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Source Reconstruction Method-Based Radiated Emission Characterization for PCBs
Ping Li, Student Member, IEEE, and Li Jun Jiang, Member, IEEE

Abstract—A technique based on a source reconstruction method (SRM) is employed to reproduce the same electromagnetic radiated emissions from printed circuit boards (PCB). This equivalent source is composed of both the electric and magnetic currents that are represented by a set of Rao–Wilton–Glisson basis functions. Planar magnetic near field is served as the input information for the SRM. The equivalent source is inversely acquired through the conjugate gradient algorithm that minimizes the norm of the residue. Magnetic field integral equation is utilized to find out the unknown coefficients of the currents. This method is directly extended to model the emissions from PCBs in shielding boxes, in which the PCBs are substituted by the reconstructed equivalent current source, the interaction between the PCBs and cavity is calculated via the method of moments. To investigate the robustness and versatility of the proposed approach, various numerical examples are presented.

Index Terms—Method of moments (MoM), planar magnetic field sampling, radiated emission modeling, Rao–Wilton–Glisson (RWG) basis, source reconstruction.

I. INTRODUCTION

Source reconstruction is a widely used near-field measurement postprocessing technique. It can be employed to undertake the near-field–far-field (NF-FF) transformation, which is much more cost efficient compared with the direct far-field measurement that requires a larger anechoic chamber. Besides, it is quite useful for antenna diagnostics and radiation pattern synthesis. Recently, this approach is harnessed to model the radiated emissions from printed circuit boards (PCBs) and electronic devices, etc. After obtaining this equivalent source, it is quite easy to conduct source diagnostics, near-field prediction, preperformance radiation evaluation regularized by the electromagnetic compatibility (EMC) standards like the radiated emission limitation at 3 m, 10 m even 30 m away from the device under test (DUT), and investigation of interactions between the PCBs and the shielding cavities.

The development of the source reconstruction method (SRM) first appeared in [1] for antenna NF-FF transformation. The equivalent source expanded by pulse basis functions is distributed over an infinite perfectly electric conductor (PEC). Numerical examples illustrate that this equivalent source is valid for the upper half-space due to the existence of the fictitious PEC plane. Followed by this idea, a number of literatures are published by other researchers in the antenna area. Among them, Alvarez et al. [2] proposed to build the equivalent source over the 3-D surface of the DUT itself, where the spherically measured electric field is used as the input information. There are many literatures related to the SRM; interested readers can refer to [3]–[9].

Inspired by the efficiency and usefulness, this approach is directly moved to an EMC region. In [10], the radiation from a PCB is reproduced by an equivalent source composed of dipole distributed over a PEC plane. Each dipole has eight degree of freedoms including the position, amplitude, orientation, etc. The unknowns are solved by a genetic algorithm (GA). Since the expensive computation of GA, the maximum number of dipoles is limited to 8 which severely constricts its accuracy and practicality. In [11] and [12], following the idea in [10], the equivalent source is also composed by either electric or magnetic dipoles. The unknowns are optimized through least square method. Only limited accuracy is achieved.

Although numerous literatures such as [13] and [14] using similar ideas have investigated the radiation from PCBs and electronic device in free space, only [15] as far as the authors’ knowledge extended the SRM to characterize the radiation from PCBs in metal cavities. The equivalent source also consists of dipoles, and the Tikhonov regularization method is applied to solve unknowns. Although the accuracy is not quite satisfactory due to the limitation of the dipole model, it enlightens us to design more robust models to emulate the radiation from PCBs in closed environments.

The aim of this paper is to study the radiation behavior of PCBs through the SRM based on planar magnetic near field. To characterize the whole 3-D radiation pattern, the magnetic tangential near-field sampling scheme is conducted over two planes: one is located over the upper half-space, another is located at the lower half-space to capture the backward radiation. First, the equivalent source is constructed for PCBs in free space, then the radiation properties of this PCB can be thoroughly investigated by this equivalent current source. Second, if we wish to study the radiation behavior of the same PCB placed inside a shielding cavity, the former reconstructed equivalent source model can be employed to replace this PCB, then the radiation from the original PCB can be studied by considering the interaction between the cavity and the equivalent source. Although there are some approximations in this situation, its main
advantage is the simplicity of the equivalent source model compared with the traditional numerical methods like the finite-element method (FEM) that needs to mesh the whole PCB, resulting a large number of unknowns. Instead, with the retrieved current model, we only need to mesh the surface of the metal cavity, the interaction between the equivalent source and the cavity is numerically solved by method of moments (MoM) wherein only a small number of unknowns are involved, which is more efficient.

