# DEVELOPMENT OF A BI-DIRECTIONAL PEDESTRIAN STREAM MODEL WITH OBLIQUE INTERSECTING ANGLE

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**Abstract**

This study establishes a mathematical model that can represent the conflicting effects of two pedestrian streams with an oblique intersecting angle in a large crowd. In a previous study, a controlled experiment in which two streams of pedestrians were asked to walk in designated directions was used to model the bi-directional pedestrian stream of certain intersecting angles. In this study, we revisit that problem and apply the Bayesian inference approach to calibrate an improved model with the controlled experiment data. We also collected pedestrian movement data from a busy crosswalk using a video observation approach. The two sets of data are used separately to calibrate our proposed model. With the calibrated model, we study the relationship between speed, density, and flow in both the reference and conflicting streams, and predict how these factors affect the interactions of moving pedestrian streams. We find that the speed of one stream not only decreases with its total density, but it also decreases with the ratio of its flow in relation to the total flow, i.e., the speed of the pedestrians decreases if their stream changes from the major to the minor stream. We also observe that the maximum disruption induced by pedestrian flow from an intersecting angle occurs when the angle is near 135°.

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## Introduction

Walking is an environmentally friendly mode of transportation. A good understanding of pedestrian activities and the effective planning of walking facilities are particularly important for densely populated Asian cities such as Hong Kong. Previous studies have used observational surveys and controlled experiments to examine one-dimensional and bi-directional pedestrian streams. Video recording has been a widely applied survey method in these studies, as it is economic, convenient, and has relatively high accuracy. The video provides a real-time record of the pedestrian movements from which it is possible to extract the position of each individual pedestrian at any moment. Bi-directional pedestrian streams are more common in daily life than one-dimensional pedestrian movements, but very few previous studies have modeled bi-directional streams. Hence, in this study, we video recorded the pedestrian movements at a busy crosswalk in Hong Kong and extracted relevant data to develop a mathematical model that reflects the relationships between macroscopic quantities related to bi-directional pedestrian flow, including the speed, density, flow, and the intersecting angle between the reference stream and the conflicting stream.

Since Hughes (2002) proposed equations governing two-dimensional pedestrian flow and pointed out the importance of the conflicting effect induced by the interactions of bi-directional pedestrian streams, studies have increasingly focused on bi-directional pedestrian flows. Compared to uni-directional pedestrian flows, bi-directional flows are more complicated but also more commonly found in various walking facilities such as crosswalks, metro stations and even shopping malls. Lam et al. (2002, 2003) investigated bi-directional pedestrian movement in several walking facilities in Hong Kong, including signalized crosswalks in various areas. Ye et al. (2008) conducted an observational experiment on several walking facilities in Shanghai, including a two-way passageway. In addition to observational surveys, controlled experiments have been widely used in the study of bi-directional pedestrian behavior, as they can be designed to cover the full range of model parameters and provide data under a variety of conditions.

However, most experimental studies on bi-directional pedestrian flows have only considered the counter-flow case, in which two streams of pedestrians walk toward each other. Some experiments have involved crossing flows with two perpendicular streams, such as those conducted by Daamen and Hoogendoorn (2003) and Helbing et al. (2005). Moreover, Wong et al. (2010) and Ando et al. (1988) looked at cases with an oblique intersecting angle between two streams of pedestrians, which are situations rarely discussed in the literature.

Many researchers have investigated the counter-flow case using data from studies of uni-directional pedestrian flows. Daamen and Hoogendoorn (2003) and Kretz et al. (2006a) also performed experiments for pedestrian counter flow in corridors of various widths. Kretz et al. (2006b) found that the performance of counter flow, in terms of macroscopic quantities such as passing time, speed, and flux, is not necessarily lower than that of situations without counter flow. They pointed out that pedestrians are able to increase their efficiency in using space to a certain degree, and thus compensate for the existence of counter flow. Another interesting finding was the phenomena of lane formation, whereby the pedestrians in the experiment always chose right-hand traffic. As this experiment was conducted in Germany and most of the participants were German, the authors suggested that it would be useful to perform similar experiments in countries with left-hand traffic, to check the correlation between vehicular traffic rules and pedestrian behavior. In terms of lane-formation, Helbing et al. (2005) observed similar self-organization phenomena in a series of experiments for bi-directional pedestrian flows in bottlenecks with differing widths.

However, the pedestrian streams in these experiments were mainly opposite to each other, and there was usually a 180° angle between the two streams. The investigation by Ando et al. (1988) was one of the few studies on bi-directional pedestrian flow to include an oblique intersecting angle. Jiang et al. (2009) proposed a reactive dynamic continuum–user equilibrium model to simulate bi-directional pedestrian flows. Xiong et al. (2011) proposed a high-order computational scheme for the Jiang et al. model that proved more efficient than the first-order methods. These two studies, although they involved little empirical data, considered the intersecting angle between the two streams, which provided useful information for further studies on the influence of intersecting angles in bi-directional pedestrian flows. Recognizing the limitations of previous research on this problem, Wong et al. (2010) conducted controlled experiments to address them.

