

Theoretical and Experimental Study of Plate-Strengthened Concrete Columns under Eccentric Compression Loading

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Abstract

Steel jacketing has been widely used for strengthening reinforced concrete (RC) columns in the past four decades. In practice, the RC columns to be strengthened are usually subjected to eccentric pre-compressed axial loads. Until now, there have been only limited studies conducted that address the stress-lagging effects between the original column and the new jacket due to the pre-existing load. In this paper, the precambered steel plate strengthening approach, which can alleviate the stress-lagging effects, was adopted to improve the axial strength and moment capacity of the preloaded RC columns subjected to eccentric compression loading. An experimental study that involved eight specimens with different eccentricities, plate thicknesses and initial precamber displacements was conducted to examine the ductility and moment-curvature response of strengthened columns and to validate the effectiveness of this approach. A theoretical model was developed to predict the axial load capacity of the plate-strengthened columns. A comparison of the theoretical and experimental results showed that the theoretical model accurately predicted the axial load-carrying capacities of the plate-strengthened columns under eccentric compression loading.

Keywords:

Reinforced Concrete Columns; Precambered Steel Plates; Strengthening; Eccentric Loads

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27 **Introduction**

28 Due to the deterioration of materials and the demand for additional strength, a large number of
29 reinforced concrete (RC) columns may need to be retrofitted or strengthened. Steel jacketing, which is
30 executed by attaching steel plates or angles onto the concrete, has been widely used to strengthen RC
31 structures due to the cost effectiveness and simple construction. Although a number of studies (Oey et
32 al. 1996; Ersoy et al. 1993; Ramírez 1996; Wu et al. 2006; Fukuyama et al. 2000; Cirtek et al. 2001;
33 Adam et al. 2007, 2008 and 2009; Giménez et al. 2009) were conducted to investigate the performance
34 of the jacketed columns under axial compression loads, only a few considered the effects of pre-
35 existing loads on stress-lagging between the concrete core and the new jacket. Ersoy et al. (1993),
36 Takeuti et al. (2008) and Giménez et al. (2009) experimentally investigated the effects of pre-existing
37 loads on the strengthening efficiency. Their test results demonstrated that the stress-lagging effects can
38 significantly decrease the ultimate axial load capacity of the strengthened columns.

39 In real applications, many columns are subjected to various degrees of eccentric compression
40 loading. The effects of RC columns strengthened by steel jackets under eccentric compression loads
41 should be investigated. Li et al. (2009) and Garzón et al. (2011) studied the behavior of steel-caged
42 columns under combined bending and axial loads. Their experimental results revealed that the steel
43 strips and angles can increase the load resistance and ductility of strengthened columns. Montuori and
44 Piluso (2009) tested thirteen RC columns strengthened by steel angles and battens under eccentric
45 loading. Their study demonstrated that both the axial load-carrying capacity and the lateral
46 deformability of strengthened concrete columns can be enhanced. Furthermore, they proposed a
47 theoretical model that was able to predict the load-carrying capacity of the strengthened columns
48 based on a kinematic mechanism. In their model, the hoops were considered simple support restraints,
49 and the longitudinal bar was modeled as a continuous beam on simple supports that were subjected to
50 a compressive axial load. With increasing axial load, the section of the bar between the two hoops
51 developed a kinematic mechanism characterized by three plastic hinges. In addition, a comparison of
52 the moment-curvature responses was performed that showed the accuracy of the model in predicting

the structural response within the whole deformation range. Our companion paper (Wang and Su 2012) presented a test of nine preloaded RC columns strengthened by precambered steel plates under eccentric loading. The test results showed that precambered steel plates could actively share the existing axial loads with the original column. Stress relief in the original concrete column and post-stress developed in the steel plates can alleviate the stress-lagging and displacement incompatibility problems. Both the axial and moment capacities of strengthened columns were enhanced. The post-yield deformation was substantially increased.

In this paper, new experimental results in terms of the ductility and moment-curvature response of strengthened RC columns with precambered steel plates under eccentric compression loads are presented. A theoretical model based on elementary structure mechanics with consideration of stress-lagging effects was developed to predict the axial load-carrying capacity of plate-strengthened columns under eccentric compression loading. The accuracy of the model was verified through a comparison of the model with experimental results obtained by the authors and by Montuori and Piluso (2009).

Theoretical model

Initial Precamber

Two stainless steel rods and bolts are used to control the initial deformation of the plates and to form the required precambered profile as shown in Fig. 1. Because the bolts at both ends of the steel plates restrain the end rotations of the plates, the initial lateral displacement (v) of the precambered plate can be approximated by a cosine function (Su et al. 2011) as expressed in Eq.(1).

$$v = \frac{\delta}{2} \left[1 - \cos \left(\frac{2\pi x}{L_{rc,pl}} \right) \right] \quad (1)$$

where δ is the initial precamber at the mid-height of the plate, $L_{rc,pl}$ is the clear height of the RC column under preloading (P_{pl}), x is the coordinate defined along the height of the column, and the

subscript pl denotes the preloading stage. Eq. (1) satisfies the boundary conditions at both ends of the steel plates, i.e., $v = 0$ and $\frac{dv}{dx} = 0$ when $x = 0$ or $x = L_{rc,pl}$.

The difference in length of the steel plate and the RC column (Δ_L) can be evaluated by Eq. (2).

$$\Delta_L = \frac{1}{2} \int_0^{L_{rc,pl}} \left(\frac{dv}{dx} \right)^2 dx \quad (2)$$

Putting Eq. (1) into Eq. (2) gives

$$\Delta_L = \frac{(\pi\delta)^2}{4L_{rc,pl}} \quad (3)$$

Material Constitutive Laws and Simplified Stress Block Model

The stress-strain relationship of concrete in compression is represented by the parabolic relationship proposed by Hognestad et al. (1955).

$$\sigma_c = f_c' \left[\frac{2\varepsilon_c}{\varepsilon_{co}} - \left(\frac{\varepsilon_c}{\varepsilon_{co}} \right)^2 \right] \quad (4)$$

where f_c' is the concrete compressive cylinder strength, σ_c and ε_c are the stress and strain of the concrete, respectively, and ε_{co} is the concrete compressive strain corresponding to f_c' .

Both the steel plates and steel bars are assumed to be elasto-plastic materials. In the initial elastic stage, the stress-strain models of steel plates and steel bars can be expressed as

$$\sigma_p = E_p \varepsilon_p \quad (5)$$

$$\sigma_s = E_s \varepsilon_s \quad (6)$$

where σ_p and ε_p are the stress and strain of steel plates, respectively, and σ_s , ε_s and E_s are the stress, strain and Young's modulus of the steel bars, respectively.

Collins and Mitchell (1987) noted that, for a column section with a constant width, the parabolic portion of the concrete stress distribution can be replaced by an equivalent rectangular block by introducing the stress block factors α and β as shown in Fig. 2, which can be calculated using Eq. (7) and Eq. (8).