The remainder of this paper is organized as follows. In Section II, the principle of the equivalent SRM is described. In Section III, numerical results are exemplified to investigate the radiated emissions from PCBs in free space. In Section IV, the equivalent source model is extended to study the radiation properties of PCBs in shielding boxes. Discussions are made in Section V. Conclusions are presented at the end of this paper.

II. PRINCIPLE OF THE SRM OVER ARBITRARY SURFACE

The electromagnetic equivalence principle [16] states that only the tangential electric field or tangential magnetic field over a surface enclosing the DUT is needed to characterize the radiated field, where the tangential electric field actually corresponds to the equivalent magnetic current, the tangential magnetic field actually corresponds to the equivalent electric current. Based on the previous statement, the SRM can be facilitated as follows: given a PCB with arbitrary geometry, bounded by a minimum conformal surface \( S' \) enclosing the PCB, an equivalent source can be built over that surface, as shown in Fig. 1. The equivalent source consists of both electric current \( J_s(r') \) and magnetic current \( M_s(r') \) which reproduce the original radiation outside that surface. Referring to the inverse source reconstruction procedure, the unknown source can be theoretically determined by the measured field data over the two planar domains \( S_1 \) and \( S_2 \).

In our method, magnetic field integral equation is employed since magnetic field sampling is preferred [23]. The general formula is written as

\[
H(r_{\text{Meas}}) = H_{J_s}(r_{\text{Meas}}) + H_{M_s}(r_{\text{Meas}})
\]

(1)

where \( H \) is the magnetic field at the sampling position, \( H_{J_s} \) is the magnetic field radiated by the equivalent current, and \( H_{M_s} \) is the magnetic field radiated by the equivalent magnetic current. If \( r_{\text{Meas}} \) and \( r' \) represent the measurement and source positions, respectively, we have

\[
H_{J_s} = \int_{S'} J_s(r') \times \nabla e^{jkr'dS'}
\]

(2)

\[
H_{M_s} = -j\frac{k_0}{\eta_0} \int_{S'} \left[ M_s(r') + \frac{1}{\sqrt{\epsilon_r}} \nabla \nabla \cdot M_s (r') \right] e^{jkr'dS'}
\]

(3)

where \( k_0 = \sqrt{\mu_0/\epsilon_0} \) is the free space wave number, \( \eta_0 = \sqrt{\mu_0/\epsilon_0} \) is the vacuum characteristic wave impedance, and \( R \) is the distance between the measurement point and source position.

The surface where equivalent currents reside is meshed into nonoverlapping triangles. The equivalent currents are expanded by Rao–Wilton–Glisson (RWG) basis functions and expressed as

\[
J_s(r') = J_0 \sum_{n=1}^{N_s} P_n \phi_n(r')
\]

(4)

\[
M_s(r') = M_0 \sum_{m=1}^{N_m} P_m^h \psi_m(r')
\]

(5)

where \( J_0 \) and \( M_0 \) are two normalization factors. \( \phi_n \) denotes the \( n \)th RWG basis function for electric current \( J_s \), \( \psi_m \) denote the \( m \)th RWG basis function for magnetic current \( M_s \), and \( P_n \) and \( P_m^h \) are the corresponding coefficients.

In order to obtain the unknown expansion coefficients in (4) and (5), a vector weighting function is introduced

\[
\omega(r) = \delta(r - r_{\text{Meas}}) \hat{e}_l
\]

(6)

where \( \hat{e}_l \) represents polarization directions of the measurement probe. In this paper, we assume that the measurement probe is a linearly polarized antenna and the measurement is along \( \hat{x} \) and \( \hat{y} \) directions at every measurement point. Thus, polarization vectors \( \hat{e}_l = \hat{x} \) or \( \hat{e}_l = \hat{y} \).

