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Abstract: Most privatizations take the form of staged privatization with multistage lockups and step-by-step unlocking of shares. A lockup prevents the shares of a company from selling to the public for a specified or unspecified period of time. This paper presents a theory and provides empirical evidence for staged privatization under market forces. Our theory is based on a specification of a lockup effect on demand, where the existence of this lockup effect is shown by our empirical analysis. With such a theory, we can analyze how various factors, such as the lockup effect, demand elasticity, performance growth and business fluctuations, affect a staged privatization process, in particular, the equilibrium speed of privatization. Our paper is the first to analyze a market-oriented multistage privatization process, instead of a fully government-controlled process. Interestingly, this reform resembles initial public offerings (IPOs). Hence, our study of the reform can shed light on our understanding of IPOs from a unique angle. Our empirical analysis provides evidence in support of our theoretical findings.

Keywords: staged privatization, lockups, lockup effect, nontradable shares, market processes

JEL Classification: P31, P21
1. Introduction

Existing studies indicate that most privatization programs in the real world are carried out as a multistage process and a temporary lockup of shares works as an instrument. Some argue that lockups work as a form of commitment from the government to the market. However, some key questions are unanswered. How is a lockup determined, or equivalently, how is the speed of privatization determined? What are the crucial factors and how do these factors affect such a staged process? Most existing studies have treated lockups as exogenously determined by the government, by an initial agreement among existing shareholders, or simply by laws and regulations. Our theory, in contrast, is based on a model of staged privatization in which the program is carried out through multistage lockups and these lockups are endogenously determined by several important factors including the lockup effect, demand elasticity, performance growth and business risks. We show that the multistage lockups are a rational behavior that defines the equilibrium speed of staged privatization.

Consider a firm that has already experienced an earlier reform that divided the firm into tradable and nontradable shares. It is now time for a further reform, which intends to transform the firm into a fully market-based firm by allowing the nontradable shares to become tradable conditional on (a) an initial lockup period and (b) the tradable and nontradable shareholders can reach an agreement on a compensation package to the tradable shareholders for unlocking the nontradable shares.

In our theoretical analysis, we first show that the lockup effect has a positive effect on the length of a lockup, the share price, and the amount of unlocked nontradable shares at a lockup’s expiration (the unlock day). Second, demand elasticity is negatively related to the length of a lockup and the share price. Also, if demand elasticity is not large, it is positively related to the sale of nontradable shares at the unlock day; but if demand elasticity is large enough, the relationship is negative. Third, a firm’s growth potential has a positive effect on the length of a lockup and the share price, but a negative effect on the sale of nontradable shares at the unlock day. Finally, a positive technology shock to a firm will have a positive effect on the length of a lockup, the sale of nontradable shares at the unlock day, and also the share price.

In our empirical analysis, our first task is to identify the lockup effect. We show empirically the existence of a lockup effect and identify the underlying factors that determine it. This empirical result supports the specification of the lockup effect in our theoretical model. Second, we show that the behavior of state-owned enterprises’ (SOEs) nontradable shareholders are determined by market principles. In particular, nontradable shareholders in SOEs and nontradable shareholders in non-SOEs are shown to choose statistically the same lockup lengths. Third, our regression results provide empirical evidence in support of predictions of
the theoretical analysis. In particular, using an ordinal logit model, we find that firms with
great growth potential (measured as high ROAs or high annual sales growth) favor longer
lockups. We also find that lockups and other commitments are complements, while lockups
and financial compensation are substitutes.

Our paper contributes to the literature in two aspects. First, we study economic reforms
by focusing on staged privatization and identify the speed of staged privatization by multistage
lockups. We show that lockups are a rational behavior by identifying the lockup effect in de-
mand for shares. We also identify those key factors that affect the lockup effect. Second, our
results can shed light on the literature of lockups in IPOs and SEOs (secondary market offer-
ings). For example, Brav and Gompers (2003) find that, for IPOs, 26% of the firms will keep
their shares longer than the 180-day lockup period and they contribute this totally to moral
hazard. For the Chinese reform, nontradable shareholders in 60% of the firms (about 700 out
of 1176 firms) commit to lock their shares longer than the required minimum of 3 years. Hence,
in both types of lockups, there is a voluntary component.

This paper is organized as follows. In Section 2, we review the related literatures and de-
scribes the Chinese reform as an example of staged privatization. In Section 3, we setup the
model. In Section 4, we present the equilibrium solution. In Section 5, we provide theoretical
analysis on our solution. In Section 6, we present empirical analysis using data on listed firms
in the Chinese stock market. Finally, we conclude the paper in Section 7. The derivation for
our solution is in the Appendix.

2. Literature Review and the Chinese Reform

Studies of staged privatization are very rare, except Li and Wang (2007) who focus on the
compensation package of the Chinese reforms. Hence, we do not have directly related studies
to refer to. In this section, we review some indirectly related literatures. Specifically, we review
two literatures in relation to our contributions: privatization methods and IPO lockups.

2.1. How to Privatize?

Privatization has become a recent hot topic. Researchers have looked at various aspects of
privatization. However, the majority of the existing studies focuses on the inefficiency of SOEs.
See the surveys by Megginson and Netter (2001), Bonin and Wachtel (2003) and Turhan
(2005). In contrast, we focus on the process that SOEs are privatized. This is an important
issue. For example, Russia’s big-bang privatization has been widely viewed as the cause of a
huge drop in Russia’s GNP following its privatization, while China has experienced fast and
continuous growth following its gradual reform in 1978. We review a few existing studies on
this issue below. Readers may see a comprehensive survey by Megginson and Netter (2001).
How should privatization be carried out? This same question can be asked of a developed economy with a mature capital market or a developing economy with an underdeveloped capital market and an unreliable legal system. Megginson-Nash-Netter-Poulsen (2004) indicate that privatizations in the real world are typically carried out through three ways: asset sales, voucher privatizations, and share issue privatizations. In an asset sales, the government sells ownership of a SOE directly to an existing private firm, an institution, or a small group of individuals. Such deals are typically made through direct face-to-face negotiations. In a voucher privatization, the government distributes vouchers (paper claims for ownership) to citizens. These vouchers are usually free or very lowly priced and are available to most citizens. In a share issue privatization (SIP), the government sells equity shares to the public. The government may sell a fraction or all of a SOE in any of these methods. Megginson-Nash-Netter-Poulsen (2004) present various arguments for a government’s choice of a privatization method, including market conditions, the political and legal environment, and firm-specific characteristics. Among these three methods, SIP is the dominant form of privatization in terms of asset value, and it is the only form that can be modelled mainly on the basis of economic rationality. In a cross-country study involving 33 countries, Megginson-Nash-Netter-Schwartz (2000) find large, positive, abnormal, long-term returns from firms privatized through SIP. Another reason that we focus on SIP is because the recent Chinese privatization since 2005 is in this form.

Should SOEs be sold all at once or in stages? In an early study on the subject, Gupta (2005) pointed out that most privatization programs begin with a period of partial privatization in which only non-controlling portions of firms are sold on the stock market. Megginson and Netter (2001) have also observed that reform programs typically consist of many small privatizations and there are few outright sales of SOEs. Jones-Megginson-Nash-Netter (1999) further observe that, among SIPs in 59 countries, only 11.5% of the firms sold all of their capital at once and less than 30% sold more than half of their capital in the initial public offering. There have been a few theoretical analyses on this issue. Bonin (1976) is a pioneer who provides a theory on step-by-step reforms in the Soviet Union in 1966 and 1973. He analyzes the effect of various combinations of incentive schemes and quantity targets on production decisions of the socialist manager. Bonin’s theory implies an interaction of constraints and rewards that provides a reasonable explanation for the reforms.

Another influential study is by Perotti (1995), who also observes that SOEs in both developed and developing countries are mostly privatized through a sequence of partial and staggered sales. In addition, Perotti finds that governments often temporarily take a risk-bearing role even well after the transfer of control to the private sector. Perotti proposes two explanations for these behaviors. One is the existence of temporary market capacity constraints (downward sloping demand). The other is based on a confidence-building strategy on the part
of the government in its willingness to retain a stake in a firm, which signals more reliable future policy. The latter may be a crucial element for the success of a privatization program.

