right) + (z - z')^2}} \] (42)

by recognizing that the symmetry can be used to reduce the fourfold integral to a threefold integral. We were able to analytically solve the integrals for the case of interest where the length \( \ell \) of the wire is longer than the diameter \( d = 2a \). The result for the tube conductor or high-frequency limit is given by

\[ L_{p_{11}} = \frac{\mu \ell}{4} \left[ \left( \frac{k^2}{480} + \frac{k^4}{1280} + \frac{1}{3600} \right) \frac{\pi}{3} + \left( \frac{1}{18} - \frac{k^4}{24} \right) \right] \]

\[ + (-2 \log(\ell) + 6 \log(2) + 2 + 2 \log(a)) \]

\[ - 4 \log(k\pi) \left( \frac{1}{\pi} + \frac{8a}{\ell \pi^2} \right) \] (43)

Evaluating (43) resulted in 0.54995 nH, which is the high-frequency limit for \( a = 0.05 \) mm and \( \ell = 1 \) mm, while the approximate formula (41) leads to 0.54775 nH. Hence, the approximate formula does not represent a low-frequency result for the full cylinder, since its value is supposed to be larger than the high-frequency result.

Figs. 14 and 15 show a comparison for the internal differential inductance using the analytical model (43) and the equivalent circuit model in Fig. 13. We note that this model is very similar to the small thickness GSI model. Hence, it is suitable for the implementation in a practical solver. We should note that the work in [79] also considers some mutual couplings and the radiation resistance for the cylinder.

D. Full 3-D Skin-Effect Models

As remarked in the introduction, we also attempted to construct a full 3-D model using the internal–external inductance approximations for general 3-D conductors. To save compute time, we made an internal 3-D differential equation model. Hence, the fundamental approach is
The first curl-Maxwell equation is
\[ \nabla \times E = -sB - \sigma_n H \]  
(44)

\[
\frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} = -(\sigma_m + s\mu)H_x \\
\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -(\sigma_m + s\mu)H_y \\
\frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} = -(\sigma_m + s\mu)H_z \\
\]
which can be rewritten as
\[ \Delta y \Delta z \left( \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial z} \right) = -(\sigma_m + s\mu) \frac{\Delta y \Delta z}{\Delta x} I_x \]  
(46a)
\[ \Delta x \Delta z \left( \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right) = -(\sigma_m + s\mu) \frac{\Delta x \Delta z}{\Delta y} I_y \]  
(46b)
\[ \Delta x \Delta y \left( \frac{\partial E_y}{\partial x} - \frac{\partial E_z}{\partial y} \right) = -(\sigma_m + s\mu) \frac{\Delta x \Delta y}{\Delta z} I_z \]  
(46c)

Resistances and inductances are now given by
\[ R_{mx} = \sigma_m \frac{\Delta y \Delta z}{\Delta x} \quad R_{my} = \sigma_m \frac{\Delta x \Delta z}{\Delta y} \quad R_{mz} = \sigma_m \frac{\Delta x \Delta y}{\Delta z} \]  
(51)

\[ I_x = \mu \frac{\Delta y \Delta z}{\Delta x} I_x \quad I_y = \mu \frac{\Delta x \Delta z}{\Delta y} I_y \quad I_z = \mu \frac{\Delta x \Delta y}{\Delta z} I_z. \]  
(52)

The global impedance matrix \( Z \) is generated by a diagonal block concatenation of \( Z_x, Z_y, \) and \( Z_z \) matrices
\[ Z = \text{blkdiag}([Z_x, Z_y, Z_z]). \]  
(53)

The matrix form of (47) is
\[ A \psi V = -ZI. \]  
(54)

where the rotational voltages are
\[ V_x = E_x \Delta x \quad V_y = E_y \Delta y \quad V_z = E_z \Delta z \]  
(48)

the currents are defined as
\[ I_x = H_x \Delta x \quad I_y = H_y \Delta y \quad I_z = H_z \Delta z \]  
(49)

and the impedances are
\[
Z_x = \frac{\Delta y \Delta z}{\Delta x} (\sigma_m + s\mu) = R_{mx} + sL_x \]  
(50a)
\[
Z_y = \frac{\Delta x \Delta z}{\Delta y} (\sigma_m + s\mu) = R_{my} + sL_y \]  
(50b)
\[
Z_z = \frac{\Delta x \Delta y}{\Delta z} (\sigma_m + s\mu) = R_{mz} + sL_z. \]  
(50c)