In their controlled experiment, Wong et al. (2010) used a modified form of Drake's model (1967) for one-dimensional traffic. In that model, the density of the streams and the intersecting angle are independent variables. As one of the few studies on bi-directional pedestrian streams with an oblique intersecting angle, the study advanced our understanding of bi-directional pedestrian streams. However, in re-evaluating the study, we found that the model could be further modified to better describe the bi-directional pedestrian movements if it included a key variable.

This paper presents the formulation of the improved model and compares that model with the original version. We also collected a new set of data at a crosswalk in Hong Kong to verify the improved model. In a real-world situation, pedestrians have their own destinations rather than assigned directions, and the pedestrians' demographic composition is a better reflection of reality than the student sample used in the controlled experiments. In the next section of this paper, we describe the data collection in this circumstance, and then demonstrate the formulation of the improved model for the bi-directional pedestrian stream. Finally, we discuss the model calibration results and the properties of the improved model.

## Data

Two sets of data were used in this study. The first was the dataset from the controlled experiment performed by Wong et al. (2010), and the other was collected from an observational survey of a busy signalized crosswalk in Hong Kong. Through a thorough comparison and study of these two data sets, we identified a key variable that could better describe the bi-directional pedestrian movements and formed the basis of formulating an improved model. The data from the controlled experiment were used to recalibrate the improved model. The data from the field observation were then used to verify that model in a real-world situation.

### ***Controlled Experiment***

The controlled experiment was conducted in a sports stadium. Volunteer students were asked to walk in designated directions, and the intersecting angles between the paths for the two streams were set at 45°, 90°, 135° and 180° (Fig. 1). The total density and the spilt ratio of the pedestrian numbers were controlled to test how these factors affected the speed of the pedestrian streams.

[Insert Figure 1 Here]

### ***Field Observation***

A new set of data was collected so that we could apply the model to a real-world situation. The site selected for video recording was the busy signalized crosswalk between Queen's Road Central and D'Aguilar Street in Central District, Hong Kong. The camera was set at the top of a nearby tall building, providing us with an ideal top view of the junction.

[Insert Figure 2 Here]

#### *Video Data Processing*

The video was taken at 25 frames per second under a PAL analogue television encoding system. The pictures, in JPEG format, were extracted from the video every 5 frames, i.e., every 0.2 s. This sampling interval ensured the smooth and complete tracking of pedestrian movements for this study. At the selected junction, the signal cycle was about 120 s, and there was a 15 s pedestrian phase in each cycle. However, we were only interested in periods during which the two pedestrian streams fully mixed, i.e., the short period, about 2 s, in the middle of each pedestrian phase. The final dataset consisted of 65 cycles, with an average of 103 pedestrians in each cycle. In total, we traced the movements of more than 6000 pedestrians in the video.

*Acquisition of Positions*

To obtain the image coordinate of each pedestrian in the region of interest (ROI), the selected video images were imported into a specially designed Visual Basic (VB) program, and the positions of pedestrians were marked manually. As shown in Fig. 3, we marked the pedestrians' heads and feet, if visible, with blue and green dots, respectively. This prepared the video data for the coordinate transformation necessary to obtain the real-world positions.

[Insert Figure 3 Here]

*Computation of Average Speed and Density*

As shown in Fig. 4, the distribution of pedestrians in the region was not homogeneous. To ensure that the computed average speed and density reflected the true relationship between speed and density, we divided the region into 18 sub-areas, each measuring 3m x 3m. The sub-areas were distributed according to the size of the crosswalk, three across the Queen's Road Central, and six along the road.

[Insert Figure 4 Here]

In total, as each sub-area gave one data point, we obtained 18 data points from each frame (picture) for data analysis.

To summarize, we counted 1160 pedestrians in the controlled experiment and 6788 in the field observation. As shown in Table 1, the average speed of pedestrians in the field observation was higher than in the experiment, but the average density of pedestrians in the field observation was lower than in the experiment.

[Insert Table 1 Here]

## Model Formulation

### *Original Model*

The model used by Wong et al. (2010) is a modification of the one-dimensional traffic model proposed by Drake et al. (1967):

$$V_r = V_f \exp(-\theta_r(\rho_r + \rho_c)^2)\exp(-\theta_c(1-\cos\varphi)\rho_c{}^2) \tag{1}$$

where

$V_r$ is the speed of the reference stream;

$V_f$ is the free-flow speed;

$\rho_r$ is the density of the reference stream;

$\rho_c$ is the density of the conflicting stream;

$\varphi$ is the intersecting angle between the two streams;

$\theta_r$ and $\theta_c$ are parameters reflecting the sensitivity of speed to density on isotropic and conflicting effects, respectively.

This model satisfies the following natural boundary conditions as stated in the original study.