Perotti’s model is different from ours. Perotti (1995) is based on the argument of government creditability in reform. By discussing and comparing with Perotti (1995), we highlight the key features in our model. First, he treated the length of the lockup as exogenous. Specifically, he assumed that a portion of the shares is sold at $t = 1$ and the rest are sold at $t = 2$. In our model, in contrast, we have multi-stage lockups (just as in reality) and the length of each lockup is endogenous (an optimal choice). At the unlock day, a nontradable shareholder optimally unlocks a portion of her shares and locks up the rest for a further optimal length of time. This setup is more consistent with reality. Such endogenously determined lockups imply an equilibrium speed of staged privatization, which is a key feature of our model. For example, if the economic environment experiences a downturn, the speed of privatization will automatically slow down, as each nontradable shareholder chooses to lock up their remaining shares for a longer period of time.

Second, in Perotti (1995), the government is the single owner and the selling shares are all primary shares. For the Chinese reform, the central government typically held half of each firm’s nontradable shares when the 2005 reform began, with the other half held by so-called legal persons—-institutions and real persons, including foreigners. Also, the shares sold were all secondary shares. Hence, a nontradable shareholder can be treated as a typical economic agent who maximizes the payoff from her shares. This behavior is confirmed in our empirical analysis. This means that privatization in our model is a market process, while Perotti’s privatization is a centrally planned process.

Third, in Perotti (1995), the government may or may not tax earnings from private shares (those shares of a SOE sold to private individuals). If the government does not sell the firm at once, its tax revenue from a firm will be lower, which may be a signal to indicate that this government has no intention of taxing earnings from the shares. Hence, partial privatization can serve as a signal that the government is a no-tax government. In a separating equilibrium, a no-tax government uses staged privatization, while a taxing government uses one-time privatization. A tax reduces the firm’s incentive to invest. Hence, this equilibrium may explain why in reality many governments privatize SOEs in stages.

In contrast, we focus on the lockup effect. With downward-sloping demand for shares (Perotti also has such demand), a lockup may increase shareholder value by pushing up demand. Hence, with a choice in deciding a lockup’s length, a shareholder strategically decides to sell a portion of her shares and lock up the rest for an optimal length of time. In reality, for the Chinese privatization, these decisions are made by each individual shareholder and different firms’ shareholders typically make different decisions. Some choose not to lock up at all after the initial lockup expires, while others choose to further lock up all their shares for many years (as long as 15 years).
There are some other theoretical models of the sequencing and staging of SOE sales, including models in Katz and Owen (1995), Boycko, Shleifer and Vishny (1996), Cornelli and Li (1997), and Biais and Perotti (2002). Katz and Owen (1995) treat a SOE as an asset for sale which the government needs to package before selling, including providing sufficient ownership for the buyer and enough subsidy for the firm. Boycko, Shleifer and Vishny (1996) study privatization by a divided government. Cornelli and Li (1997) present an auction model, in which the optimal privatization scheme uses the number of shares sold as an instrument to attract the most efficient investors. Finally, Biais and Perotti (2002) analyze a political process of privatization in a democracy. There are also some empirical studies that test these theories, including studies by Perotti and Guney (1993), Dewenter and Malatesta (1997), Jones-Megglinson-Nash-Netter (1999) and Megginson-Nash-Netter-Poulsen (2004). But none of their models resembles the most recent Chinese privatization program that we study. Also, one key difference is that they study a fully government-controlled privatization process, while we study a market-oriented privatization process.

There have also been some studies of the optimal speed of privatization. For example, Katz and Owen (1993) focus on labor issues in privatization, including temporary unemployment during privatization, improvement in labor productivity and reemployment of the redundant labor at the end of the privatization process. These concerns imply an optimal speed of privatization. In the recent Chinese reform, however, apart from the minimum lockup of three years for large shareholders, the speed of privatization is decided largely by the market. Except that one third of shares is at the hands of the central government, other shareholders are expected to behave according to market principles. Since all these shares are secondary shares, the decision on selling or locking them up no longer has political and social implications, so we expect the central government to follow market principles as well in its handling of its shares. As shown in Table 1, our empirical analysis indeed confirms that SOEs’ nontradable shareholders and non-SOEs’ nontradable shareholders choose statistically the same lockup lengths. This finding is also consistent with Boubakri et al.’s (2005) finding from their 39-country study that “...the government relinquishes control over time to the benefit of local institutions, individuals, and foreign investors.”

To our knowledge, our paper is the first to analyze a market-determined privatization process. As a market-oriented process, we deal with an equilibrium speed of privatization rather than an optimal speed of privatization. We do not attempt to propose an optimal way of privatization, nor justify which observed approach is the best. Instead, we take the actual steps in the recent Chinese privatization as given and analyze shareholder behavior and market reactions during the reform process. We provide both theoretical predictions and empirical analysis for this reform process.
2.2. The Chinese Reform

Take Chinese economic reform as an example of staged privatization. In the earlier reform starting in 1991, every listed firm (including non-SOEs), except four, is divided into two types of shares: tradable shares (T-shares) and nontradable shares (N-shares), where the T-shares are tradable on the stock market but the N-shares are not. We call this a share-issuing privatization (SIP). With such divided shares, the Chinese capital market in 2005 is defined by a split-share structure, with about one third of domestically listed shares being T-shares and the rest being N-shares. This means that two thirds of the Chinese market capitalization is in nontradable shares, most of which are held by the central government, local governments and state-owned institutions.

In the recent reform, the government in April 2005 announced that it would allow all N-shares to become T-shares after an initial lockup, conditional on the holders of the T-shares (T-holders) and the holders of the N-shares (N-holders) in each firm reaching an agreement on a compensation package by which the N-holders compensate the T-holders for the dilution. This is called the split-share reform (SS reform). When the reform program was launched in 2005, there was no additional issuance of primary shares and the SOEs were already public companies with one third of equity in T-shares. After an initial lockup period, all the N-shares for the other two thirds of equity become tradable. This reform process is still ongoing and it may take many years to complete. It is the most thorough reform and perhaps the final phase of China’s three-decade economic reform endeavor.

Although the government requires a minimum initial lockup, most firms voluntarily set their initial lockups (the first lockup) longer than the requirement. Furthermore, after the first lockup expires, many firms choose to lock most of their shares for a further period (the second lockup), even though they are free to trade all their shares at that time. Some firms even lock all their shares without selling any when the first lockup expires. Lockups can thus be treated as voluntary market-oriented behavior and as optimal choices made by rational profit-maximizing shareholders. In fact, our empirical analysis in this study confirms that lockups for all the listed firms have indeed been market-based behavior.

\footnote{See Sun and Tong (2003) for a detailed description of China’s reform programs since 1978. In particular, between 1991 and 2000, “…many weak enterprises were sold off through auctions and corporate transformation while some large- and medium-sized SOEs were transformed into publicly listed firms on the stock market. Only relatively strong SOEs were eligible to go public.” This reform led to the establishment of China’s two stock exchanges around 1991.}
2.3. IPO Lockups

On lockups, an initial lockup of 180 days is standard in IPOs. The literature on the IPO lockup is related to our study on lockups in privatization. We mention a few such studies to highlight some features in our model.

From a large sample of close to 2,000 IPOs from 1988 to 1997, Field and Hanka (2001) identify an unlock effect from a negative return of -1.5% at the unlock day. This is an abnormal return, which challenges the most extreme version of the efficient market hypothesis. This abnormal return implies a downward sloping demand curve for shares, since the abnormal return is associated with an abnormally high trading volume. Their unlock effect is the same as our lockup effect in essence, since both implies a change in demand resulting from a change in the supply of shares.