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$$\alpha = \left[\frac{\varepsilon_{cu}}{\varepsilon_{c0}} - \frac{1}{3} \left(\frac{\varepsilon_{cu}}{\varepsilon_{c0}} \right)^2 \right] / \beta \quad (7)$$

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$$\beta = \left(4 - \frac{\varepsilon_{cu}}{\varepsilon_{c0}} \right) / \left(6 - \frac{2\varepsilon_{cu}}{\varepsilon_{c0}} \right) \quad (8)$$

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Preloading stage

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The preloading force is resisted by concrete and steel bars before flattening the precambered steel

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plates. The equilibrium equation of the RC column before flattening the plates can be obtained from

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the sum of the internal forces.

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$$P_{pl} = \alpha \beta b (c_{pl} - 2d_b) f_c' \left(\frac{2\varepsilon_{c,pl}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,pl}^2}{\varepsilon_{c0}^2} \right) + E_{sc} A_{sc} \varepsilon_{c,pl} \left(\frac{c_{pl} - d'}{c_{pl}} \right) - E_{st} A_{st} \varepsilon_{c,pl} \left(\frac{d - c_{pl}}{c_{pl}} \right) \quad (9)$$

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The equation obtained from taking moments about the tension steel is

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$$P_{pl} e' = \alpha \beta b (c_{pl} - 2d_b) f_c' \left(\frac{2\varepsilon_{c,pl}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,pl}^2}{\varepsilon_{c0}^2} \right) \left(d - \frac{\beta c_{pl}}{2} \right) + E_{sc} A_{sc} \varepsilon_{c,pl} \left(\frac{c_{pl} - d'}{c_{pl}} \right) (d - d') \quad (10)$$

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where b is the width of the column section as shown in Fig. 2, d and d' are the depths of the tension

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steel and the compression steel measured from extreme compression fiber, respectively, d_b is the

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diameter of the bolt hole, E_{sc} and E_{st} are the Young's moduli of the compression steel bar and tension

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steel bar, respectively, A_{sc} and A_{st} are the total cross-sectional areas of the compression steel bars and

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tension steel bars, respectively, and e' is the distance between the load point and the tension steel. The

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depth of the compression zone (c_{pl}) and the concrete strain at extreme compression fibers ($\varepsilon_{c,pl}$) in the

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preloading stage can be obtained from Eqs. (7), (8), (9) and (10).

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The axial stiffness of the RC column ($K_{rc,pl}$) and a steel plate (K_p) can be determined by Eq.(11) and

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Eq.(12), respectively.

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$$K_{rc,pl} = \frac{E_c A_c}{L_{rc,pl}} \quad (11)$$

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$$K_p = \frac{E_p A_p}{L_p} \quad (12)$$

where E_c and E_p are the values for the Young's moduli of concrete and steel plates, respectively, A_c is the cross-sectional area of the RC column considering the cracked section, A_p is the cross-sectional area of a steel plate and L_p is the undeformed length of the steel plate.

Post-stressing stage

When the precambered steel plates are flattened, the preloading force is resisted by concrete, steel bars and steel plates. Fig. 3 shows the lengths and deformations of the plates and the RC column at three different loading stages, i.e., the undeformed stage, the preloading stage and the post-stressing stage. By progressively tightening the bolts on both sides of the column, the precambered steel plates are gradually flattened. Due to the arching action, a post-compressive force ($P_{p,ps}$) is generated in the steel plates, and an equal magnitude de-compressive force is generated in the RC column. Using Hooke's law, the total post-stressed force provided by the plates is

$$P_{p,ps} = 2K_p \Delta_{p,ps} \quad (13)$$

where $\Delta_{p,ps}$ is the axial shortening of the steel plate while tightening the bolts when compared to the original undeformed state, and the subscript ps denotes the post-stressing stage.

The de-compressive force in the RC column can be written as

$$P_{p,ps} = K_{rc,pl} \Delta_{rc,ps} \quad (14)$$

where $\Delta_{rc,ps}$ is the increase in length of the RC column during the post-stressing stage, as shown in Fig. 3.

The difference in lengths of the steel plate and RC column in the preloading stage can be expressed as

$$\Delta_L = L_p - L_{rc,pl} \quad (15)$$

According to the displacement compatibility model (Fig. 3), the difference in the lengths of the steel plate and RC column in the preloading stage is equal to the sum of the axial stretching of the RC column ($\Delta_{rc,ps}$) and the axial shortening of the steel plates ($\Delta_{p,ps}$). Hence,

$$\Delta_L = \Delta_{rc,ps} + \Delta_{p,ps} \quad (16)$$

Substituting Eq. (13) and Eq. (14) into Eq. (16) gives

$$\Delta_L = \frac{K_{rc,pl} + 2K_p}{K_{rc,pl}} \Delta_{p,ps} \quad (17)$$

Putting Eq. (17) into Eq. (13), the post-compressive force in the plates can be obtained by

$$P_{p,ps} = \Delta_L \frac{2K_p K_{rc,pl}}{2K_p + K_{rc,pl}} \quad (18)$$

Meanwhile, the stress of steel plates ($\sigma_{p,ps}$) at the post-stressing stage can be expressed by

$$\sigma_{p,ps} = \frac{P_{p,ps}}{2A_p} \quad (19)$$

By considering vertical force equilibrium, the preloading force is resisted by the concrete, the steel bars and the steel plates. Hence,

$$P_{pl} = \sigma_{c,ps} A_c + \sigma_{s,ps} A_s + 2\sigma_{p,ps} A_p \quad (20)$$

where $\sigma_{c,ps}$, $\sigma_{s,ps}$ and $\sigma_{p,ps}$ are the axial stresses in the concrete, the steel bars and the steel plates in the post-stressing stage, respectively, and A_s is the total cross-sectional area of the vertical steel bars.

We assume that there is no bond slip between the steel bars and the concrete. Hence,

$$\varepsilon_{c,ps} = \varepsilon_{s,ps} \quad (21)$$

By considering the equivalent rectangular stress block, the equilibrium equation of the strengthened column can be obtained from the sum of the internal forces.

$$P_{pl} = \alpha\beta b(c_{ps} - 2d_b) f'_c \left(\frac{2\varepsilon_{c,ps}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,ps}^2}{\varepsilon_{c0}^2} \right) + E_{sc} A_{sc} \varepsilon_{c,ps} \left(\frac{c_{ps} - d'}{c_{ps}} \right) - E_{st} A_{st} \varepsilon_{c,ps} \left(\frac{d - c_{ps}}{c_{ps}} \right) + 2A_p \sigma_{p,ps} \quad (22)$$

The equation obtained from taking moments about the tension steel is

$$P_{pl} e' = \alpha\beta b(c_{ps} - 2d_b) f'_c \left(\frac{2\varepsilon_{c,ps}}{\varepsilon_{c0}} - \frac{\varepsilon_{c,ps}^2}{\varepsilon_{c0}^2} \right) \left(d - \frac{\beta c_{ps}}{2} \right) + E_{sc} A_{sc} \varepsilon_{c,ps} \left(\frac{c_{ps} - d'}{c_{ps}} \right) (d - d') + 2A_p \sigma_{p,ps} d'' \quad (23)$$

where d'' is the distance from the center of the compression block of the steel plate to the tension steel.