Substituting (2) and (3) into (1) with (4) and (5), and taking inner products of the integral equation with the weighting function in (6), the following matrix equations are obtained:

\[
\begin{pmatrix}
\bar{H}_x & \bar{H}_y
\end{pmatrix} =
\begin{pmatrix}
\bar{Z}(e_x, J_s) & \bar{Z}(e_x, M_s)
\end{pmatrix}
\begin{pmatrix}
P
\end{pmatrix}
\]

(7)

\[
\begin{pmatrix}
H_x & H_y
\end{pmatrix} =
\begin{pmatrix}
H_{x,1}, H_{x,2}, \ldots, H_{x,N_x}
\end{pmatrix}^T
\]

(8)

\[
\begin{pmatrix}
H_x & H_y
\end{pmatrix} =
\begin{pmatrix}
H_{y,1}, H_{y,2}, \ldots, H_{y,N_y}
\end{pmatrix}^T
\]

(9)
where $N_x$ and $N_y$ are the number of measurement points in $\hat{x}$ and $\hat{y}$ directions, respectively. $\mathbf{H}_x$ and $\mathbf{H}_y$ are column vectors. Superscript $T$ denotes transpose. The block matrices at the first row of the impedance matrix $\mathbf{Z}$ represent the contribution to the $x$ components of the electric field from the equivalent currents. The block matrices at the second row of the impedance matrix $\mathbf{Z}$ represents the contribution to $y$ components of the electric field from the equivalent currents. Column vectors $\mathbf{P}^x$ and $\mathbf{P}^y$ are unknown coefficients. Then, a conjugate gradient (CG) method can be employed to solve the matrix equation in (7).

Since this inverse radiation problem is ill conditioned, the two normalization factors $J_0$ and $M_0$ were introduced to improve the conditioning of the matrix equation system. A better conditioned system can be achieved by choosing $J_0$ and $M_0$ according to $M_0/J_0 = \eta_0$, where either $M_0$ or $J_0$ can choose freely. To further alleviate the singularity of impedance matrix, the adjoint of the impedance matrix $\mathbf{Z}^{\text{adj}}$ is premultiplied to the both sides of (7). It can be simplified as

$$
(\mathbf{Z}^{\text{adj}} \mathbf{Z}) \mathbf{P} = \mathbf{b} \tag{10}
$$

and iteratively solved by the CG method.

In this paper, the tangentially sampled near magnetic field data are obtained from a full wave commercial software FEKO [19]. Since this paper focuses on theoretical investigation, employing synthetic data is more favorable for theoretical investigation.

III. NUMERICAL RESULTS FOR PCBs IN THE FREE SPACE

To verify the feasibility of the SRM for PCBs placed in the free space, two numerical examples are included. The first one is a canonical bent transmission line driven by a voltage source; the second one is more complex PCB containing loop lines.

A. Bent Transmission Line

A 15 by 10 cm$^2$ PCB with a bent transmission line with the dimension $t = 10$ cm and $w = 7$ cm in Fig. 2 is numerical studied. The port 1 of the bent line is driven by a 1 V voltage source operating at 100 MHz, the two sampling planes located at $z = 1$ cm and $z = -1$ cm, respectively. The tangential magnetic near fields over two planes at $z = 1$ cm and $z = -1$ cm are obtained by FEKO simulation with sampling resolution $\Delta x = \Delta y = 1$ cm. The measurement size in $x$-direction is $30$ cm $= 0.1 \lambda$, and in $y$-direction is also $30$ cm $= 0.1 \lambda$. In total, 1922 measurement points are obtained. The near-filed sampling size is chosen based on the numerical investigation to capture the radiation from the edges of the PCB, especially when the PEC traces are close to the board edges. The equivalent current source including both electric and magnetic currents is distributed over the PCB surface which is triangulated into 780 patches. The resultant matrix equation has 2340 unknowns.