To derive a standard 3-D differential model, we start again from Maxwell equations (17) and (18). Differential-circuit-oriented models have been derived in various forms [5], [80]. The first curl-Maxwell equation is
Multiplying both sides of (55) by the cross section orthogonal to \( k = x, y, z \), respectively, yields

\[
\begin{align*}
\Delta y \Delta z \left( \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \right) &= \frac{\Delta y \Delta z}{\Delta x} (\sigma + s\varepsilon)V_x, \\
\Delta x \Delta z \left( \frac{\partial H_z}{\partial x} - \frac{\partial H_x}{\partial z} \right) &= \frac{\Delta x \Delta z}{\Delta y} (\sigma + s\varepsilon)V_y, \\
\Delta x \Delta y \left( \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \right) &= \frac{\Delta x \Delta y}{\Delta z} (\sigma + s\varepsilon)V_z.
\end{align*}
\]

which can be rewritten as

\[
\begin{align*}
\Delta I_x - \Delta I_y &= Y_x V_x, \\
\Delta I_x - \Delta I_z &= Y_y V_y, \\
\Delta I_y - \Delta I_z &= Y_z V_z.
\end{align*}
\]

where the total voltages are defined as

\[
V_x = E_x \Delta x \quad V_y = E_y \Delta y \quad V_z = E_z \Delta z
\]

and the admittances are

\[
\begin{align*}
Y_x &= \frac{\Delta y \Delta z}{\Delta x} (\sigma + s\varepsilon) = G_x + sC_x \\
Y_y &= \frac{\Delta x \Delta z}{\Delta y} (\sigma + s\varepsilon) = G_y + sC_y \\
Y_z &= \frac{\Delta x \Delta y}{\Delta z} (\sigma + s\varepsilon) = G_z + sC_z
\end{align*}
\]

where the conductances and capacitances are

\[
\begin{align*}
G_x &= \sigma \frac{\Delta y \Delta z}{\Delta x}, \\
G_y &= \sigma \frac{\Delta x \Delta z}{\Delta y}, \\
G_z &= \sigma \frac{\Delta x \Delta y}{\Delta z}, \\
C_x &= \varepsilon \frac{\Delta y \Delta z}{\Delta x}, \\
C_y &= \varepsilon \frac{\Delta x \Delta z}{\Delta y}, \\
C_z &= \varepsilon \frac{\Delta x \Delta y}{\Delta z}.
\end{align*}
\]

We again neglect the displacement currents in the conductor. In this case, admittances (59a) reduce to their ohmic part.

The global admittance matrix is generated by a diagonal block concatenation of \( Y_x, Y_y, \) and \( Y_z \) matrices

\[
Y = \text{blkdiag}([Y_x, Y_y, Y_z]).
\]

The conventional Yee cell can be used to relate currents and voltages. More specifically, when enforcing (57)

\[
A_i I + B I_s = Y V.
\]

Equations (54) and (63) allow us to identify the voltage drops \( V \) in terms of the surface currents \( I_s \). The surface voltage drops can be isolated from the vector \( V \) by an output selector matrix \( L^T = B \) to render

\[
V_s = L^T V = B^T (A_i Z^{-1} A_V + Y)^{-1} B I_s.
\]

Finally, the surface impedance is given by

\[
Z_s = B^T (A_i Z^{-1} A_V + Y)^{-1} B.
\]

Hence, this model is again embedded in the total system (30) as is done for the thin conductor model.

### E. Identification of Surface Currents \( I_s \)

Next, we refer to the to the conductor element in Fig. 16, which has been discretized using a single Yee cell. The electrical quantities are numbered for clarity.
The discrete form of curl-$H$ Maxwell's equation (57), when enforced at node 1, reads

$$\begin{align*}
(I_{z17} - I_{zbc} - I_{y15} + I_{ybc}) = Y_s V_{x1},
\end{align*}$$

(66)

where $I_{zbc}$ and $I_{ybc}$ denote $z$- and $y$-oriented currents, external to the computational domain. Assuming a fine mesh is used in proximity to the surface, they can be substituted by surface currents. Indeed, the surface current density is defined as

$$\begin{align*}
I_s = \hat{n} \times H_t \hat{t}.
\end{align*}$$

(67)

Both external currents in (66) can be treated as surface currents. The surface current $I_{zx}$ can be defined as

$$\begin{align*}
I_{zx} \hat{x} = \hat{n} \times I_{zbc} \hat{z} = -I_{zbc} \hat{x}.
\end{align*}$$