1. When $\varphi = 0$, there is effectively only a single stream of pedestrians.
2. The interaction effect due to the conflicting pedestrian stream should be symmetrical across the 180° intersecting angle.
3. When the walking facility is nearly empty, the speed of the reference pedestrian stream should approach the free-flow speed, i.e., $V_r \to V_f$ when $\rho_r, \rho_c \to 0$.
4. When the walking facility is nearly empty, the flow of the reference pedestrian stream should approach zero, that is, $q_r \to 0$ when, $\rho_r, \rho_c \to 0$, because $q_r = V_r \rho_r$.
5. When the walking facility is nearly empty, the addition of a pedestrian in the reference or the conflicting stream does not affect the speed of the reference stream, i.e., $\partial v_r / \partial \rho_r \to 0$ and $\partial v_r / \partial \rho_c \to 0$, when $\rho_r, \rho_c \to 0$.

In this model, an exponential term is added to describe the conflicting effect from the opposite stream. The conflicting effects from the opposite stream mainly depend on the density of the conflicting stream, and on the intersecting angle between the two streams: i.e., the direction of the opposite stream. The conflicting effect is symmetrical across 180°.

As the two streams are actually each other's conflicting stream, we can also represent the speed of the conflicting stream as in Eq. (2):

$$V_c = V_f \exp(-\theta_r (\rho_r + \rho_c)^2) \exp(-\theta_c (1 - \cos\varphi) \rho_r{}^2) \tag{2}$$

Dividing Eq. (1) by Eq. (2), we obtain:

$$\frac{V_r}{V_c} = \exp\left(\theta_c \left(1 - \cos\varphi\right)\left(\rho_r^2 - \rho_c^2\right)\right) \tag{3}$$

$$\frac{V_r}{V_c} = \exp\left(\theta_c\left(1-\cos\varphi\right)\left(\rho_t^2\left(1-\frac{2\rho_c}{\rho_t}\right)\right)\right), \tag{4}$$

where $\rho_t$ represents the total density, i.e., the sum of $\rho_r$ and $\rho_c$. This indicates that the ratio between the speed of the two streams is governed by the density difference between the two streams. If $\rho_r > \rho_c$, i.e., $\frac{\rho_c}{\rho_t} < 0.5$, then $\frac{V_r}{V_c} > 1$. This means that the stream with a higher density will suffer a relatively lower conflicting effect from the other stream, so that it can achieve a higher speed, and vice versa.

Both the experimental data and the field data agree with the model that $\frac{V_r}{V_c}$ is generally larger than 1, when the density ratio $\frac{\rho_c}{\rho_t}$ is less than 0.5. However, as shown in Table 2, the correlation between these two quantities is quite weak in both sets of data, i.e., there is no noticeable increase in the conflicting effect as the density of the conflicting stream rises. On the other hand, we find that there is a much stronger correlation between the speed ratio and the flow ratio, $\frac{q_r}{q_c}$, such that the flow of one stream is the product of its speed and density, i.e., $q_r = V_r\rho_r$, $q_c = V_c\rho_c$ and $q_t = q_r + q_c$.

[Insert Table 2 Here]

As shown in Table 2, the correlation between speed ratio and flow ratio is more significant. This suggests that the density difference may not be a good way to represent the speed in bi-directional pedestrian stream movements, as the density of one stream is a static quantity and does not reflect the movement of the stream. However, the conflicting effect induced by the opposite stream is dependent not only on the density of the conflicting stream itself, but also on the movements of both streams. Therefore, to better model the conflicting effect between the two opposite streams, we adopt a momentum term, *flow* (density × speed, analogous to mass × speed in a physical system), that reflects the relative movement momentum between the two streams and the density difference. This improved model is discussed in the next section.

### *Improved Model*

Our modification to the previous model is as follows:

$$V_r = V_f \exp\left(-\theta(\rho_r+\rho_c)^2\right)\exp\left(-\beta\left(1-\frac{V_r\rho_r}{V_r\rho_r+V_c\rho_c}\right)\left(1-\cos\alpha\varphi\right)(\rho_r+\rho_c)\right) \qquad (5)$$

$$V_c = V_f \exp\left(-\theta(\rho_r+\rho_c)^2\right)\exp\left(-\beta\left(1-\frac{V_c\rho_c}{V_r\rho_r+V_c\rho_c}\right)\left(1-\cos\alpha\varphi\right)(\rho_r+\rho_c)\right) \qquad (6)$$

where $V_r$, $V_f$, $\rho_r$, $\rho_c$ and $\varphi$ are defined in equation (1), $\theta$, $\beta$ and $\alpha$ are coefficients, and $\frac{V_r\rho_r}{V_r\rho_r+V_c\rho_c}$ is the flow ratio (flow = density·speed, the momentum term), with $\frac{V_r\rho_r}{V_r\rho_r+V_c\rho_c}$ = 1, when both $\rho_r = 0$ and $\rho_c = 0$.

The improved model satisfies the same boundary conditions as the original model. It can also be reduced to a one-dimensional Drake model when the intersecting angle $\varphi = 0$.

### *Bayesian Inference*

Bayesian inference is a method of statistical deduction in which Bayes' theorem is used to calculate how the prior distribution changes according to new evidence. This method is a modeling approach for parameter estimation that integrates prior and current information. The ultimate aim of Bayesian inference is to obtain the posterior distribution of all unknowns, i.e., the parameters of interest.