The depth of the compression zone (c_{ps}) and strain of concrete ($\varepsilon_{c,ps}$) in the post-stressing stage can be obtained by solving Eqs. (7), (8), (22) and (23).

167 *Ultimate Load Capacity*

168 Assuming that the compression steel has been yielded, the equilibrium equation can be obtained from
 169 the sum of the internal forces.

$$170 \quad P_u = \alpha\beta b(c_u - 2d_b)f'_c + A_{sc}f_{scy} - A_{st}f_{st} + 2P_{pcu} - 2P_{ptu} \quad (24)$$

171 The equation obtained from taking moments about the tension steel is

$$172 \quad P_ue' = \alpha\beta b(c_u - 2d_b)f'_c(d - \frac{\beta c_u}{2}) + A_{sc}f_{scy}(d - d') + 2P_{pcu}d_{pcu} - 2P_{ptu}d_{ptu} \quad (25)$$

173 where f_{scy} is the compression steel yield strength, f_{st} is the stress in the tension steel, P_{pcu} and P_{ptu} are
 174 the forces defined in Fig. 4(c), and d_{pcu} and d_{ptu} are the distances from the center of force P_{pcu} and
 175 force P_{ptu} to the tension steel, respectively.

176 Force P_{pcu} at the ultimate load is

$$177 \quad P_{pcu} = t_p f_{py} c_{pu} \quad (26)$$

178 where t_p is the thickness of plate, f_{py} is the yield strength of steel plate, and c_{pu} is the depth of the
 179 neutral axis measured from the extreme compression fiber of the steel plate, as shown in Fig. 4(b),
 180 which can be calculated by

$$181 \quad c_{pu} = \begin{cases} h & (\text{Case1, } c_{pu} \geq h) \\ (\varepsilon_{cu} - \varepsilon_{c,ps} + \varepsilon_{p,ps})/\phi & (\text{Case2, } c_{pu} < h) \end{cases} \quad (27)$$

182 where h is the width of steel plate and ϕ is the change of curvature of RC column between the post-
 183 stressing stage and the ultimate load stage, which can be expressed as

$$184 \quad \phi = \phi_2 - \phi_1 = \frac{\varepsilon_{cu}}{c_u} - \frac{\varepsilon_{c,ps}}{c_{ps}} \quad (28)$$

185 where ϕ_1 is the curvature of RC column at the post-stressing stage and ϕ_2 is the curvature of the RC
 186 column at the ultimate load stage, as shown in Fig. 4(a).

187 According to the assumption of curvature compatibility between the RC column and steel plates, the
 188 force P_{ptu} is

$$189 \quad P_{ptu} = t_p E_p \left(h - \frac{\varepsilon_{cu} - \varepsilon_{c,ps} + \varepsilon_{p,ps} - \varepsilon_{py}}{\phi} \right) \left[\varepsilon_{py} - \left(\frac{\varepsilon_{cu} - \varepsilon_{c,ps} + \varepsilon_{p,ps}}{\phi} - h \right) \phi \right] \quad (29)$$

The depth of the compression zone (c_u) and ultimate load-carrying capacity (P_u) can be obtained from Eqs. (7), (8), (24) to (29). If a tension failure occurs, the tension steel yields, and Eq. (24) applies with $f_{st}=f_{sty}$.

A Brief Description of Experimental Study

Because the detailed experimental procedure for preloaded RC columns strengthened with precambered steel plates subjected to eccentric loading has been presented in our companion paper (Wang and Su 2012), only a brief description of the test procedure is given in this paper. The new experimental results on ductility and moment-curvature response of eight precambered steel plate-strengthened column specimens, involving a new specimen ESC3-3, are presented.

All the tested concrete columns have the same dimensions and reinforcement arrangements. Fig. 5 shows the reinforcement and steel plate details. Specimens ESC1-1, ESC2-1 and ESC3-1 were control specimens without any strengthening measures to demonstrate the structural performance of RC columns prior to strengthening. The other five specimens were strengthened by precamber steel plates with varying initial precamber and plate thicknesses. Table 1 shows the average concrete cube and cylinder compressive strengths (f_{cu} and f_c') as well as the design parameters for each specimen. Table 2 summarizes the material properties of the steel reinforcements and steel plates. All plate-strengthened columns were subjected to preloading before the plates were flattened, which was equal to 30% of the ultimate axial load capacity of the corresponding control column. For the plate-strengthened specimens, the axial load was applied under a force control with a loading rate of 2 kN/sec. After tightening the bolts and flattening the precambered plates, the applied load was changed to a displacement control with a displacement rate of 0.5 mm/min. The test was terminated when the post-peak load reached 80% of the peak load.

Before installing the steel plates, 65 mm × 65 mm steel angles were welded to both ends of the plates, as shown in Fig. 6. The gaps between the steel angles and the concrete at the bottom and top of the steel plates were filled with an injection plaster, forming a layer of bedding between the steel

217 angles and the concrete. The post-stress procedure described in Wang and Su (2012), which can avoid
218 warping or buckling of the steel plates during decompression of the RC column by flattening the
219 precambered steel plates, was adopted.

220

221 **Results and Discussion**

222 *Ultimate Load Capacity and Bending Strength*

223 Table 3 summarizes the ultimate axial load capacities of all of the specimens. Compared with the
224 control column in each of the groups, the strengthened specimens show various degrees of
225 strengthening from 13.9% to 64.0%. In group A, the ultimate load capacities of Specimens ESC1-2,
226 ESC1-3 and ESC1-4 are increased by 27.1%, 64.0% and 44.6%, respectively. In group B, the ultimate
227 load capacity of Specimen ESC2-2 is enhanced by 13.9%. In group C, the ultimate capacity of
228 Specimen ESC3-3 is increased by 49.0%.

229 According to the proposed theoretical model described in the previous sections, the predicted axial
230 load capacity (P_{pre}) of the specimens was determined by Eq. (22) and Eq. (23). During the calculations
231 of the ultimate load capacity of the RC columns, the extreme fiber compression strain of concrete ε_{cu}
232 was assumed to be 0.003 (Park and Paulay, 1975), and the gross sectional area of the concrete (A_c) did
233 not include the areas of the bolt holes. The predicted axial load capacity of the specimens is presented
234 in Table 3. Comparing the theoretical and experimental axial load capacities reveals that the proposed
235 design procedure is generally able to conservatively estimate the actual axial load capacities of the
236 plate-strengthened columns under eccentric compression loading with an average overestimation of
237 2.1%.

238 Due to the eccentricity of the applied axial load, a bending moment is always generated. The
239 ultimate moment (M_u) at the mid-height of the column is composed of the primary moment (M_p),
240 which is calculated based on the nominal eccentricity, and the secondary moment (M_s) caused by the
241 $P-\Delta$ effect; both are summarized in Table 3. The definitions of the primary, secondary and ultimate
242 moments can be found in Wang and Su (2012). In Group A, the secondary moment of the

strengthened columns ESC1-2, ESC1-3 and ESC1-4 due to the P - Δ effect increased by 27.6%, 49.2% and 37.3%, respectively. In Group B, the secondary moment of the strengthened column due to the P - Δ effect increased by 24.9%. In Group C, the secondary moment of the strengthened column due to the P - Δ effect increased by 188.3%. It is evident that the bending moment of Specimen ESC3-3 is the largest among the eight specimens due to the largest lateral displacement and degree of eccentricity as listed in Table 3.