Brav and Gompers (2003) propose three possible motivations for IPO lockups: a signal for firm quality, a commitment device to alleviate moral hazard problems, and a mechanism for underwriters to extract additional compensation from the issuing firm. Arguments for these three potential effects are as follows. First, with a cost of lockup on insiders, a lockup can truthfully signal firm quality in equilibrium. Second, by committing to hold a large portion of the firm for a period, the insiders can convince shareholders that they will act in the best interests of shareholders. Third, a lockup agreement does not prevent an insider to sell shares within the lockup period if the lead underwriter consents. In this case, the insider can do a block trade through the lead underwriter or perform a SEO. In either case, the underwriter makes additional fees by either making a market in the firm’s shares for the block transaction or else underwriting a SEO. Hence, the IPO lockup can allow underwriters to earn additional fees.

Brav and Gompers (2003) find (1) empirical support for the commitment hypothesis, (2) no support for insiders signalling their quality by locking themselves in for a longer period of time, and (3) little evidence in support of the view that investment banks impose a longer lockup as a way to extract additional compensation from the firm’s insiders. Interestingly, like Field and Hanka (2001), they identify an unlock effect from a negative return of -2% at the unlock day, which again implies a downward sloping demand curve for shares.

In contrast to Brav and Gompers (2003), Brau et al. (2005) find empirical support for a signal effect. They find that an IPO lockup will be longer when the degree of asymmetric information is higher and the level of idiosyncratic risk is lower. Also, Leland and Pyle (1977) develop a model in which the fraction of equity retained by insiders at IPO serves as a signal of quality. In a separating equilibrium, owners of firms with different firm values choose different percentages of shares to hold. Further, in Hughes’ (1986) signalling model, the entrepreneur uses both share retention and direct disclosure of private information as signals. Finally, Courteau (1995) has a voluntary lockup model, in which insiders use the IPO lockup as a
signaling device. Due to the cost of lockup on the insiders, a separating equilibrium can be found at which the length of the IPO lockup reveals firm quality, with high-quality firms signalling their quality by a longer lockup.

Instead of an unlock effect, our empirical identification of the lockup effect in a special situation adds to the literature. We show the existence of the lockup effect, which boosts demand when it is announced by a shareholder. We empirically identify its dependence on various factors. Such a lockup effect is also found by empirical studies in the literature. For example, Ang and Brau (2003) find that insiders are more likely to commit to a longer IPO lockup as a positive signal to compensate for a negative signal of their selling shares.

In the most recent Chinese privatization program, lockups for the listed firms resemble lockups in IPOs. However, there are several crucial differences. First, while shares on sale at an IPO are mainly primary shares, the locked shares (called N-shares) in the Chinese reform are all secondary shares. A nontradable shareholder's motive for her shares is much clearer than a firm's motive for an IPO. Hence, we can treat an N-holder as a share value maximizer, while a firm's motive for its IPO is questionable. Second, in an IPO lockup, the lead underwriter can and often release locked shares early. In the case of the Chinese reform, a lockup is strictly enforced by the law and an early unlock of a lockup is completely impossible. Also, an extension of a lockup is completely voluntary and, once it is announced by an N-holder, it is legally binding. Third, many N-holders offer multi-stage lockups in the case of the Chinese reform, while the IPO lockup is a one-time lockup. The multi-stage lockups allow us to identify the lockup effect empirically, while identifying the lockup effect from an initial IPO lockup is difficult due to its dependence on many factors, especially on the fact that the firm has never been public before. Fourth, the minimum lockup of three years for a large N-holder (with 10% shares or more) in the Chinese reform is much longer than an IPO lockup of 180 days. This means that an N-holder faces much bigger risk but, at the same time, she has more incentive to improve the company. In fact, N-holders in a Chinese firm are always among the biggest five shareholders and data shows that the percentage of N-shares held by the biggest five N-holders is statistically indifferent from the percentage of shares held by the biggest five shareholders. These shareholders control most of the important management positions. Their commitments can assure the market that they will continue to contribute to the firm rather than expropriate minority shareholders by cashing out their investment soon after the reform. Hence, lockups in the Chinese reform may be an effective way in controlling moral hazards. Finally, in an IPO, the private firm becomes a public firm; however, for the Chinese reform, when the reform policy is announced, the firm has already been a public firm for many years. This has implications on the purpose of a lockup, especially on the role of lockup as a signal.
3. The Model

The Reform Program

Consider a firm that has already experienced an earlier reform. The reform divided the firm into tradable and nontradable equity shares, by which a portion $\theta_0$ of the shares has been sold to the public and these shares are tradable (T-shares) and the rest are nontradable (N-shares). For the case of China, the average value of $\theta_0$ is about one third.

It is now time $(t = 0)$ for a further reform. The time period is $[0, \infty)$. There are two groups of shareholders: one group consists of T-share holders (T-holders) who hold the T-shares and the other group consists of N-share holders (N-holders) who hold the N-shares of the firm. The reform intends to transform the firm into a fully market-based firm, by which the N-shares are allowed to become tradable conditional on (a) an initial lockup period and (b) that the T-holders and the N-holders can reach an agreement on a compensation package for the N-holders to compensate the T-holders for unlocking the N-shares.

Shareholder Strategy

In our model, each firm has one representative T-holder (he) and one representative N-holder (she). After the reform policy is announced at $t = 0$, the two holders negotiate a compensation package by Nash bargaining. A compensation agreement comes with two components: financial compensation and an initial lockup. The financial compensation is a lump-sum transfer $m_0$ of money from the N-holder to the T-holder. In reality, due to cash constraints, financial compensation comes with various forms, such as a payment in shares. An initial lockup of length $l_0$ means that the N-shares are not tradable in period $[0, l_0)$; they are allowed to become tradable at and after $t = l_0$. We call this the initial stage or first-stage lockup.

After $t = l_0$, the N-holder is completely free to make decisions on her shares without a need for an agreement from the T-holder. In our model, the N-holder will choose to let a portion $\theta_1$ of her N-shares to become T-shares and lock up the rest for a further length $l_1 \geq 0$, which is called the second-stage lockup. We assume no more lockups after $t = l_0 + l_1$.

\[ 0 \quad \begin{array}{c} \theta_0 \text{ amount in T-shares} \\ \text{First-Stage Lockup Period} \end{array} \quad \begin{array}{c} \theta_0 + \theta_1 \text{ amount in T-shares} \\ \text{Second-Stage Lockup Period} \end{array} \quad l_0 + l_1 \]

Figure 1. The Stages of Privatization
Demand for Shares

The total supply of shares is 1 and \( \theta_0 \) is the amount of T-shares at \( t = 0 \), with \( \theta_0 \in [0, 1] \). Demand for the firm’s equity is \( x_t(p_t, q_t, l_t) \), where \( x_t \) is the demand for shares, \( p_t \) is the stock price, \( q_t \) is a performance or quality index, and \( l_t \) is the remaining lockup length from \( t \) onwards. To focus on certain key characteristics of the demand, let

\[ x_t(p_t, q_t, l_t) = \tilde{A}_t z(p_t) q_t, \]

where \( \{\tilde{A}_t\} \) is a Markov random process representing demand fluctuations and \( z(p_t) \) is a conventional downward-sloping demand function. At time \( t \), \( \tilde{A}_t \) is known; but, before \( t \), \( \tilde{A}_t \) is random.

We need to add a lockup effect into demand. We assume that demand will increase proportionally by amount \( \phi(\theta_N) \), depending on the amount \( \theta_N \) of shares being locked up. Specifically, let \([1 + \phi(\theta_N)] \tilde{A}_t z(p_t) q_t \) be the demand at \( t \) if an amount \( \theta_N \) of the company’s shares is locked up forever. Further, we will use a simple functional form: \( \phi(\theta_N) = \beta \theta_N \), where \( \beta \geq 0 \) is a constant. Then, if all the shares \( (\theta_N = 1) \) is locked up, \( \beta \tilde{A}_t z(p_t) q_t \) is the extra amount of demand. Hence, we call \( \beta \) the lockup effect. More generally, consider a finite lockup length \( l \).