The convergence rate using the CG method is listed in Fig. 3. It can be clearly observed that the convergence rate is indeed improved by the premultiplication of the conjugate complex transpose of the impedance matrix $\mathbf{Z}$. The reconstructed equivalent source is shown in Fig. 4. It is noted that the strongest current is around the transmission line; it means that this technique can be applied to hot spot detection that is quite useful for EMC/EMI diagnostics. Next, this equivalent source is further employed to calculate the radiated emissions at 3 and 10 m away. The results are shown in Fig. 5. Excellent agreement between the predicted results and the reference is achieved. For hot spot detection or EMC diagnostics, near-field data are more meaningful. Hence, it is necessary to study the qualification of the reconstructed source for near-field prediction. The electric near-field planar plane at $z = 3$ cm is shown in Fig. 6. Great similarity is achieved.

B. More Complex PCB With Loop Structures

A PCB with loops and straight lines is characterized in this part. The physical layout of this circuit board is shown in Fig. 7. Both the two loop lines and the three straight traces are, respectively, driven by a voltage source operating at a randomly picked frequency of 1 GHz. The dimension of the PCB is $l_1 = 15$ cm and $l_2 = 10$ cm with the height $h = 0.2$ cm. The tangential magnetic near fields over two planes at $z = 1$ cm and $z = -1$ cm are obtained by FEKO simulation with sampling resolution...
Fig. 5. Magnitude of electric field at 3 and 10 m away from the PCB calculated by the equivalent source, the reference is obtained by FEKO simulation. (a) E-field in the XOZ plane. (b) E-field in the YOZ plane. (c) E-field in the XOY plane.

Fig. 6. Magnitude of electric field (in decibel) at plane $z = 3$ cm. The left column is calculated by the reconstructed equivalent source, the right column is calculated by FEKO. (a1) and (a2) X-component. (b1) and (b2) Y-component. (c1) and (c2) Z-component.

Fig. 7. Physical layout of the PCB with loops and straight lines. The ports from $P_1$ to $P_5$ are driven by a voltage source, while the ports from $P_6$ to $P_{10}$ are terminated by a 100-$\Omega$ resistor.

Fig. 8. Convergence rate versus the iteration number when solving (10). Case 1: The normalization factor $M_0/J_0 = 1$. Case 2: The normalization factor $M_0/J_0 = \eta_0 = 120\pi$.

Fig. 9. Reconstructed equivalent current density (in decibel) distribution over the PCB surface. (a) Equivalent electric current. (b) Equivalent magnetic current.

$\Delta x = 1$ cm and $\Delta y = 1$ cm. The total number of sampling points is 2542 with sampling size in $x$-direction 40 cm and in $y$-direction 30 cm.

Again, the equivalent current source is over the PCB surface which is discretized into 1028 small triangular patches. In total, 3084 unknowns are involved. The CG method is employed to solve (10). To study the effect of the normalization factor $M_0/J_0$ to the convergence rate, two cases are considered. One is $M_0/J_0 = 1$; another is $M_0/J_0 = 120\pi$. The result is shown in Fig. 8. It can be explicitly noted that the proper defined normalization factor will accelerate the convergence speed efficiently.

The resulting equivalent current source is shown in Fig. 9. The outline of the magnetic loop can be clearly observed, and the strong current can be found near the straight line region. One possible reason is the coupling between the loop and the straight line. The magnitude of the electric field at 3 and 10 m away from the PCB is calculated by the equivalent source, as shown in Fig. 10. Very excellent agreement is achieved again. To further
illustrate the accuracy and reliability of our method, the feature selective validation (FSV) method [20]–[22] is employed. The FSV has been adopted as a standard comparison method to validate the computational electromagnetics computer modeling and simulation. The confidence histograms and the grade-spread value for the comparison of Fig. 10 are shown in Fig. 11. These confidence histograms indicate that our model is very reliable and accurate.

IV. NUMERICAL RESULTS IN SHIELDING BOXES

In this part, the former constructed equivalent source model is directly applied to replace the original PCB placed inside shielding cavities. Two numerical examples are investigated. The first one is a PCB placed inside a PEC cavity without a shielding cover. The second one is a PCB placed inside a shielding cavity with few narrow slots.

A. PCB Board in Fig. 2 Is Placed Inside a Cavity Without a Cover.

The PCB presented in Fig. 2 is again utilized to study the feasibility of the proposed method when PCBs are located inside closed environments, as shown in Fig. 12. Traditional numerical methods such as an FEM and the finite-difference time-domain method must discretize both the PCB and box, which will result a huge number of unknowns when the PCB is very complex. It is much computation expensive or even impossible to solve the resulted matrix. However, our method only needs to discretize the surface of the box using MoM which only leads to a small number of unknowns.