Deformation and Ductility

The deformability factor (λ), proposed by De Luca et al. (2011), was adopted to evaluate the deformation performance of the strengthened columns, which is defined as

$$\lambda = \Delta_f / \Delta_u \quad (30)$$

where Δ_u is the axial shortening at the ultimate load and Δ_f is the axial shortening at the failure load, which is equal to 75% of the ultimate load. In Group A, compared to the control column, the axial shortening at the failure load of the strengthened columns ESC1-2, ESC1-3 and ESC1-4 improved by 61.7%, 160.2% and 103.9% respectively, as shown in Fig. 7(a), and the deformability factor of the strengthened columns increased by 27.3%, 61.4% and 65.9% respectively. In Group B, compared to the control column, the axial shortening at the failure load of the strengthened column improved by 49.0%, as shown in Fig. 7(b), and the deformability factor of the strengthened column increased by 31.8%. In Group C, compared to the control column, the axial shortening at the failure load of the strengthened column improved by 222.2%, as shown in Fig. 7(c), and the deformability factor of the strengthened column increased by 93.0%. Thus, the plate thickness plays an important role in increasing the deformability of the strengthened columns, whereas the initial precamber and eccentricity do not have a substantial effect on the displacement ductility of columns.

The displacement ductility factor (η) is introduced to evaluate the ductility performance of the strengthened columns. The load-axial shortening responses of the specimens shown in Fig. 7 can be idealized as a bi-linear curve (Fig. 8). The displacement ductility factor (Su et al. 2010) is defined as the ratio of the axial shortening at peak load (Δ_u) to the notional yield displacement (Δ_y); thus,

$$\eta = \Delta_u / \Delta_y \quad (31)$$

As shown in Table 4, the displacement ductility factors range from 1.36 (for Specimen ESC2-1) to 1.94 (for Specimen ESC3-3). For each of the groups, the displacement ductility factor of the control columns was the lowest. Compared with Specimens ESC1-4 ($\delta = 6$ mm), the displacement ductility factor of Specimens ESC1-3 ($\delta = 10$ mm) was increased by only 3.4%. Hence, the increase in the initial precamber cannot effectively enhance the displacement ductility. Compared with Specimens ESC1-2 and ESC1-3 ($e = 30$ mm), the displacement ductility factors of Specimens ESC2-2 ($e = 70$ mm) and ESC3-3 ($e = 100$ mm) were increased by only 1.4% and 5.4%, respectively. Hence, the displacement ductility is not sensitive to the eccentricity of the applied load. Using thicker plates ($t_p = 6$ mm) for Specimen ESC1-3 instead of thinner plates ($t_p = 3$ mm) for Specimens ESC1-2, the displacement ductility of ESC1-3 was increased by 30.5 %. Hence, using thicker plates can effectively improve the ductility of strengthened columns.

Moment-curvature Responses

Fig. 9(a) and Fig. 9(b) show the effects of eccentricity on the moment-curvature relationship of the columns. For the specimens strengthened by 3 mm plates, the moment-curvature relationship of Specimen ESC2-2 under 70 mm eccentricity was elastic until the moment reached 13.1 kNm, which was 19.3% larger than the moment of Specimen ESC1-2 under 30 mm eccentricity. Specimen ESC2-2 failed when the curvature was $26.2 \times 10^{-3} \text{ m}^{-1}$, which was 12.9% larger than that of Specimen ESC1-2. For the specimens strengthened by 3 mm plates, the moment-curvature relationship of Specimen ESC3-3 under 100 mm eccentricity was elastic until the moment reached 27.2 kNm, which was 97.1% larger than the moment of Specimen ESC1-3 under 30 mm eccentricity. Specimen ESC3-3 failed when the curvature was $38.3 \times 10^{-3} \text{ m}^{-1}$, which was 13.7% larger than that of Specimen ESC1-3.

Fig. 9(c) shows the effects of the plate thickness on the moment-curvature relationship of the columns. Under the condition of 30 mm eccentricity, the moment-curvature relationship of Specimen ESC1-3 strengthened by the plates that were 6 mm thick was elastic until the moment and curvature

reached 13.8 kNm and $5.8 \times 10^{-3} \text{ m}^{-1}$, respectively, which were 21.1% and 99.7% larger than the moment and curvature of Specimen ESC1-2 strengthened by the plates that were 3 mm thick. Specimen ESC1-3 failed when the ultimate curvature was $32.3 \times 10^{-3} \text{ m}^{-1}$, which was 38.1% larger than that of Specimen ESC1-2.

Fig. 9(d) shows the effects of the initial precamber on the moment-curvature relationship of the columns. The moment-curvature relationship of Specimen ESC1-3 with 10 mm initial precamber was elastic until the moment reached 13.8 kNm, which was 8.7% larger than that of Specimen ESC1-4 with 6 mm initial precamber. Both of them had the same curvature ($5.7 \times 10^{-3} \text{ m}^{-1}$) during the elastic stage. Specimen ESC1-3 failed when its curvature was $32.3 \times 10^{-3} \text{ m}^{-1}$, which was 4.5% larger than that of Specimen ESC1-4. The results demonstrated that the ductility of the column was mainly affected by the plate thickness rather than the eccentricity and the initial precamber, and a larger plate thickness can provide better ductility.

Comparison with Available Experimental Results

Montuori and Piluso (2009) tested eight RC columns strengthened with steel cages subjected to eccentric compression loads. The steel cage consisted of steel angles and battens. The strengthened columns can be divided into three different types according to the function of steel angles. The ultimate capacity of the strengthened columns was evaluated using the proposed theoretical model. The stress-strain relationship of confined concrete used by Montuori and Piluso (2009) was adopted in our theoretical calculation.

Table 5 compares the ultimate load capacity presented in Montuori and Piluso (2009) with that obtained from the theoretical model. As shown in the table, all the theoretical load capacities (P_{pred}) agree well with the experimental ultimate load capacities ($P_{Mon,exp}$). The average discrepancy of $P_{Mon,exp}/P_{pred}$ is only 2%. Meanwhile, comparing the theoretical results ($P_{Mon,pred}$) proposed by Montuori and Piluso (2009) with the theoretical results obtained from our proposed model, the average discrepancy of $P_{Mon,pred}/P_{pred}$ is also 2%. Hence, the proposed theoretical model is of a similar accuracy when compared with that from Montuori and Piluso (2009).

321 **Conclusions**

322 The paper presents a study on the strengthening of RC columns using precambered steel plates. The
323 theoretical and experimental findings are summarized as follows:

324 (1) The experimental results show that precambered plates can share the existing axial load in the
325 original column. Stress-lagging effects can be alleviated by controlling the initial precambered profile
326 of the steel plates.

327 (2) External steel plates can considerably enhance the axial strength and deformation capacity of
328 plate-strengthened columns under eccentric compression loading.