To perform Bayesian inference, we used the WinBUGS software to estimate the proposed model. According to Ioannis Ntzoufras (2009), Bayesian statistics regard all unknown parameters as random variables, so prior distribution must be defined initially. Assuming that the prior distribution for all of the parameters to be estimated is normal, the prior mean $\mu$ and variance $\sigma^2$ should be specified for each parameter. When we strongly believe that the estimate mean is accurate, the variance can be set relatively low and great uncertainty concerning to the prior mean can be represented by large variance. No prior information is available when we first apply the proposed model to the controlled experiment data. Therefore, a prior distribution that will not influence the posterior distribution should be specified to let the data speaks for themselves: i.e., a non-informative prior distribution should be adopted. In practice, the variance $\sigma^2$ is set very large ($\sigma^2$ =10000) such that the prior distribution contributes negligible information to the posterior distribution.

To evaluate the goodness-of-fit and to check the performance of the models, we used the deviance information criterion (DIC) and the posterior p-value to assess both the statistical fit and the prediction of the proposed model. The DIC is useful in Bayesian model selection as it measures how well the model fits and considers penalties on number of parameters. Generally, the model with low DIC value is preferred (Spiegelhalter et al., 2002). The posterior p-value checks the goodness-of-fit by comparing the model's predictive data to the observed data. This assumes that if experiments with the same parameters were replicated in the future we would obtain another set of observed data. If the model is appropriate for the observed data, the replicated data should be very close to the observed data. Hence, the difference between the two sets of data will reveal the goodness-of-fit of the model. The posterior p-value is defined as the probability that the replicated data is more extreme than the observed data. Therefore, the closer the posterior p-value is to 0.5, the better the fit of the model (Gelman et al., 2004).

Besides these statistics in the Bayesian framework, we also adopted the mean absolute percentage error (MAPE), the root mean square error (RMSE), and the relative root mean square error (RRMSE) as statistics to evaluate the goodness-of-fit for the models.

## Results and Discussion

Table 3 presents the calibration results of the two models for the controlled experiment data.

[Insert Table 3 Here]

In Table 3, it can be seen that the value of free-flow speed $V_f$ is 1.074 m/s (0.95 CIs: 1.065, 1.083), and the parameter of isotropic effect $\theta$ is 0.062 (0.95 CIs: 0.058, 0.066) in the improved model. These values are similar to those in the original model. The calibrated value of $\beta$ is 0.072 (0.95, CIs: 0.064, 0.080), and $\alpha$ is 1.271 (0.95, CIs: 1.208,1.336), which is between 1 and 2, indicating that the intersecting angle between the two streams has a negative influence on speed, and this conflicting effect is maximized when the intersecting angle is between 90° and 180°. The DIC value for the improved model is far less than that of the original model, and the posterior *p*-value (the closer to 0.5, the better the model fit) and other statistical indexes of the improved model also indicate that the improved model results in a better fit of the experimental data.

There is no doubt that the controlled experiment is a very good sample of bi-directional pedestrian stream movements with oblique intersecting angles. The volunteers were asked to walk in designated directions, and a variety of densities and intersecting angles were tested. Hence, the data collected from the controlled experiment are of good quality. However, no experiment is the same as a real-world situation. The data from the observational survey are less controllable than those in the experiment, as we cannot control the density of the crowds or the directions the pedestrians walk. However, these data are a better reflection of reality.

To test the model's applicability to a real-world situation, we adopt the Bayesian method to further calibrate the model with the field data collected from the observational survey. For the parameters reflecting the interactions between pedestrians, θ, β, and α, we use the posterior distribution from the controlled experiment to provide prior distribution, as shown in Table 4.

[Insert Table 4 Here]

However, for the free-flow speed ($V_f$), no prior information is available. As pedestrians in the crosswalk walk much faster than the volunteers in the experiment, the free-flow speed clearly depends on the environment in which the data are collected. To assess the free-flow speed, we extract the data points (on the speed of the reference stream) that had low total density ($\rho_r + \rho_c < 1$) from both the experiment and the field survey, and perform a *t*-test. We find that the means of the speed for these two situations ($\rho_r + \rho_c < 1$) are significantly different (at a 0.1% level). The two means are 1.074 m/s for the experiment and 1.307 m/s for the field survey. The mean value for the controlled experiment (1.074 m/s) is the same as the calibrated free-flow speed shown in Table 3. The mean value for the field data is 30% greater than that in the experiment. Therefore, the free-flow speed should be revised for the model in accordance with the field data.

Finally, we calibrate the model for the field data and compare statistics to those of the controlled experiment, as shown in Table 5.

[Insert Table 5 Here]

Table 5 shows that the free-flow speed increases to 1.326 m/s when the field data is used to update the model to account for the people hurrying through the crosswalk. This free-flow speed is consistent with that measured in an empirical study reported by Lam et al. (2002), which examined a signalized crosswalk in Hong Kong. The posterior *p*-value indicates that the model generally fits the field data. Although the mean absolute percentage error and the relative root mean square error have a roughly 10% increase, this is still reasonable when considering the large variability of the field data (standard deviation = 0.5 m/s) compared to the experimental data (standard deviation = 0.2 m/s).