(68)

Similarly, the surface current $I_{xy}$ is

$$\begin{align*}
I_{xy} \hat{x} = \hat{n} \times I_{ybc} \hat{z} = I_{ybc} \hat{x}.
\end{align*}$$

(69)

Thus, (66) can be rewritten as

$$\begin{align*}
(I_{z17} - I_{y15} + I_{zx} + I_{xy}) = Y_s V_{x1}.
\end{align*}$$

(70)

The same can be done for the other equations, leading to (63) and, finally, to (65). Having cast the surface currents in a vector $I_s$, the matrix $B$ is defined as

$$\begin{align*}
B_{ij} = \begin{cases} 
1, & \text{if the jth surface current contributes to Kirchhoff current law at node i} \\
0, & \text{otherwise}.
\end{cases}
\end{align*}$$

(71)

In the models in Sections IV-B and IV-D, a decrease of the cell thickness toward the surfaces was used. It is clear that this is a very important aspect for an efficient solution. However, this is a much more complex aspect for general 3-D shapes using frequency-domain time domain (FDTD)-type differential equation models.

V. SURFACE-EQUIVALENCE-BASED SKIN-EFFECT MODEL

Our comparison would be incomplete without considering the surface-integral-equation-based skin-effect models. Surface skin-effect models avoid unknowns placed inside the conductors as is done for the VFI model. A local approximation with an impedance model leads to the same very high-frequency limited applicability since the planar approximation is suitable only for a very strong skin effect. Hence, the conventional impedance boundary condition is not suitable for electromagnetic couplings for chip, package, and PCB board problems.

Theoretically, the PMCHWT method introduced in Section I can give rigorous evaluation to the skin-effect losses for piecewise homogeneous conductors in a surface integral formulation. However, problems may occur in a numerical implementation due to the high contrast between the conductor and the surrounding dielectric, which makes the resultant linear system highly unbalanced. Also, this leads to a solution based on four integral equations, two in region 1 and two in region 2 in Fig. 1.

Here, we consider a new simplified surface formulation called GIBC condition given in [32] and [53]. This new approach is specifically tailored to the solution of the skin-effect problem for lossy materials. The GIBC solution considered here uses a cEFIE as given in Section II including the skin-effect part.

According to the equivalence principle [42], [43], the EFIE for closed shapes can be written as

$$\begin{align*}
\hat{i} \cdot \mathcal{L}_{1E}(\mathbf{r}, \mathbf{r}') J(\mathbf{r}') + \hat{i} \cdot \mathcal{K}_{1E}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') = -\hat{i} \cdot \mathbf{E}(\mathbf{r})
\end{align*}$$

(72)

where $\mathcal{L}_{1E}$ is the electric field integral operator based on the outer region 1 of the closed object for EFIE. We have an alternate form of the cEFIE in (5) and (12), where the operator form can be written as

$$\begin{align*}
\mathcal{L}_{1E}(\mathbf{r}, \mathbf{r}') J(\mathbf{r}') = i \omega \mu_0 \int_{S} \left( \mathbf{I} + \frac{\nabla \nabla^\top}{k^2} \right) G_\nu(\mathbf{r}, \mathbf{r}') \cdot J(\mathbf{r}') \, ds'.
\end{align*}$$

(73)

where $G_\nu$ is the free-space Green’s function based on region $\nu$, $\mathcal{K}_{1E}$ is the magnetic field integral operator based on the outer region 1 for EFIE. It is defined as

$$\begin{align*}
\mathcal{K}_{1E}(\mathbf{r}, \mathbf{r}') J(\mathbf{r}') = -\int_{S} \nabla G_\nu \times J(\mathbf{r}') \, ds'.
\end{align*}$$

(74)

To avoid the magnetic field excitation, the magnetic field integral equation of the interior region is employed as follows:

$$\begin{align*}
\hat{i} \cdot \mathcal{K}_{2H}(\mathbf{r}, \mathbf{r}') J(\mathbf{r}') + \hat{i} \cdot \mathcal{L}_{2H}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') = 0
\end{align*}$$

(75)
where

\[ \mathcal{L}_{\text{SH}}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') = \frac{1}{\eta_0} \mathcal{L}_{\text{EF}}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') \]  

(76)

\[ \mathcal{K}_{\text{SH}}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') = - \mathcal{K}_{\text{EF}}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}'). \]  