329 (3) Thicker steel plates and larger initial precamber can enhance the ultimate load capacity of
330 columns, and a larger plate thickness can also improve the axial deformation capacity and ductility of
331 columns significantly.

332 (4) The bending moment capacity of a column is significantly affected by the degree of eccentricity
333 because the larger degree of eccentricity can increase the lateral displacement at the mid-height of
334 columns and, hence, increase the secondary moment caused by the $P-\Delta$ effect.

335 (5) An original theoretical model was developed based on the principles of force equilibrium and
336 the displacement compatibility between the steel plates and the RC column. The experimental and
337 theoretical results showed a good agreement with each other. The comparison between the available
338 test results of Montuori and Piluso (2009) and the predicted theoretical results was also presented. This
339 study demonstrates that the theoretical model is able to accurately predict the axial load-carrying
340 capacity of the plate-strengthened columns under eccentric compression loading.

341

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Table 1. Summary of strengthening details

Group	Specimen	f_{cu} (MPa)	f'_c (MPa)	E_c (GPa)	L_{rc} (mm)	e (mm)	t_p (mm)	δ (mm)	P_{pl} (kN)
[A]	ESC1-1	31.3	25.6	23.8	600	30	-	-	-
	ESC1-2	31.9	25.8	23.9	600	30	3	10	101
	ESC1-3	31.6	25.9	23.9	600	30	6	10	101
	ESC1-4	32.7	26.1	24.0	600	30	6	6	101
[B]	ESC2-1	33.3	27.8	24.8	600	70	-	-	-
	ESC2-2	32.0	25.7	23.8	600	70	3	10	63
[C]	ESC3-1	29.7	24.2	23.1	600	100	-	-	-
	ESC3-3	32.6	26.5	24.2	600	100	6	10	43

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Table 2. Material properties of reinforcements and steel plates

Steel Plate		
Thickness	f_{yp} (MPa)	E_p (GPa)
3 mm	301	215
6 mm	327	219

Reinforcement bars		
Specimen	f_y (MPa)	E_s (GPa)
T10	497	198
T12	516	198
R6	464	186
R8	437	187

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Table 3. Comparison of the theoretical and experimental results

Group	Specimen	ζ_u (mm)	M_p (kN m)	M_s (kN m)	M_u (kN m)	P_{exp} (kN)	P_{pred} (kN)	P_{exp}/P_{pred}
[A]	ESC1-1	5.51	10.08	1.85	11.93	336	329	1.02
	ESC1-2	5.53	12.81	2.36	15.17	427	390	1.09
	ESC1-3	5.01	16.53	2.76	19.29	551	545	1.01
	ESC1-4	5.23	14.58	2.54	17.12	486	471	1.03
[B]	ESC2-1	9.62	14.63	2.01	16.64	209	208	1.01
	ESC2-2	10.55	16.66	2.51	19.17	238	227	1.05
[C]	ESC3-1	10.11	14.30	1.45	15.75	143	143	1.00
	ESC3-3	17.36	24.10	4.18	28.28	213	222	0.96

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Note: P_{exp} is the test result, P_{pred} is the predicted result.

Table 4. Summary of deformability and ductility factors

Group	Specimen	Δ_y (mm)	Δ_u (mm)	Δ_f (mm)	λ	η
[A]	ESC1-1	0.71	0.97	1.28	1.32	1.37
	ESC1-2	0.87	1.23	2.07	1.68	1.41
	ESC1-3	0.85	1.56	3.33	2.13	1.84
	ESC1-4	0.67	1.19	2.61	2.19	1.78
[B]	ESC2-1	0.28	0.38	0.49	1.29	1.36
	ESC2-2	0.30	0.43	0.73	1.70	1.43
[C]	ESC3-1	0.14	0.21	0.27	1.29	1.50
	ESC3-3	0.18	0.35	0.99	2.83	1.94

Table 5. Comparison of ultimate load capacities from Montuori and Piluso (2009) and the present proposed theoretical model

Specimen	$P_{Mon,exp}$ (kN)	$P_{Mon,pred}$ (kN)	P_{pred} (kN)	$P_{Mon,exp}/P_{pred}$	$P_{Mon,pred}/P_{pred}$
A-R1	513.95	527.02	507.08	1.01	1.04
B-R1a	703.23	683.62	683.38	1.03	1.03
B-R1b	662.71	649.75	656.56	1.01	0.99
C-R1	498.74	495.15	480.55	1.04	1.03
D-R1	545.19	553.24	528.23	1.03	1.05
D-R2	568.98	583.22	563.18	1.01	1.04
D-R3	483.63	453.84	462.86	1.04	0.98
E-R1	713.24	713.80	705.28	1.01	1.01
Mean	-	-	-	1.02	1.02

Note: $P_{Mon,exp}$ is the test result and $P_{Mon,pred}$ is the predicted result, both from Montuori and Piluso (2009).

467 **A List of Figure Captions**

468 Figure 1. The configuration of the proposed strengthening method

469 Figure 2. Stress-block factors

470 Figure 3. Lengths and deformations of RC column and steel plates at various loading stages

471 Figure 4. Strain distribution and calculation; (a) Concrete strain distribution; (b) Steel plate strain
472 distribution; (c) Steel plate stress distribution in the theoretical calculation

473 Figure 5. Reinforcement and precambered steel plates; (a) RC details; (b) Steel plate details

474 Figure 6. Joint at the end of precambered steel plates; (a) Anchor bolts details; (b) Steel angle details

475 Figure 7. Load-axial shortening curves; (a) Group A; (b) Group B; (c) Group C

476 Figure 8. Definition of displacement ductility factor

477 Figure 9. Moment-curvature responses of columns; (a) $t_p=3$ mm, $\delta=10$ mm; (b) $t_p=6$ mm, $\delta=10$ mm; (c)
478 $e=30$ mm, $\delta=10$ mm; (d) $t_p=6$ mm, $e=30$ mm