As the model's form is a set of structural equations, it is not straightforward to compute the speed of one stream with a given $\rho_r$ and $\rho_c$. Therefore, Fig. 5 provides the design charts for finding the speed of the reference stream that corresponds to $\rho_r$ and $\rho_c$ under different intersecting angles. Fig. 5 also shows the relationships between the speed of the reference stream and its density, when the density of the conflicting stream is kept constant. Generally, when the density of the conflicting stream is low ($\rho_c < 1$), the speed of the reference stream first decreases very slightly (from 1.3 to 1.2 m/s) as the density of the reference stream gradually increases from 0 to 1 ped/m$^2$, because the total density is also low and the interaction between pedestrians is weak at this stage. The reference stream's speed reduces more significantly as the total density builds, and the conflicting effect from the opposite stream grows as the number of interactions between pedestrians increases. Finally, the decline becomes stable when the reference stream's density increases to the point that it becomes the major stream. In contrast, when the density of the conflicting stream is relatively high ($\rho_c > 1$), it skips the first phase that was seen in the previous situation. The speed of the reference stream drops sharply at the beginning, as the conflicting stream is absolutely the major stream when the reference stream has very low density. Thus, the conflicting effect from the opposite stream is tremendous at the starting stage. The gradient gradually reduces as the density of the reference stream increases.

[Insert Figure 5 Here]

Fig. 5 also shows the effects on stream speed induced by different intersecting angles. When the intersecting angle increases from 0° to 90°, the pedestrians actually have the same destination, i.e., the opposite side of the crosswalk, although they may enter the crosswalk area from different points. The smaller the intersecting angle, the less difference there is between their directions. Hence, speed reduces as angle increases. However, when the intersecting angle exceeds 90° and continues to increase between 90° and 180°, the speed no longer decreases steadily with the increase of the intersecting angle. The worst situation occurs when the intersecting angle is 135°. We use Fig. 6 to illustrate this phenomenon. When the intersecting angle between the two streams is 90° (Fig. 6(a)), each stream of pedestrians is walking orthogonally to the other, and the pedestrians can easily find gaps in the conflicting stream. When the intersecting angle is 180° (Fig. 6(b)), the formation of self-organized lanes helps to reduce the conflicting effect induced by the opposite stream. However, when the intersecting angle is 135° (Fig. 6(c)), there is no obvious gap in the conflicting stream, and individual pedestrians must zigzag to avoid others coming the other way. Such interactions between pedestrians of different streams reduce their walking speeds.

To illustrate this flow-density relationship, a straightforward comparison between situations with different intersecting angles is shown in Fig. 7. Fig. 7 also shows that the optimum total density under different intersecting angles is about 2.0 ~ 3.0 ped/m$^2$, with a maximum flow of about 1.8 ~ 2.1 ped/m/s (for different intersecting angles). This value is slightly higher than the value reported in Wong et al. (2010). It is not surprising that pedestrians walk through a crosswalk faster than students cross a sports stadium in an experiment.

## Conclusions

Expanding on Drake's model, we developed a mathematical model to represent the movements of bi-directional pedestrian flows, which introduces the flow ratio and the intersecting angle as attributes that influence the speed of the streams. Two sets of data were collected, one from a controlled experiment and the other from an observational survey. Bayesian inference was adopted in the parameter calibration. The empirical data was used to calibrate the model as it completely and homogeneously covers the different possible intersecting angles and the different levels of flows. The calibrated parameters of the controlled experiment were used as the prior data in the substantial calibration of the field data. The field data was then used to update the model to reflect real-world situations.

Compared to the previous model, the new model achieves a better fit for experimental data, and continues to satisfy the same boundary conditions as the original model. The updating process with the field data also improves the model to reflect real-world situations. The new model reflects the reality that the speed of the streams in bi-directional pedestrian movements depends not only on the density of each stream, but also on the factors of the flow speed in both streams and the intersecting angle between the two streams. Therefore, the new model is more comprehensive in representing the interactions of bi-directional pedestrian flows. Finally, the new model also shows that the conflicting effect induced by the intersecting angle maximizes when the angle is near 135°. At this angle, pedestrians must pay more attention to avoid pedestrians in the conflicting flow, as there is neither lane formation nor a straightforward gap between streams in such situations.

These findings build on previous controlled experiments that focused on bi-directional pedestrian streams with oblique intersecting angles. Data on the flows of streams are added to data from the previous experiments to better describe the movements and interactions of flows. The result is an improved form of model for bi-directional pedestrian flows. The use of on-site observation helps us to better understand the difference between experimental and real situations, and this improves the model. The results are consistent with similar observations by other researchers. However, more observational surveys on different walking facilities should be conducted to make the model even more congruent with actual pedestrian behavior. Once we have a comprehensive understanding of bi-directional pedestrian flows, we can further extend the study to multi-directional pedestrian flows, in which the interactions between streams can be quite different from the bi-directional ones.