(77)

From (75) and (77), the GIBC can be shown inside the following equation:

\[ [\mathbf{t} \cdot \mathcal{L}_{\text{IE}}(\mathbf{r}, \mathbf{r}') - \mathbf{t} \cdot \mathcal{K}_{\text{IE}}(\mathbf{t} \cdot \mathcal{L}_{\text{2H}})^{-1} \cdot \mathbf{t} \cdot \mathcal{K}_{\text{2H}}] J(\mathbf{r}') = -\mathbf{t} \cdot \mathbf{E}(\mathbf{r}) \]  

(78)

or

\[ [\mathbf{t} \cdot \mathcal{L}_{\text{IE}}(\mathbf{r}, \mathbf{r}') - Z_s] J(\mathbf{r}') = -\mathbf{t} \cdot \mathbf{E}(\mathbf{r}) \]  

(79)

where \( Z_s \) is the GIBC as shown in Fig. 2

\[ Z_s = \mathbf{t} \cdot \mathcal{K}_{\text{IE}}(\mathbf{t} \cdot \mathcal{L}_{\text{2H}})^{-1} \cdot \mathbf{t} \cdot \mathcal{K}_{\text{2H}}. \]  

(80)

Hence, after eliminating the magnetic current, we obtained an integral equation in terms of the electric current only. It will be noted that \( \mathcal{K}(\mathbf{r}, \mathbf{r}') M(\mathbf{r}') \) has the singular term \( \pm \mathbf{M}/2 \) when \( \mathbf{r} = \mathbf{r}' \). This term was canceled in PMCHWT method but does exist in GIBC formulation. This special handling has to be made according the norm direction to determine the exact sign in the equation.

The GIBC method employs the conventional delta gap feed model, for a finite size cross section, where the feed current is only applied to the surface. This is different from VFI, which feeds the current throughout the cross section. Hence, the surface feeding scheme used in the GIBC could generate certain error in the result. However, we found that the results for a large number of cells agreed very well with the VFI model.

In Section VI, we give results where we compare the GIBC with using the VFI model.

VI. NUMERICAL EXPERIMENTS

The class of problems of interest has been outlined in Section I. Clearly, we are considering problems for which the current path changes with frequency or time, depending on the type of solver used. The fact that it is difficult to obtain accurate broadband skin-effect results for many 3-D structures with simplified models was pointed out to the first author by Pinellos before the end of the last century [81]. This is confirmed by this work. Considerable understanding of the issues has been gained by comparing the solutions using the different approaches presented in this paper.

In all experiments, we assume the conductors to be copper embedded in air. The external air environment in all our solutions is represented using an EFIE model described in Section II. However, we exclude the dielectric part of the model to make certain that the skin-effect results are not masked by resonances due to the external geometry.

One of the 3D-VFI model solvers used is from Archambeault and Connor (IBM Raleigh), while another one is an experimental Matlab PEEC solver. Both PEEC solvers use a full 3D-VFI model with current flow in all three directions inside the conductors. For the 3D-VFI solution, we use the conventional PEEC meshing with cell divisions in each direction where we use the usual half-size cell at the edge and the surfaces as is conventionally done for PEEC solvers. Two other new experimental Matlab solvers used in our comparison are the GSI solver for thin and thick conductors. Finally, the GIBC solver used in the comparison is based on the surface formulation in [53]. Most computations are done on a single processor with an 8-GB machine. Hence, this limits some of the very high-frequency accuracy of the results since a larger number of small cells are required for this case.

The examples used are interesting wide-frequency range problems where the current path is not predetermined. Specifically, the horseshoe (HShoe) problem consists of one layer shown in Fig. 17, but it has wide flat corners, which leads to a current redistribution with frequency. The geometrical data for this example are shown in Table 1. The additional data required are \( w = 10 \mu m \) and the gap width is \( w_c = 0.2 \mu m \). We use two different models for the contacts area at the inside of the gap. In one case, we

![Fig. 17. Geometry of U-shaped test problem called HShoe.](image-url)

Table 1 Dimensions for the Two Example Problems in Micrometers

<table>
<thead>
<tr>
<th>Prob</th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>HShoe1</td>
<td>x_g</td>
<td>x_W</td>
<td>x_L</td>
<td>y_g</td>
<td>y_W</td>
<td>y_L</td>
<td>d</td>
<td>Z_g</td>
</tr>
<tr>
<td>400</td>
<td>80</td>
<td>320</td>
<td>400</td>
<td>80</td>
<td>320</td>
<td>18</td>
<td>50</td>
<td>5</td>
</tr>
<tr>
<td>LShape5</td>
<td>400</td>
<td>80</td>
<td>320</td>
<td>400</td>
<td>80</td>
<td>320</td>
<td></td>
<td>50</td>
</tr>
</tbody>
</table>
use a 0.4-μm section at the contact surface with a conductivity which is 10^4 times larger than copper. This leads to contacts at the entire cross-section surface. Alternatively, point contacts are used at the front of the gap, as shown in Fig. 17.