Let \( \rho \) be the rate of time preferences. Since \( \rho \int_0^{\infty} e^{-\rho t} dt = 1 \), the instant discount factor is \( \rho e^{-\rho l} \). Then, if an amount \( \theta_N \) of shares is locked up for a length of \( l \), the extra amount of demand at \( t \) should be

\[ \beta \theta_N \tilde{A}_t z(p_t) q_t \int_0^l e^{-\rho \tau} d\tau = \tilde{A}_t z(p_t) q_t \beta \theta_N (1 - e^{-\rho l}). \]

Hence, if an amount \( \theta_N \) of shares is locked up for a length \( l \), the demand at \( t \) is

\[ x_t = \tilde{A}_t z(p_t) q_t \left[ 1 + \beta \theta_N (1 - e^{-\rho l}) \right]. \]  

This demand for shares is downward sloping in price and it includes demand shocks and the lockup effect.

Figure 2. Our Demand Curve
Many existing studies support downward sloping demand for shares, such as Perotti (1995), Field and Hanka (2001) and Brav and Gompers (2003). In our case, if the demand curve is horizontal, the N-holder will not lock up her shares in stages. That is, the existence of lockups is a proof of a downward sloping demand curve. Besides, we also include the lockup effect as a component in the demand slope. This lockup effect will be identified in our empirical study in Section 6.

A Compensation Agreement

The reform policy includes a provision that the T-holders and N-holders must come to an agreement on compensation, otherwise the N-shares can never become tradable. We assume that the two parties will negotiate and settle with an agreement by Nash bargaining. That is, the Nash bargaining solution defines the compensation package.

Parametric Functions

For convenience of discussion, we choose some popular functions. Specifically, let

\[ z(p) = p^{-\alpha}, \quad (2) \]

where \( \alpha \) is demand elasticity. Assume also that quality \( q_t \) grows at a constant rate \( \gamma \), i.e.,

\[ q_t = e^{\gamma t}. \quad (3) \]

Assume further:

\[ \{A_t\} \text{ is an i.i.d. process with mean } E(A_t) = A_0, \quad (4) \]

where \( A_0 > 0 \) is a given constant. Finally, assume that the firms do not pay any dividends. That is, dividends are included in firm value.

4. The Solution

In this section, we derive the solution. We solve the model backward.

At \( t = l_0 + l_1 \)

Assume that the N-holder will unlock all her shares at \( t = l_0 + l_1 \). Then, the total supply of shares is 1 and the price is determined by

\[ \bar{A}_{l_0 + l_1} z(p_{l_0 + l_1}) q_{l_0 + l_1} = 1. \]

Then, with the demand function in (2), the equilibrium share price is

\[ p_{l_0 + l_1} = \left( \bar{A}_{l_0 + l_1} q_{l_0 + l_1} \right)^{1/\alpha}. \]
At \( t = l_0 \)

The initial lockup length at \( t = 0 \) is \( l_0 \). At \( t = l_0 \) or just before \( t = l_0 \), the N-holder needs to decide what to do with her shares. Given the market demand in (1), the N-holder has two choices \((\theta, l_1)\) to make: decide to unlock an amount \( \theta \) of her shares and decide to lock up the rest for a further length \( l_1 \). At \( t = l_0 \), with the total supply of shares being \( \theta_0 + \theta_1 \), the share price \( p_0 \) is determined by

\[
\tilde{A}_0 z(p_0)q_0 \left[ 1 + \beta(1 - \theta_0 - \theta) \left( 1 - e^{-\rho_0} \right) \right] = \theta_1 + \theta_0.
\]

With the demand function in (2), the equilibrium share price is

\[
p_0 = \left( \frac{\tilde{A}_0 q_0}{\theta_0 + \theta_1} \left[ 1 + \beta(1 - \theta_0 - \theta) \left( 1 - e^{-\rho_0} \right) \right] \right)^{1/\alpha}.
\]

Then, the N-holder’s problem at \( t = l_0 \) is

\[
\pi_N(l_0) \equiv \max_{\theta, l_1} p_0 \theta_1 e^{-\rho_{l_0}} + E_{t_0} (\tilde{p}_{l_0 + l_1} (1 - \theta_0 - \theta) e^{-\rho(l_0 + l_1)}),
\]

where \( \tilde{p}_{l_0 + l_1} \) is random at \( t = l_0 \), \( e^{-\rho_{l_0}} \) is the discount factor of future income, and \( E_{t_0} \) is the expectation operator conditional on available information at \( t = l_0 \). That is,

\[
\pi_N(l_0) \equiv \max_{\theta, l_1} \theta_1 \left( \frac{\tilde{A}_0 q_0}{\theta_0 + \theta_1} \left[ 1 + \beta(1 - \theta_0 - \theta) \left( 1 - e^{-\rho_0} \right) \right] \right)^{1/\alpha} e^{-\rho_{l_0}}
\]

\[
+ (1 - \theta_0 - \theta) E_{t_0} \left( \tilde{A}_0 q_0 \left[ 1 + \beta(1 - \theta_0 - \theta) \left( 1 - e^{-\rho_0} \right) \right]^{1/\alpha} \right) e^{-\rho(l_0 + l_1)}.
\]

At \( t = 0 \)

At \( t = 0 \), the total supply of shares is \( \theta_0 \) and the total value of the T-shares is \( \pi_T = p_0 \theta_0 \). With the N-shares being locked up for a length \( l_0 \), the equilibrium condition is

\[
A_0 z(p_0)q_0 \left[ 1 + \beta(1 - \theta_0) \left( 1 - e^{-\rho_0} \right) \right] = \theta_0.
\]

With the demand function in (2), the equilibrium share price is

\[
p_0 = \left( \frac{A_0 q_0}{\theta_0} \left[ 1 + \beta(1 - \theta_0) \left( 1 - e^{-\rho_0} \right) \right] \right)^{1/\alpha}.
\]

Hence, the wealth of the T-holder is

\[
\pi_T(l_0) = \left( \frac{A_0 q_0}{\theta_0} \left[ 1 + \beta(1 - \theta_0) \left( 1 - e^{-\rho_0} \right) \right] \right)^{1/\alpha} \theta_0.
\]

Before the reform, the N-shares were supposed to be locked up forever. Hence, if there is no agreement, the N-shares can never be unlocked. In this case, \( l_0 = \infty \). Let the price of the T-shares just before the reform be \( p_T \). Then, the alternative equilibrium condition to (6) is
\[ A_0 z(p_T) q_0 [1 + \beta (1 - \theta_0)] = \theta_0, \]

which implies

\[ p_T = \left( \frac{A_0 q_0 [1 + \beta (1 - \theta_0)]}{\theta_0} \right)^{1/\alpha}. \]

Hence, the wealth of the T-holder before the reform is

\[ \pi_T = \left( \frac{A_0 q_0 [1 + \beta (1 - \theta_0)]}{\theta_0} \right)^{1/\alpha} \theta_0. \]

We obviously have \( \pi_T(l_0) < \pi_T \) for any \( l_0 < \infty \).

During a lockup, the N-shares can be traded in the internal market, where the N-holders can trade among themselves. Suppose that the price of the N-shares in the internal market just before the reform is \( p_N \). Then, before the reform, the N-holder’s wealth is

\[ \pi_N = p_N (1 - \theta_0). \]

In practice, price \( p_N \) is generally the equity book value, which is typically only a small fraction of \( p_T \). We assume that, after the reform, the N-holders will no longer use the internal market to trade their N-shares.