[Insert Figure 6 Here]

[Insert Figure 7 Here]


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## LIST OF FIGURE AND TABLE CAPTIONS

**Table 1** Summary of data

| Dataset | Controlled Experiment | Field Observations |
|---|---|---|
| Total Pedestrian No. | 1160 | 6788 |
| Average Speed (m/s) | 0.74 | 1.15 |
| Standard Deviation of Speed (m/s) | 0.2 | 0.5 |
| Average Density (ped/m$^2$) | 2.07 | 0.63 |
| Standard Deviation of Density (ped/m$^2$) | 0.51 | 0.33 |

**Table 2** Comparison between experimental data and field data

| $\frac{V_r}{V_c}$ | Controlled Experiment Data | Field Data |
|---|---|---|
| Maximum | 2.46 | 8.01 |
| Minimum | 0.61 | 0.11 |
| Mean | 1.16 | 1.10 |
| Standard Deviation | 0.28 | 0.62 |
| Correlation between $\frac{V_r}{V_c}$ & $\frac{\rho_c}{\rho_t}$ | -0.099 | 0.038 |
| Correlation between $\frac{V_r}{V_c}$ & $\frac{q_c}{q_t}$ | -0.368 | -0.331 |

**Table 3** Comparison of the original and improved models

| | Controlled Experiment | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Original Model | | | | Improved Model | | | |
| Sample Size | 5487 | | | | 3459 | | | |
| | | Estimate | (95% BCIs) | | | Estimate | (95% BCIs) | |
| Calibrated Parameters | $V_f$ | 1.076 | 1.067 | 1.085 | $V_f$ | 1.074 | 1.065 | 1.083 |
| | $\theta_r$ | 0.079 | 0.075 | 0.082 | $\theta$ | 0.062 | 0.058 | 0.066 |
| | $\theta_c$ | 0.025 | 0.019 | 0.031 | $\beta$ | 0.072 | 0.064 | 0.080 |
| | | | | | $\alpha$ | 1.271 | 1.208 | 1.336 |
| DIC | -4520.32 | | | | -7754.05 | | | |
| Posterior p-value | 0.5275 | | | | 0.5110 | | | |
| MAPE | 17.7% | | | | 17.4% | | | |
| RMSE | 0.1703 m/s | | | | 0.1686 m/s | | | |
| RRMSE | 19.1% | | | | 18.9% | | | |

**Table 4** Informative prior distribution for parameters to be estimated

| | Mean | Standard Deviation |
|---|---|---|
| θ | 0.062 | $2.18\times10^{-03}$ |
| β | 0.072 | $4.27\times10^{-03}$ |
| α | 1.271 | 0.032 |

**Table 5** Comparison of statistics

| | | Controlled Experiment | | | Field Observation | | |
|---|---|---|---|---|---|---|---|
| Sample Size | | 3459 | | | 1737 | | |
| | | Estimate | (95% BCIs) | | Estimate | (95% BCIs) | |
| Calibrated Parameters | $V_f$ | 1.074 | 1.065 | 1.083 | 1.326 | 1.312 | 1.341 |
| | θ | 0.062 | 0.058 | 0.066 | 0.065 | 0.061 | 0.069 |
| | β | 0.072 | 0.064 | 0.080 | 0.078 | 0.070 | 0.086 |
| | α | 1.271 | 1.208 | 1.336 | 1.214 | 1.149 | 1.275 |
| Posterior *p*-value | | 0.5110 | | | 0.5028 | | |
| MAPE | | 17.7% | | | 28.8% | | |
| RMSE | | 0.1703 m/s | | | 0.3400 m/s | | |
| RRMSE | | 19.1% | | | 30.9% | | |