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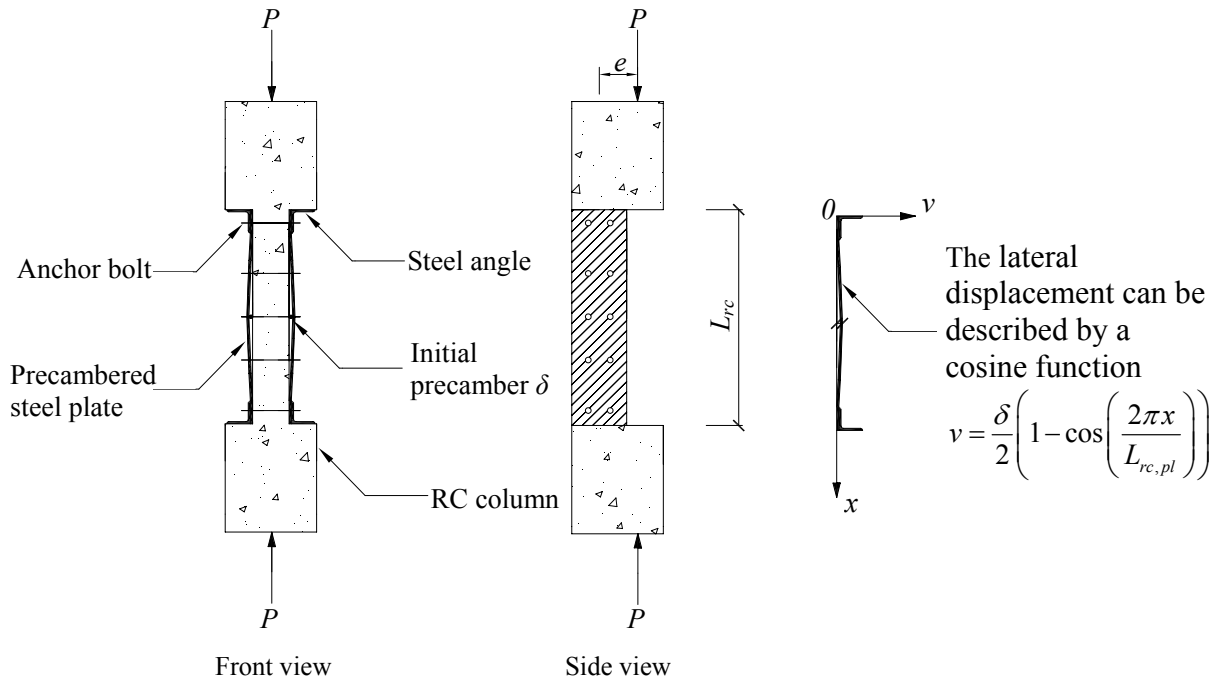


Fig. 1. The configuration of the proposed strengthening method

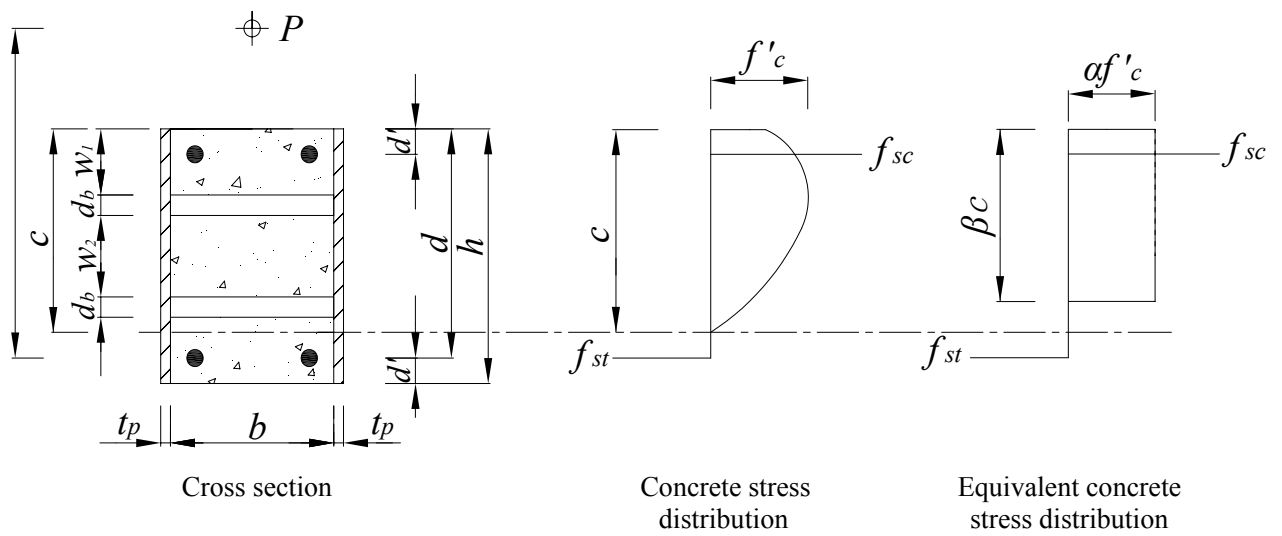


Fig. 2. Stress-block factors

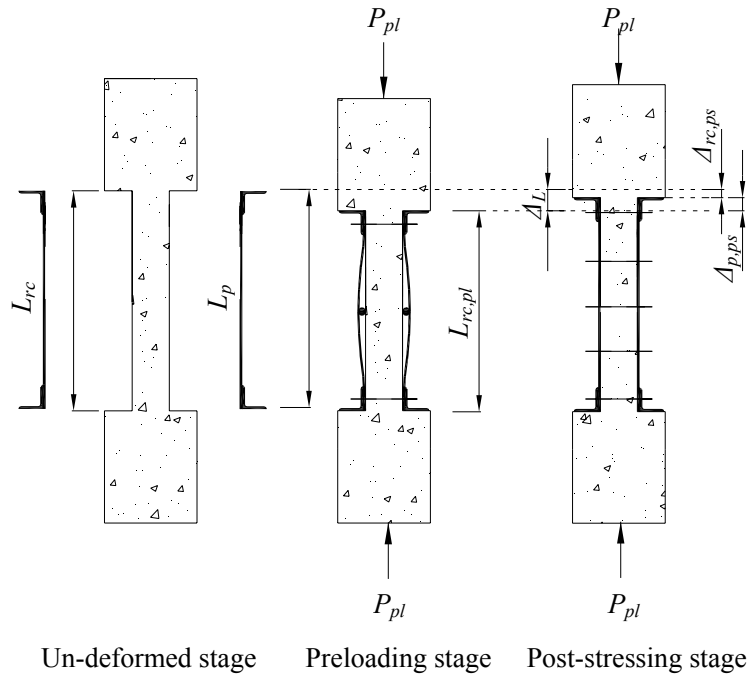


Fig. 3. Lengths and deformations of RC column and steel plates at various loading stages