That is, at \( t = 0 \), before the reform, the T-holder and N-holder’s incomes are respectively \( \pi_T \) and \( \pi_N \); after the reform, the T-holder and N-holder’s incomes are respectively \( \pi_T(l_0) \) and \( \pi_N(l_0) \), which are defined respectively in (7) and (5). We assume that \( l_0 \) is contractable.\(^3\)

Then, the Nash bargaining solution implies the following two steps. First, the two parties agree on the optimal \( l_0 \), which is determined by

\[ W^* = \max_{l_0 \geq 0} E_0 [\pi_T(l_0) + \pi_N(l_0)]. \]

Here, we have used equal weights in social welfare for the two groups. An alternative is to use population shares as weights; but, the population shares may not be appropriate weights in this case since the influence of each group in a bargaining solution is not proportional to its population share. However, such weights are not crucial to our analysis. Second, the two parties agree to a sharing rule as defined by the Nash bargaining solution:

\[ \Pi_T = \pi_T + \frac{1}{2} (W^* - \pi_T - \pi_N), \quad \Pi_N = \pi_N + \frac{1}{2} (W^* - \pi_T - \pi_N), \]

where \( \Pi_T \) and \( \Pi_N \) are the final incomes of the two groups after they reach a settlement. This sharing rule defines financial compensation. Before this compensation, the N-holder has

\(^3\) In the case of the Chinese reform, this lockup length \( l_0 \) is indeed guaranteed by the law. The government actually enforces any announced lockup strictly. Also, renegotiation on \( l_0 \) is not allowed.
income \( \pi_N(l_0^*) \). This means that the N-holder has to pay the following monetary amount to the T-holder:

\[
\pi_N(l_0^*) - \Pi_N = \pi_N(l_0^*) - \pi_N - \frac{1}{2} \left[ \pi_T(l_0^*) + \pi_N(l_0^*) - \pi_T - \pi_N \right] = \frac{\pi_N(l_0^*) - \pi_N + \pi_T - \pi_T(l_0^*)}{2}.
\]

After the reform, the N-holder gains \( \pi_N(l_0^*) - \pi_N \geq 0 \) and the T-holder loses \( \pi_T - \pi_T(l_0^*) \geq 0 \). Hence, the compensation is positive and it consists of half of the N-holder’s gain plus half of the T-holder’s loss. Finally, due to cash constraints, this compensation is often paid in terms of N-shares in practice. At \( t = 0 \), just after the reform, each N-share is worth \( p_0^* \). Hence, the number of shares paid by the N-holder to the T-holder should be:

\[
\text{compensation in N-shares} = \frac{\pi_N(l_0^*) - \pi_N + \pi_T - \pi_T(l_0^*)}{2p_0^*}.
\]

**A Parametric Solution**

As shown in the Appendix, given the parametric functions in (2)–(4), under condition \( \alpha \rho \geq \gamma \), the solution \( (l_0^*, i_1^*, \theta_1^*) \) from (5) and (8) is determined by the following three equations jointly:

\[
E_0 \left[ \frac{\tilde{A}_{n+1}}{A_0} \right]^{\frac{1}{\alpha}} \left[ \frac{\theta_1 \rho [1 + \beta (1 - \theta_0 - \theta_1)]}{\alpha \rho - \gamma} - \frac{\theta_1}{\alpha (\theta_0 + \theta_1)} + 1 - \theta_0 - \theta_1 \right]^{\frac{1}{\alpha}} \left[ \frac{\theta_1}{\beta (\alpha (\theta_0 + \theta_1))} + 1 - \theta_0 - \theta_1 \right]^{\frac{2}{\alpha \rho}} = 0,
\]

\[
l_1 = \frac{1}{\rho} \ln \left[ \frac{1}{\theta_1} + \beta \left( \frac{1}{\theta_0 + \theta_1} - 1 \right) (1 - e^{-p_0^*}) \right]^{\frac{1}{\alpha}} E \left[ \tilde{A}_{n+1} \right]^{\frac{1}{\alpha}} + (1 - \theta_0 - \theta_1) e^{\left[ \frac{1}{\alpha} - 1 \right]} E_0 \left[ \frac{\tilde{A}_{n+1}}{A_0} \right]^{\frac{1}{\alpha}} (9)
\]

\[
\frac{\alpha \rho - \gamma}{(1 - \theta_0) \beta \rho} = \frac{1}{\theta_0 + \beta} \left( \frac{1}{\theta_0 - 1} \right) (1 - e^{-p_0^*}) \left[ \frac{2}{\alpha} \right]^{\frac{1}{\alpha}} e^{-\frac{2}{\alpha} p_0^*}.
\]
Here, \( l^*_1 \) actually has an explicit solution. Hence, we can first solve for \( \theta^*_1 \) from the first equation; then get \( l^*_1 \) from the second expression; and finally solve for \( l^*_0 \) from the last equation.

5. Theoretical Analysis

This section provides analysis of staged privatization. We intend to show the dynamic steps of the privatization process and illustrate how various factors affect this process.

Our theoretical analysis will be mainly based on graphics. Since our interest is in qualitative results (trend, tendency, and direction of change), graphic illustrations are sufficient. We will supplement this graphic analysis by empirical analysis in the next section. For the purpose, we choose the following parameter values as the benchmark:

\[
\theta_0 = 40\%, \quad \alpha = 10, \quad \beta = 30, \quad \rho = 5\%, \quad \gamma = 20\%, \quad A_0 = A_v = A_{v+h} = 1.
\]

Since much of our conclusions are not sensitive to the choice of the benchmark values, our choice of the above parameter values are quite rough in the sense that we simply choose round numbers without paying attention to decimal points. When the reform program started in 2005, the amount of T-shares was close 40%; we hence set \( \theta_0 = 0.4 \). Also, Kaul et al.’s (2000, p.911) study on the Canadian stock market empirically estimates the demand elasticity to be 10.5; we hence take \( \alpha = 10 \). Further, since Chinese manufacturing firms have been growing at an annual rate of about 20% in the past 20 years, we hence take \( \gamma = 20\% \).

We are interested in the lockup effect \( \beta \), demand elasticity \( \alpha \), expected growth \( \gamma \), and business fluctuations \( A_0 / A_{v+h} \). Hence, given the above parameter values, we let each of the four parameters, \( \beta, \alpha, \gamma \) and \( A_0 \), change to see the effects of the change on the first lockup \( l^*_0 \), the second lockup \( l^*_1 \), the share price \( p^*_0 \) and the amount \( \theta^*_1 \) of converting N-shares to T-shares at the unlock date \( t = l^*_0 \). We have drawn four sets of figures, corresponding to the four parameters \( \beta, \alpha, \gamma \) and \( A_0 \), with each set corresponding to the four endogenous variables \( l^*_0, l^*_1, p^*_0 \) and \( \theta^*_1 \).

The Lockup Effect

We first consider the impact of a lockup, as represented by \( \beta \), on shareholders’ choices. Figure 3 shows four charts for \( l^*_0, l^*_1, p^*_0 \) and \( \theta^*_1 \) as functions of \( \beta \). Figure 3 indicates that a larger lockup effect, as defined by a larger \( \beta \), will imply longer lockups and a larger amount \( \theta^*_1 \) of unlocking N-shares at the unlock day of the first lockup. These make sense. First, since a larger lockup effect increases the benefit from a lockup, the N-holder tends to lock up her shares longer. Second, a longer lockup implies a higher current share price, which explains a
larger amount $\theta_1$ of converting N-shares to T-shares at the same time. Our empirical analysis in the next section supports this theoretical finding.

Hence, a larger lockup effect means that a market-based privatization takes a longer time to complete as both lockups are longer; but, at the same time, N-holders will unlock their shares faster as the amount of unlocking shares at each unlock day is larger.

**Demand Elasticity**

We now consider the impact of demand elasticity, as represented by $\alpha$, on shareholders’ choices. First, as shown in Figure 4, an increase in demand elasticity implies shorter lockups. This makes sense. A longer lockup boosts demand and hence pushes up the price. However, if the buyers are very sensitive to a price rise, the sellers may not benefit from a higher price; consequently, the N-holder may choose a shorter lockup, which implies a lower share price.

Second, as shown in Figure 4, the amount $\theta_1^*$ of converting N-shares to T-shares is initially increasing and later decreasing. With larger demand elasticity, as the lockup is shorter at $t = l_0$, the current share price is lower. Hence, the N-holder must weigh the benefit of an early sale (without a time discount on future earnings) against a lower current price. If demand elasticity is not too large, the N-holder prefers to sell more early; if demand elasticity is too
large, the current price is too low so that the N-holder prefers to lock more shares, where a shorter lockup also reduces the cost of a future sale since the next opportunity to sell is sooner.

Hence, more elastic demand for shares means that a market-based privatization takes a shorter time to complete as both lockups are shorter; but N-holders may or may not unlock their shares faster depending on how elastic the demand is.