**Fig. 1** (a) Intersecting Angle = 90° ; (b) Intersecting Angle = 135°


**Fig.2** Location of the selected site

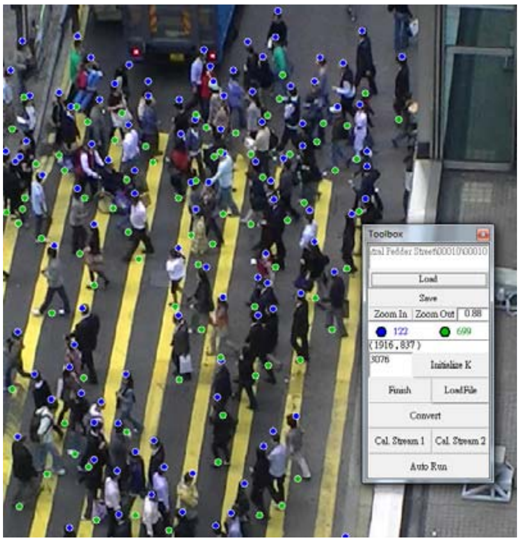


**Fig. 3** The interface of the VB program for acquisition of the coordinates

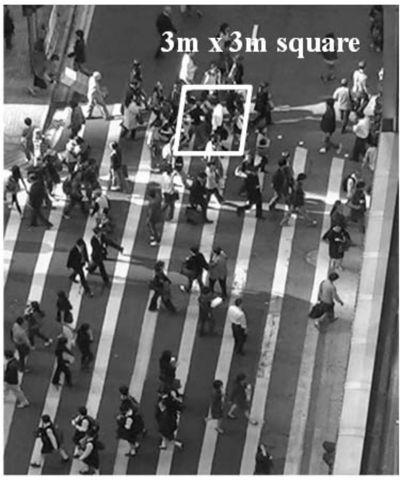


**Fig. 4** Distribution of pedestrians in the region

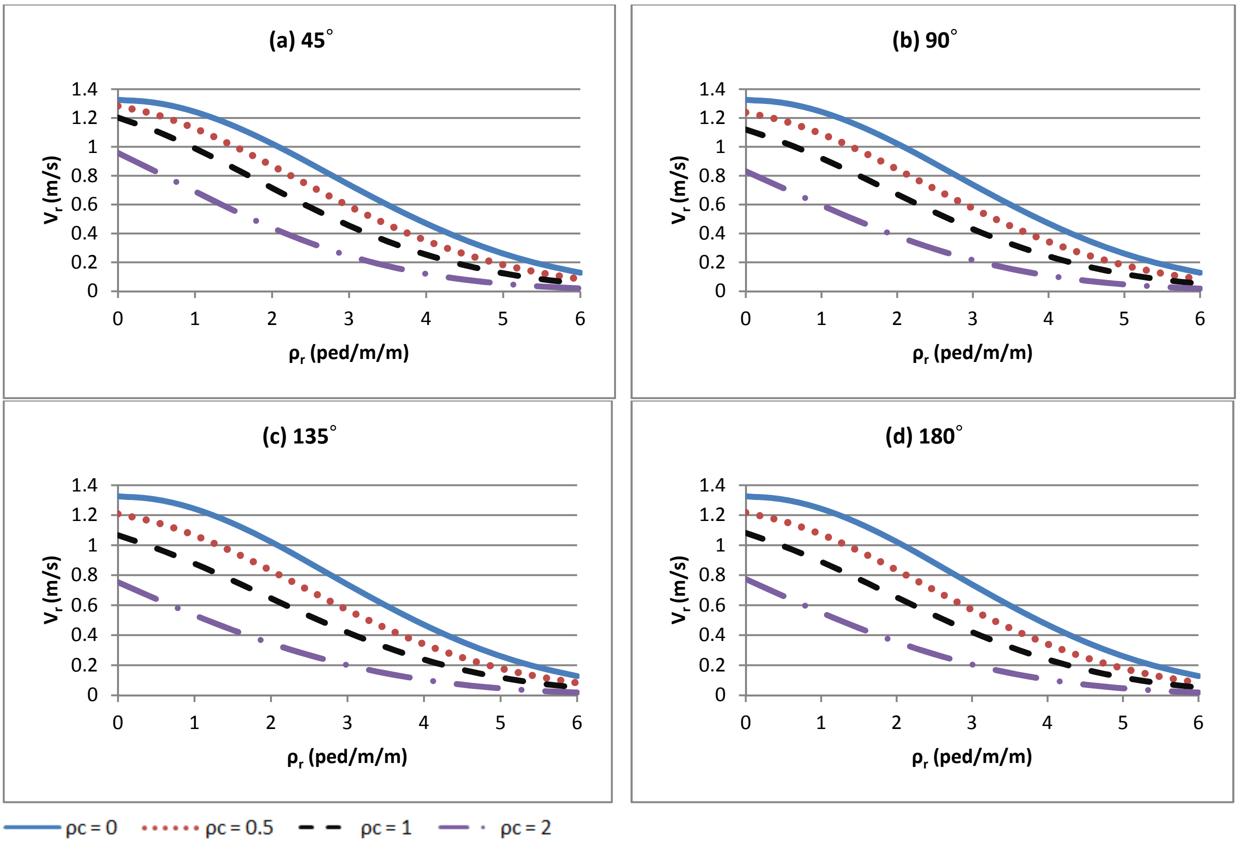


**Fig. 5** Relationship between the speed of the reference stream and the density of the reference stream at different intersecting angles: (a) 45 degree, (b) 90 degree, (c) 135 degree, and (d) 180 degree