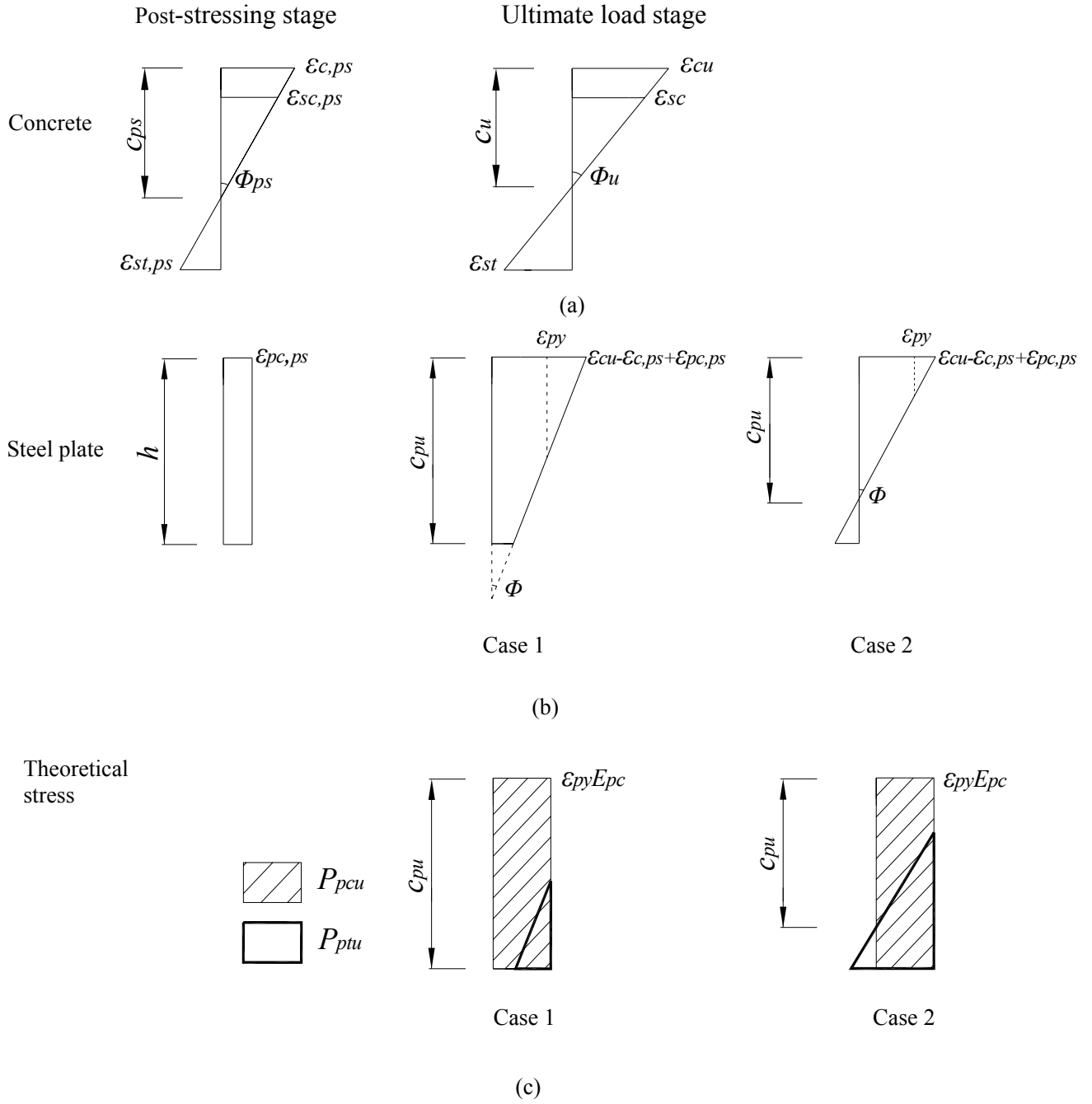


Fig. 4. Strain distribution and calculation; (a) Concrete strain distribution; (b) Steel plate strain distribution; (c) Steel plate stress distribution in the theoretical calculation

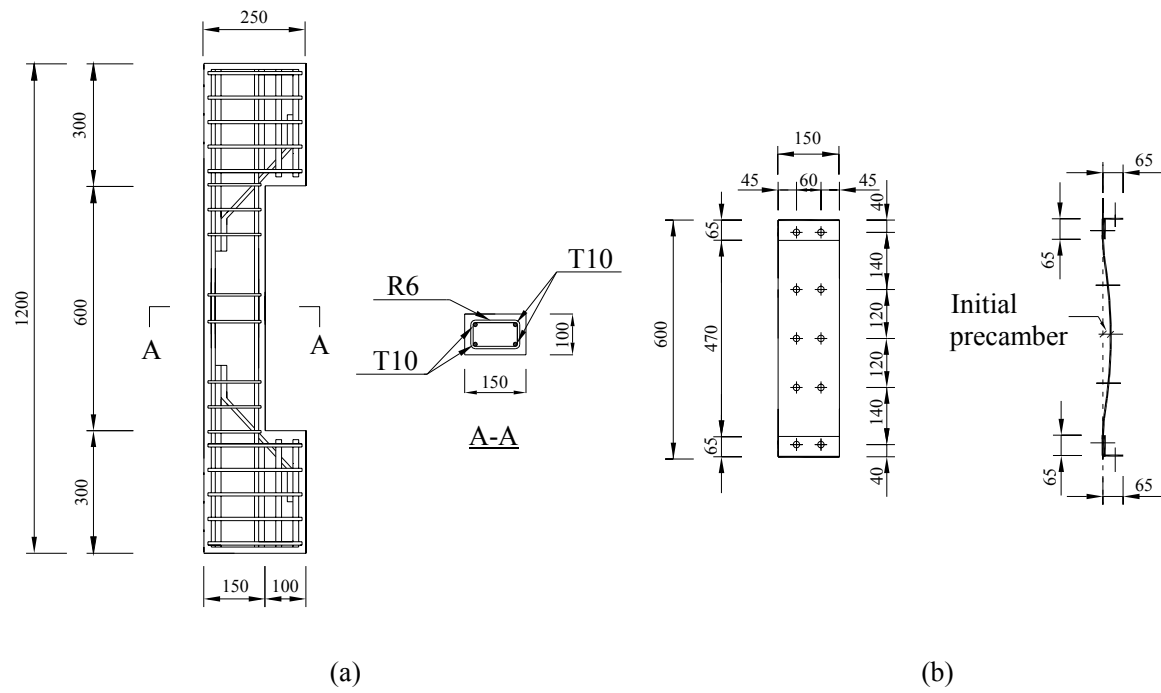


Fig. 5. Reinforcement and precambered steel plates; (a) RC details; (b) Steel plate details

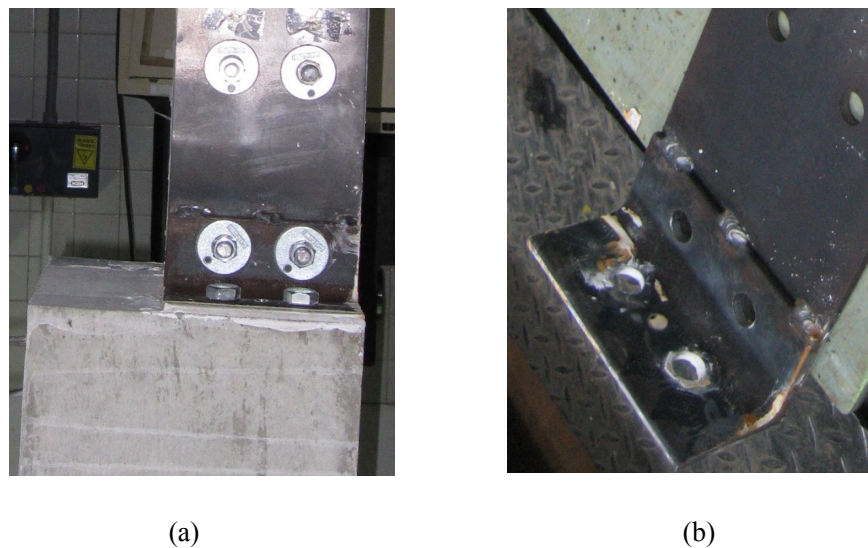
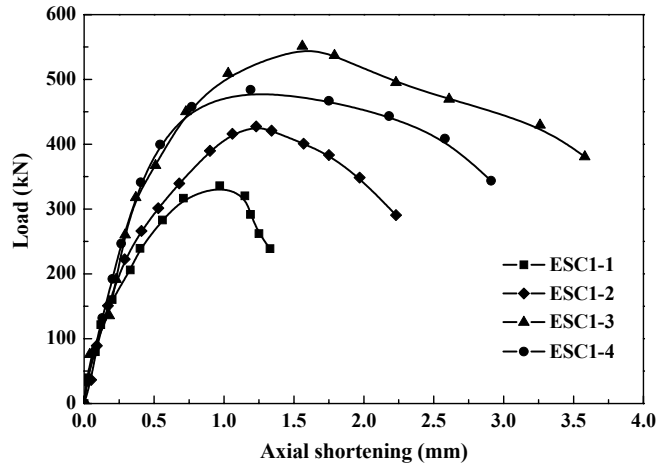
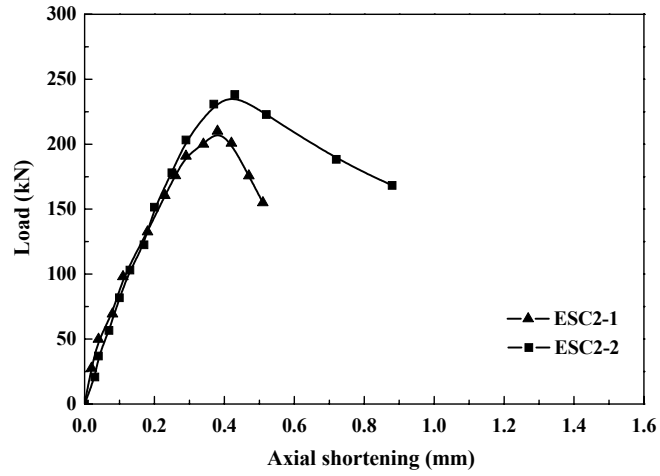