![Graphs showing the effects of demand elasticity on lockup periods and share prices](image)

**Figure 4. The Effects of Demand Elasticity**

**Performance Growth**

We now consider the impact of performance growth, as represented by $\gamma$, on shareholders’ choices. As shown in Figure 5, an increase in performance growth implies longer lockups. This makes sense. Performance growth implies a better future and encourages a shareholder to delay her selling shares. At the same time, performance growth implies a higher share price. However, a higher share price does not necessarily lead to a higher current sale of shares when the firm is expected to grow fast. Our empirical analysis supports this theoretical finding.

Hence, larger expected growth means that a market-based privatization takes a longer time to complete as both lockups are longer; also, N-holders will unlock their shares more slowly as the amount of unlocking shares is smaller.
Business Fluctuations

We now consider the impact of business fluctuations, as represented by $A_0 / A_{0+i}$, on shareholders’ choices. We will hold $A_0 = A_{0+i} = 1$ so that $A_0$ represents business fluctuations.
Figure 6 indicates that, if the economy at $t = l_0$ is better (as indicated by a larger $A_{q0}$), the N-holder will sell a larger proportion $\theta_1$ of her shares and lock the rest for a longer period $l_1$. With a longer lockup, the demand at $t = l_0$ is greater and the share price is higher, making a sale of shares more profitable.

Hence, a positive economic shock at a unlock day (at $t = l_0$) slows down the speed of a market-based privatization as the N-holder decides to lock her shares for a longer time (a larger $l_1^*$); but, at the same time, the N-holder will unlock her shares faster as the amount of unlocking shares becomes larger.

Finally, we wonder if it is possible that shareholders choose not to sell any shares at a unlock day if the economic situation at that time is very bad. Specifically, we want to find circumstances in which $\theta_1^*$ is zero. In reality, we do observe many such cases in early 2009 as the subprime crisis was in the worst situation. Indeed, our model can easily produce such a result. For example, if $A_{l0} / A_{l0+l1} = 20\%$ (bad current economic situation), $\rho = 3\%$ (not much discount on future) and $\gamma = 30\%$ (good growth potential), we find that $\theta_1^* = 0$ (no current sale) and $l_1^* = 1.13$ (a relatively short lockup). Generally speaking, when performance growth is high, the discount on future is low and the market at an unlock day is very bad, the N-holder may choose to lock all her shares at the unlock day.

6. Empirical Analysis

6.1. Main Empirical Results

We have three main empirical results. First, we show the existence of the lockup effect. This provides support for our theoretical model since our model is based on the existence of this effect. Specifically, we find that the 3-day cumulative abnormal return (CAR) around the
starting day of the initial lockup is positively correlated with the weighted length of lockups, and further that the positive CAR will not disappear in at least 60 days. These facts show that a lockup from N-holders can increase market demand for stock shares permanently.

Second, we show that SOEs’ N-holders and non-SOEs’ N-holders choose statistically the same lengths in lockups. Further, we show that decisions made by these two groups of N-holders depend on the same factors. Hence, under the same conditions, the two groups will make statistically the same decisions. In our theoretical model, we argue that the privatization process is market-based. One concern with this argument is that the N-holders of SOEs, many of whom are government-related, may not behave according to market principles. It turns out that our empirical comparison of the decisions made by the two groups support our argument.

Third, we show that the length of a lockup is positively corrected with a firm’s growth potential. Specifically, a firm with a higher ROA or higher sales growth is likely to commit to longer lockups. This evidence is consistent with our theoretical finding in Figure 5.

6.2. Definition of Variables

China’s stock market has total 1,411 firms at the end of 2008. All the listed firms (including non-SOEs), except four, are subject to the split-share policy that divides a firm’s shares into T-shares and N-shares. After the announcement of the recent reform (the SS reform) in 2005, only those firms that are considered to be eligible by the government are allowed to start their SS reform (hence, good firms start their reform early). By the end of 2007, 1,260 firms started their SS reform. Since those firms that started their reform in 2008 and after are all so-called “special treatment” firms (with negative profits for consecutively two years), we include only those firms that started their reform before 2008. We also exclude a few firms with data problems. In the end, our sample comprises of 1176 firms, with 768 SOEs and 408 non-SOEs. Since we include 90% of all the listed firms, our sample is large enough to have a complete picture of the stock market. Our data covers years from 2004 to 2007 and it mainly comes from two sources. Data on stock prices, operation returns, and corporate governance variables is from the China Stock Market and Accounting Research Database (CSMAR), while data on CEO background is manually collected from firms’ annual reports. Since all the listed firms except four face the reform, we have no sample selection bias.

In practice, lockup is the only form of commitment that is legal and protected by the law. Other forms of commitment, such as promises of assets restructuring, repurchase of tradable shares and dividend payment, are not legally enforceable. Also, for lockups, the SS reform has a three-tier minimum lockup requirement on the initial/first lockup. It is ruled that, in the first year after a firm starts its SS reform, any N-holder of the firm can sell an amount up to 0% of total shares in the secondary market; in the first two years, 5%; in the first three years, 10%; and finally, after three years, no more restriction. We find that in more than 700 firms,
about 60% of our sample, at least one N-holder of a firm chooses to commit an initial lockup longer than the required minimum. For example, Guangdong Shaoneng Group Limited commits to an initial lockup of 10 years. Table 1 provides a summary on lockups.

In our regression analysis, the lockup variable is the key. It is the dependent variable in the first step of our regressions and an independent variable in the second step. For robustness, we apply three methods to measure lockups. First, we define the lockup indicator. The lockup indicator takes value 0 if all N-holders of a firm strictly follow the three-tier minimum lockup requirement (minimum lockup); value 1 if at least one N-holder commits to a lockup that is longer than the requirement but not more than two years longer (medium lockup); and, value 2 if at least one N-holder commits to a lockup that is two years longer than the requirement (long lockup). Second, we define the lockup dummy, which takes value 1 if at least one N-holder of a firm commits to a lockup longer the minimum lockup, and 0 otherwise. Third, we define the weighted lockup. Since the lockups are voluntary lockups, they are generally different with different N-holders of the same firm; also, the same N-holder may choose different lockups at different times. Hence, we use the unlocked amount of shares as the weight for the corresponding lockup length to calculate the weighted lockup.

The definition of other variables is as follows:

**ROA**: return of assets. It is a company’s annual earnings divided by its total assets. It is in percentage.

**Firm size**: it is the logarithm of the book value of total assets.

**Sales growth rate**: it is a firm’s difference of annual total sales between this year and last year divided by its last year’s annual total sales. It is in percentage.

**N-share ratio**: it is the ratio of the number of N-shares to the total number of shares of a firm. It is in percentage.

**SOE dummy**: it takes value 1 if the firm is a SOE; 0 otherwise.

**Other commitments**: it takes value 1 if a firm not only employs lockups but also another commitment, such as a promise of assets restructuring, repurchase of tradable shares, and dividend payment; 0 otherwise.

**Compensation rate**: the number of shares paid by N-holders to T-holders for every ten T-shares held by the T-holders.

**Idiosyncratic risk**: it is based on a beta-adjusted market model regression. We choose the starting day of the first lockup as the event day and regress daily stock returns for the 120-day (-180 to -60) event window against market portfolio. The idiosyncratic risk equals the mean square error of this market model regression for each firm.
**CEO's political connection**: it takes value 1 if the current CEO once worked in the government; 0 otherwise.

**CAR**: cumulative abnormal return. The 3-day CAR is the beta-adjusted 3-day (-1 to 1) cumulative abnormal return around the starting day of the first lockup.

### 6.3. Summary Statistics

In Table 1, for each of the variables, the ROA, firm size and annual sales growth, we divide the sample into two subsamples (Groups 1 and 2) by the median; for the N-share ratio, we divide the sample by 50%; for the two dummies, “whether a firm is SOE” and “whether a firm uses other commitments beside lockups,” we divide the sample by the value of the dummy. We use univariate comparisons to exam differences in the lockup indicator between the two subsamples.