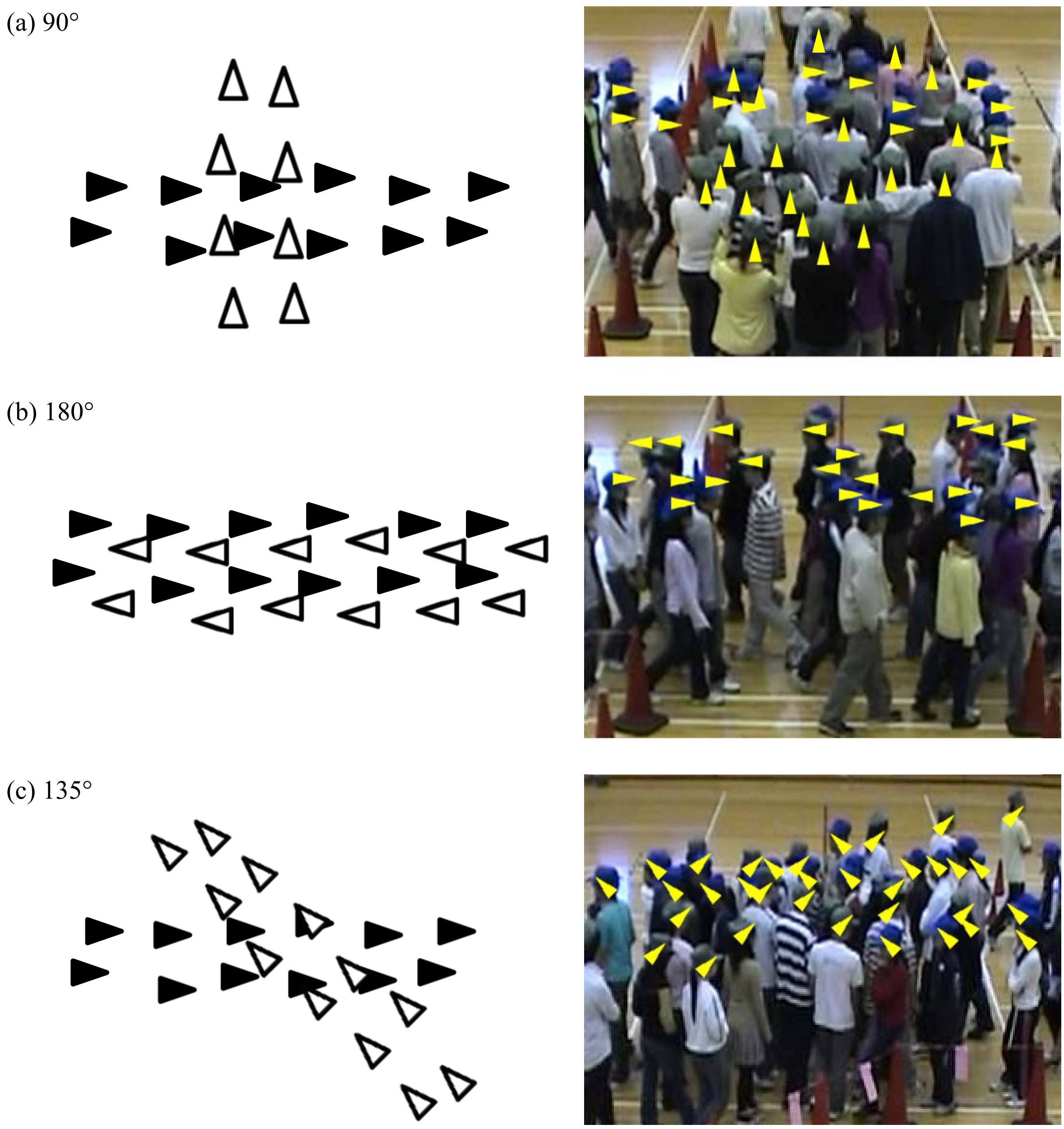


**Fig. 6** Illustration of conflicting with different intersecting angle

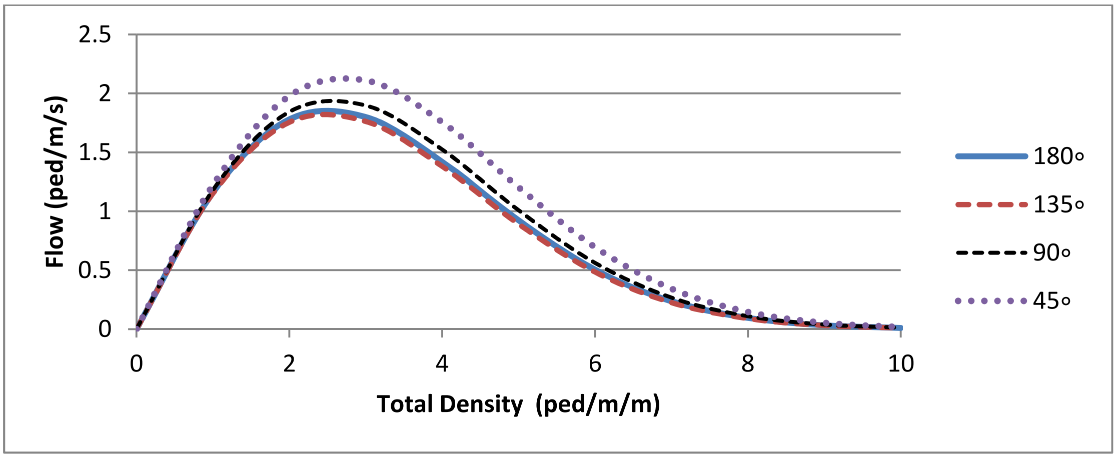


**Fig. 7** Flow-Total Density relationship under different intersecting angles ($\rho_r$=$\rho_c$)