In this example, the cavity is triangulated into 1078 patches with 1617 unknowns. It is just quite easy to solve such a small number of unknowns. After obtaining the unknown coefficients of the induced current over the PEC cavity, the total radiated emission can be easily acquired by adding the fields from the induced current and the equivalent source constructed by the SRM. In Fig. 13, the magnitude of the electric fields at 3 and 10 m is presented. Good agreements are achieved.

B. PCB Board in Fig. 7 Is Placed Inside a Cavity With Two Slots

A more complex case is investigated by putting the PCB in Fig. 7 into a cavity with slots as shown in Fig. 14. Also, the same procedure is repeated as that in Section IV-A. Fig. 15 shows the magnitude field at 3 and 10 m. It shows that not only
Fig. 13. Magnitude of electric field at 3 and 10 m away from the PCB calculated by the equivalent source, the reference is from FEKO simulation. (a) E-field in the XOZ plane. (b) E-field in the YOZ plane. (c) E-field in the XOY plane.

Fig. 14. PEC shielding cavity with two slots. The dimensions are as follows: \( l_1 = 25 \) mm, \( l_2 = 65 \) mm, \( l_3 = 150 \) mm, \( w = 20 \) mm, \( S_1 = 40 \) mm, \( a = 150 \) mm, \( b = 200 \) mm, and \( c = 50 \) mm.

Fig. 15. Magnitude of electric field at 3 and 10 m away from the PCB calculated by the equivalent source, the reference is obtained by FEKO simulation. (a) E-field in the XOZ plane. (b) E-field in the YOZ plane. (c) E-field in the XOY plane.

V. DISCUSSION

To apply the SRM technique to study the radiation from PCBs in the free space or shielding box, certain issues have to be taken care of. The sampling area should be larger than PCBs to capture the radiation from edges since the contribution from the radiation near edges is strong especially when there are PEC traces. To verify this statement, the PCB with a bended line in Section III is restudied. In this case, the contribution from the edge radiation is quite strong since the trace is very close to the PCB edge. We reduce the near-field sampling area to the same as the size of PCB. The fields at 10 m away from the PCB are calculated and shown in Fig. 18. Poor agreements with the reference simulated from FEKO are explicitly noted, which is attributed to the inability to include the radiation from edges. Based on our numerical investigation, the measurement...
Fig. 17. Magnitude of magnetic near field (in decibel) inside the cavity at a plane z = 3 cm. The left column is calculated using the equivalent source to replace the original PCB, the right column is calculated by FEKO using the original PCB. (a1) and (a2) X-component. (b1) and (b2) Y-component. (c1) and (c2) Z-component.

Fig. 18. Magnitude of the electric field computed from the equivalent source reconstructed from the near-field data sampled at a small area, the references are simulated from FEKO. (a) E-field in the XOZ plane. (b) E-field in the YOZ plane. (c) E-field in the XOY plane.

size should be 1.5–2 times larger than the PCB size, which will guarantee a reasonable accuracy.

In this paper, due to the theoretical study motivation and our limited equipment condition, we used the simulated field data instead of practically measured data to verify the developed algorithm. The sampled tangential magnetic near-field data are obtained from the full-wave commercial software FEKO [19]. To complement the study described in this paper, a near-field scanner is needed to sample the tangential magnetic field. The needed configuration would be similar to that introduced in [23].

VI. CONCLUSION

The SRM is applied to model the electromagnetic emission from PCBs, where the unknown equivalent current distributed over the surface of PCB itself is solved based on the planar measured magnetic field. To investigate the radiation behavior of PCBs inside a PEC cavity, the equivalent source reconstructed in the free space through the SRM is applied to substitute the original PCB. Hence, the interaction between the real PCB and the cavity is approximated by using the equivalent source. The advantage of this is the significant reduction of the computational cost. Numerical results demonstrated the feasibility and accuracy of our method when there is merely moderate coupling between the PCB and the cavity.

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REFERENCES

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