Table 1 first shows that the lockup indicator of a firm whose ROA is higher than the median is statistically higher than that of a firm whose ROA is lower than the median. That is, a firm with a higher ROA tends to have longer lockups. Second, the same result holds if we replace the ROA by sales growth. That is, a firm with higher growth potential tends to have longer lockups. These two results are consistent with our theoretical finding in Figure 5. Third, we find that the lockup indicator of a firm whose size is larger than the median is statistically indifferent from that of a firm whose size is smaller than the median. That is, firm size will not affect the decision on lockups. Fourth, we find that the lockup indicator of a firm whose relative size of N-shares is larger than 50% is statistically indifferent from that of a firm whose relative size of N-shares is smaller than 50%. That is, the relative size of N-shares does not affect the decision on lockups. Fifth, we find that the lockup indicator of a SOE is statistically indifferent from that of a non-SOE. That is, a SOE’s N-holders and a non-SOE’s N-holders choose statistically the same lengths in lockups. This lends support to the assumption in our theoretical model that all N-holders will behave according to market principles. Finally, we find that the lockup indicator of a firm that uses other commitments is statistically larger than that of a firm that does not. That is, a firm that uses other commitments tends to have longer lockups. In other words, lockups and other forms of commitment are complements.

In summary, firms with high ROAs, high sales growth or using other commitments tend to have long lockups. Firm size, the relative size of N-shares and whether or not a firm is a SOE do not affect decisions on lockups.

Furthermore, in order to show that the above results are not affected by outliers, we apply the Wilcoxon test to test the differences in medians. We use the two-tailed Wilcoxon rank-sum test to check whether or not the median changes significantly from one subsample to the other by the Wilcoxon z-statistic (Kazmier and Pohl, 1984). Table 1 shows that all the above results are robust to outliers.
Table 1. Descriptive Statistics on the Lockup Indicator

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<tr>
<th></th>
<th>Group 1</th>
<th>Group 2</th>
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<td>Observations</td>
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<tr>
<td></td>
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6.4. Regression Analysis

Although the univariate analysis in the last section provides some primary evidence to our main conclusions, we need more sophisticated regressions to derive our main conclusions. In this section, we investigate the general relationship between lockups and firm characteristics.

We first use the lockup indicator as the dependent variable. We use the ROA to represent firm quality and use a firm’s sales growth to represent a firm’s growth potential. Since lockups, financial compensation and other commitments may work together as a compensation package for T-holders, we need to control reform-specific variables, such as the compensation rate and other commitments. In addition, we include some other control variables: the N-share ratio is to control for the expected future supply of shares, the idiosyncratic risk variable is to control for firm-specific risks, the SOE dummy that indicates whether or not the government is the firm’s ultimate controller is to control for government interventions, and finally the dummy for CEO’s political connection is to control for government connections. In order to
mitigate the endogeneity problem, all accounting indices, such as the ROA, annual sales growth rate and total assets, are instrumented by their one-year lag values.

We use a cross-section ordinary logit model to run regressions and the results for the full sample are in Panel 1 of Table 2. First, we find that firms with higher ROAs commit to longer lockups, which confirms with our theoretical prediction in Figure 5. Further, by marginal analysis, if the ROA increases by one standard deviation (s.d.), the probability of a firm committing to medium lockups will increase by 4%, the probability of a firm committing to long lockups will increase by 2%, and the probability of a firm committing to minimum lockups will decrease by 5%.

Second, we find that firms with higher annual sales growth have longer lockups. Further, if the sales growth rate increases by one s.d., the probability of a firm committing to median lockups will increase by 3%, to long lockups will increase by 1.2%, and to minimum lockups will decrease by about 4%.

Third, we find that lockups and financial compensation are substitutes. In other words, if N-holders lock their shares longer, they will pay T-holders less in financial compensation. Further, if the compensation rate decreases by one s.d. (0.9 units), the probability of a firm committing to median lockups will increase by 2%, and to long lockups will increase by 1%.

Fourth, we find that lockups and other commitments are complements. Firms that use other commitments tend to have longer lockups. Specifically, the probability of a firm that uses other commitments having medium lockups is 9% higher, and the probability of a firm that uses other commitments having long lockups is 5% higher, in comparison with a firm that uses lockups only.

Fifth, we find that SOEs and non-SOEs have the same behaviors. That is, government-controlled firms behave the same way as market-determined firms. Further, it is argued that politically connected managers may take actions preferred by the government in order to facilitate their future careers. Hence, we distinguish those CEOs who have political connections from those who do not in order to control for government influence. Again, we cannot find evidence that SOEs and non-SOEs behave differently during the SS reform.

In summary, our full-sample regressions indicate that firms with high ROAs or high sales growth will commit to long lockups, no matter whether they are SOEs or non-SOEs.

<table>
<thead>
<tr>
<th>Table 2. Ordinary Logit Regressions on the Lockup Indicator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Independent Variable</td>
</tr>
<tr>
<td>----------------------</td>
</tr>
<tr>
<td>ROA</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Sales growth rate</td>
</tr>
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</table>
In the above regression analysis, we use the full sample consisting of all the listed firms. We now divide the sample into two subsamples with a SOE subsample containing SOEs only and a non-SOE subsample containing non-SOEs only. We use these two subsamples to do robustness checks. One question is: can we have the same results as before from the SOE subsample and also from the non-SOE subsample? The results are presented in Panels 2 and 3 of Table 2. We find that all the main results hold for both subsamples. Also, in both subsamples, the compensation rate works as a substitute for lockups, while other commitments work as complements for lockups. For the SOE subsample, a high ROA and annual sales growth rate increases the probability of a firm choosing either medium or long lockups. For the non-SOE subsample, a high ROA also has a positive effect on the probability of a firm choosing either medium or long lockups; but, the effect of a high sales growth rate is significant only at the 15% level. These subsample robustness checks confirm that our main results hold for both SOEs and non-SOEs, and that N-holders in SOEs and non-SOEs decide their lockups based on the same factors.

The SS reform began in April 2005, and by the end of 2007 about 1260 firms, more than 90% of the listed firms, had joined the reform. One question is whether or not firms that join