The second problem in Fig. 18 consists of an L-shaped conductor with a close ground plane. At high frequencies, the current will take the path which minimizes the inductance right under the L-shaped wire. However, at low frequencies, the current takes the path of least resistance, which is across the plane. The lengths of the L conductors is relatively small for this redistribution to be very large. Hence, the change in inductance is relatively modest. Much larger changes in L and R have been obtained for longer lines [82].

The dimensions for the L-shaped problem (Fig. 18) are also given in Table 1 for both the thin conductor LShape5 and the thick conductor LShape18. We chose a very wide conductor of 80 μm, since it leads to a more interesting current behavior. The short to ground at the end of the top L-shaped conductor is accomplished with a strip which extended over the entire width from the L-shaped conductor to the ground plane, as is shown in Fig. 18. The strip thickness is the same as the L-shaped conductor.

A. Thin Conductor Results

In this section, we utilize the thin-GSI model in Section IV-B. The first problem is the 1-μm-thick and 10-μm-wide HShoe1. We compare thin GSI with the 3D-VFI solver results. The results are given in Figs. 19 and 20 for the inductance and resistance. Further, for the thin L-shaped conductor problem, both the ground-plane and

![Fig. 18. Shorted L-shaped conductor over ground-plane model.](image1)

![Fig. 19. Inductance for HShoe with 1-μm conductor with thin GSI solver.](image2)

![Fig. 20. Resistance for HShoe with 1-μm conductor with thin GSI solver.](image3)

![Fig. 21. Inductance for the thin LShape conductor over the ground plane.](image4)
the L-shaped conductors are chosen to be $5 \mu m$ thick. We again compare the 3D-VFI solution with the thin GSI solution. We observe that the agreement between the solutions in Figs. 21 and 22 is good. The difference in the high-frequency resistance is due to the limited number of cells used in the 3D-VFI solution.

**B. Thick Conductor Results**

Our comparison with thicker conductor problems is first for the thick GSI solution in Section IV-D. The result we give is for the HShoe problem where the conductor thickness is $6 \mu m$ with a width of $10 \mu m$. The results shown in Fig. 23 for the resistance and in Fig. 24 for the inductance were obtained with a simplified approximate form of the approach in [83]. It is evident that the approach does not yield a satisfactory solution.

For the second thick conductor example, we compare GIBC solution with the 3D-VFI results for the HShoe problem with the thick $d = 6 \mu m$ version in Fig. 25 for the resistance and in Fig. 26 for the inductance. The agreement is good with the exception of the very high-frequency response, which is due to the limited number of cells used for the 3D-VFI model.

**C. Comparison of the Results**

A key problem with comparing different models is the fact that the programming languages and computers used are quite different for the different models. Also,
used internal–external inductance model \[75\], \[78\] may be insufficient for this type of problems, while it works well for relatively thin planes and wires as used in Sections IV-B and IV-C, respectively. This is not surprising since it is an electrical current-only approximation to the surface IE boundary conditions.

The results from the surface GIBC model compare very well with the VFI model. However, the GIBC model requires a representation with a sufficiently large number of cells to yield meaningful answers. This is different for the VFI model, which works well with a somewhat reduced accuracy.

**VII. CONCLUSION**

Hopefully, we added clarity to the complex issue of skin-effect loss modeling of large problems. Both time and frequency domains require wideband models, especially for the large class of realistic SI/PI/NI problems. We found that the key issue is that accurate skin-effect models for these applications are compute time and memory consuming. We found that the VFI model worked well even for a modest number of cells or subdivisions. We also found that the most challenging problems are large volume conductors with frequency- or time-domain-dependent current paths. From this work, we conclude that the best model depends on the particular geometry and many other issues like the meshing used.

**Acknowledgment**

The authors would like to thank the reviewers for their valuable comments.

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**REFERENCES**


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