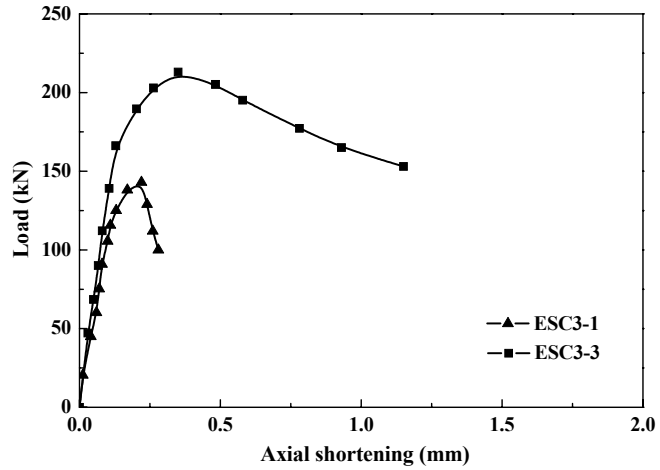
Fig. 6. Joint at the end of precambered steel plates; (a) Anchor bolts details; (b) Steel angle details



(a)



(b)



(c)

Fig. 7. Load-axial shortening curves; (a) Group A; (b) Group B; (c) Group C

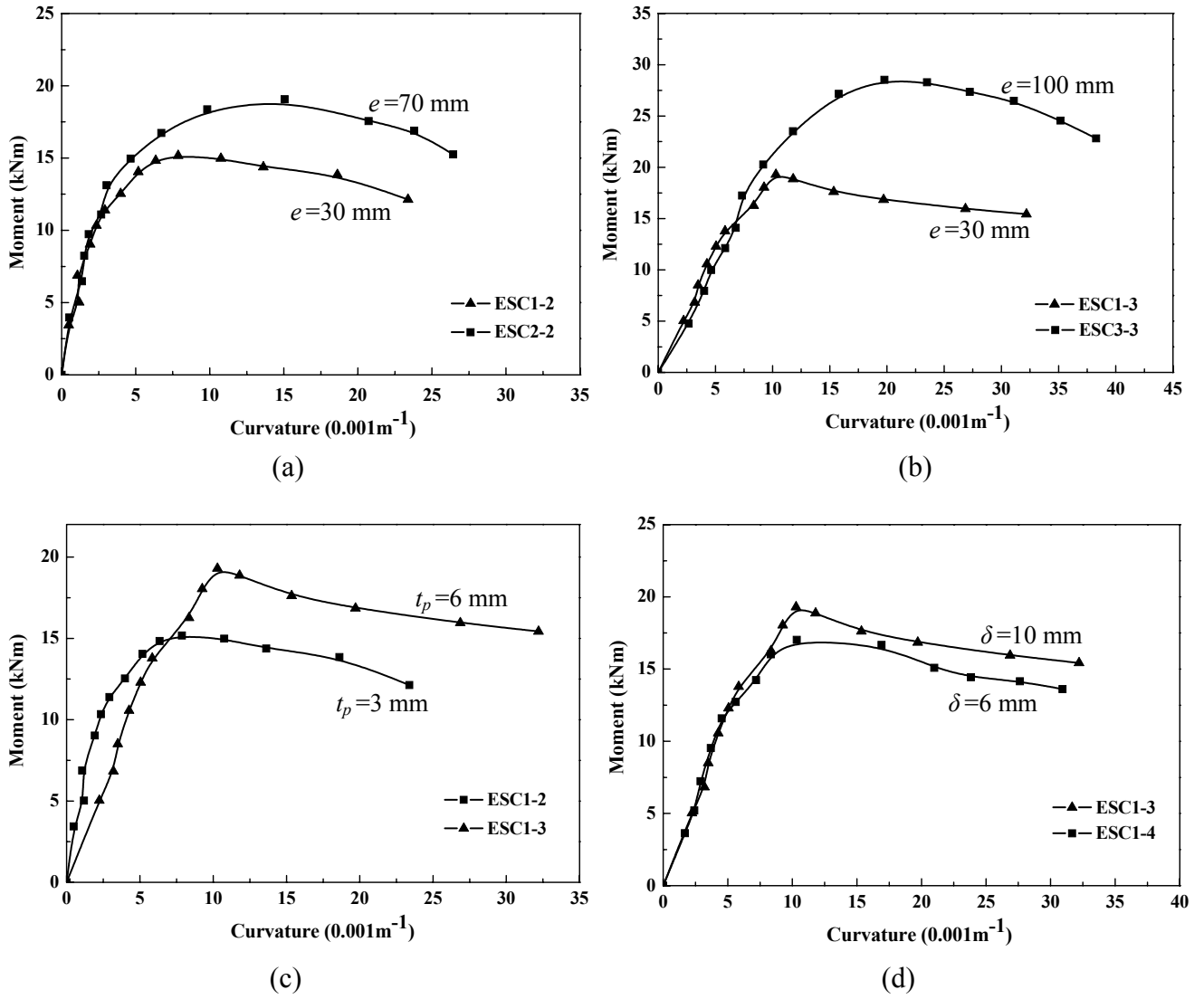


Fig. 8. Moment-curvature responses of columns; (a) $t_p=3$ mm, $\delta=10$ mm; (b) $t_p=6$ mm, $\delta=10$ mm; (c) $e=30$ mm, $\delta=10$ mm; (d) $t_p=6$ mm, $e=30$ mm