<p>| | | | | | | |</p>
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<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Compensation rate</td>
<td>-0.132*</td>
<td>-0.246**</td>
<td>-0.040</td>
<td>-0.172**</td>
<td>-0.300***</td>
<td>-0.103</td>
</tr>
<tr>
<td></td>
<td>(0.0738)</td>
<td>(0.1024)</td>
<td>(0.1114)</td>
<td>(0.0760)</td>
<td>(0.1031)</td>
<td>(0.1180)</td>
</tr>
<tr>
<td>N-share ratio</td>
<td>-0.008</td>
<td>-0.005</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.005</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(0.0058)</td>
<td>(0.0072)</td>
<td>(0.0104)</td>
<td>(0.0058)</td>
<td>(0.0071)</td>
<td>(0.0104)</td>
</tr>
<tr>
<td>Other commitments</td>
<td>0.621***</td>
<td>0.479***</td>
<td>0.920***</td>
<td>0.621***</td>
<td>0.475***</td>
<td>0.926***</td>
</tr>
<tr>
<td></td>
<td>(0.1453)</td>
<td>(0.1733)</td>
<td>(0.2767)</td>
<td>(0.1454)</td>
<td>(0.1735)</td>
<td>(0.2767)</td>
</tr>
<tr>
<td>Firm size</td>
<td>0.014</td>
<td>-0.026</td>
<td>0.065</td>
<td>-0.002</td>
<td>-0.038</td>
<td>0.058</td>
</tr>
<tr>
<td></td>
<td>(0.0636)</td>
<td>(0.0767)</td>
<td>(0.1239)</td>
<td>(0.0641)</td>
<td>(0.0772)</td>
<td>(0.1257)</td>
</tr>
<tr>
<td>CEO’s political connection</td>
<td>0.013</td>
<td>0.322</td>
<td>-0.432</td>
<td>0.008</td>
<td>0.286</td>
<td>-0.409</td>
</tr>
<tr>
<td></td>
<td>(0.3660)</td>
<td>(0.4740)</td>
<td>(0.6049)</td>
<td>(0.3661)</td>
<td>(0.4728)</td>
<td>(0.6087)</td>
</tr>
<tr>
<td>SOE dummy</td>
<td>-0.060</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.1305)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Year 2006</td>
<td>-0.351**</td>
<td>-0.254</td>
<td>-0.477*</td>
<td>(0.1599)</td>
<td>(0.2006)</td>
<td>(0.2673)</td>
</tr>
<tr>
<td>Year 2007</td>
<td>-0.631**</td>
<td>-0.782**</td>
<td>-0.468</td>
<td>(0.2568)</td>
<td>(0.3540)</td>
<td>(0.3862)</td>
</tr>
<tr>
<td>Cut point 1</td>
<td>-0.884</td>
<td>-2.226</td>
<td>0.581</td>
<td>-1.660</td>
<td>-2.860</td>
<td>-0.230</td>
</tr>
<tr>
<td></td>
<td>(1.4525)</td>
<td>(1.8357)</td>
<td>(2.8551)</td>
<td>(1.4841)</td>
<td>(1.8666)</td>
<td>(2.9211)</td>
</tr>
<tr>
<td>Cut point 2</td>
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<td>0.680</td>
<td>3.260</td>
<td>1.158</td>
<td>0.058</td>
<td>2.467</td>
</tr>
<tr>
<td></td>
<td>(1.4543)</td>
<td>(1.8353)</td>
<td>(2.8598)</td>
<td>(1.4845)</td>
<td>(1.8648)</td>
<td>(2.9235)</td>
</tr>
<tr>
<td>Prob &gt; Chi2</td>
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<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>Number of observations</td>
<td>1176</td>
<td>768</td>
<td>408</td>
<td>1176</td>
<td>768</td>
<td>408</td>
</tr>
</tbody>
</table>
the reform in different years will have different lockups. That is, does the timing of the starting year affect decisions on lockups? Also, controlling for the starting year may help control for economy-wide technology shocks. As shown in Figure 6, our theory indicates that technology shocks play an important role in determining a firm's choice on lockups. We hence add two dummy variables. With year 2005 as the base year, we add a dummy for year 2006, which takes value 1 in year 2006 and 0 otherwise. Similarly, we also add a dummy for 2007. Our regression results after controlling for the starting year are presented in Panel 4 of Table 2. We find that our main results still hold. Specifically, the coefficients for the ROA, sales growth, and other commitments are significantly positive, and the coefficient for the compensation rate is significantly negative. We find that both coefficients for Year 2006 and Year 2007 dummies are significantly negative and that the marginal effect of Year 2006 is smaller than that of Year 2007. Specifically, if a firm starts the reform in 2006, the probability of the firm having medium lockups will decrease by about 6% and the probability of the firm having long lockups will decrease by about 3%, in comparison to a firm that starts the reform in 2005. Also, if a firm starts the reform in 2007, the probability of the firm having medium lockups will decrease by about 12% and the probability of the firm having long lockups will decrease by about 4%, in comparison to a firm that starts the reform in 2005. In summary, the later a firm starts the reform, the lower probability it will have longer lockups. One explanation for this fact is that lockups are used as a signal for quality or to show the holder's confidence in the firm; after the market has gained enough knowledge of the listed firms, the signal effect is reduced; but, other effects, such as moral hazards and downward sloping demand, may still present.

In Panels 5 and 6 of Table 2, we present the results for the SOE and non-SOE subsamples. The main results remain. In particular, we find that choices of lockups made by N-holders of SOEs and N-holders of non-SOEs depend on the same firm characteristics, such as firm quality and growth potential, even after we control for the starting year. Notice that, in the SOE subsample, the coefficient for Year 2006 becomes insignificant. This means that there is no significant difference in lockups among SOEs between those starting the reform in 2005 and those starting in 2006. But, for SOEs that started the reform in 2007, they do have some tendency for shorter lockups. In the non-SOE subsample, the coefficient for Year 2007 becomes insignificant and the coefficient for sales growth is significant at the 15% level only. Although we do not report the detailed results here, we have also run all the model specifications in Table 2 by a binary logit model using the lockup dummy as the dependent variable. We find that all the main results remain.

In the remaining part of this section, we test whether or not lockups have a positive effect on demand for shares. We use the beta-adjusted 3-day CAR around the starting day of the first lockup to measure the effect. In the above regressions, we have showed that firms with different characteristics will commit to different lockups. Hence, we need to control for this en-
dogeneity problem. If lockups are measured by the lockup indicator as in the above, technically we can use the sample selection model proposed by Maddala (1983) or Heckman et al. (1999) to measure the effect of this endogenous independent variable on demand. However, the sample selection model is more convenient in comparing different choices for the same firm, but we are interested in comparing different firms. Hence, we transform discrete lockup choices to a continuous weighted lockup measure and use 2SLS (2-stage least squares) regressions to check the lockup effect. We use this weighted lockup as the dependent variable and use the left-censored Tobit model to run regressions on all the model specifications in Table 2. We find that all our main results still hold.

The regression results from the 2SLS regressions are reported in Table 3. Following the first-stage regression, the weighted lockup is instrumented by the compensation rate, N-share ratio, other commitments, firm size, ROA, idiosyncratic risk, sales growth rate, and CEO’ political connection. First, as shown in Panel 1 of Table 3, the predicted weighted lockup can significantly increase demand for shares, indicating the existence of the lockup effect. Second, we find that the CAR is negatively related to the N-share ratio, indicating that the market worries about a negative effect of the potential large supply of shares on the share price in the future.

In Panel 2 of Table 3, we present the regression results after controlling for the starting year of reform. As expected, the predicted weighted lockup still has a positive effect on demand. In both of the model specifications in Table 3, we first employ Sargan and Basmann tests to test over-identification restrictions and then employ Durbin and Wu-Hausman tests to test the exogeneity of the instrumented variables. The p-values of Sargan and Basmann tests are larger than 0.2 and statistically insignificant. This means that we can exclude some of the instruments from the model and the chosen instruments are valid. With this fact, we then use two kinds of Hausman tests to test whether or not the coefficient of the weighted lockup is biased under ordinary least squares (OLS) regressions. Because the p-values for both tests are near zero, we strongly reject the hypothesis that the weighted lockup variable is exogenous. This means that our use of 2SLS regressions to control the endogeneity problem is well deserved.

Finally, in order to show that the CAR is caused by a long-run lockup effect as opposed to temporal price pressure, we calculate the CAR using different event widows with the starting day of the first lockup as the event day. We find that the CARs within 60 days after starting the reform are always significantly positive, indicating that the positive abnormal return is not a temperate phenomena.
Table 3. 2SLS Regressions on the 3-day CAR

<table>
<thead>
<tr>
<th>Independent variable</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weighted lockup</td>
<td>0.105**</td>
<td>0.073*</td>
</tr>
<tr>
<td></td>
<td>(0.0489)</td>
<td>(0.0409)</td>
</tr>
<tr>
<td>Compensation rate</td>
<td>0.113**</td>
<td>0.105***</td>
</tr>
<tr>
<td></td>
<td>(0.0472)</td>
<td>(0.0335)</td>
</tr>
<tr>
<td>N-share ratio</td>
<td>-0.036**</td>
<td>-0.026**</td>
</tr>
<tr>
<td></td>
<td>(0.0154)</td>
<td>(0.0129)</td>
</tr>
<tr>
<td>Firm size</td>
<td>-0.105***</td>
<td>-0.070**</td>
</tr>
<tr>
<td></td>
<td>(0.0379)</td>
<td>(0.0330)</td>
</tr>
<tr>
<td>ROA</td>
<td>-0.011*</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(0.0055)</td>
<td>(0.0053)</td>
</tr>
<tr>
<td>Idiosyncratic risk</td>
<td>5.308*</td>
<td>2.719</td>
</tr>
<tr>
<td></td>
<td>(3.1930)</td>
<td>(2.5830)</td>
</tr>
<tr>
<td>Year 2006</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0558)</td>
<td></td>
</tr>
<tr>
<td>Year 2007</td>
<td>0.438***</td>
<td>(1.400)</td>
</tr>
<tr>
<td>Constant</td>
<td>2.632***</td>
<td>1.755**</td>
</tr>
<tr>
<td></td>
<td>(0.9120)</td>
<td>(0.8330)</td>
</tr>
<tr>
<td>Sargan (score)</td>
<td>2.877</td>
<td>2.776</td>
</tr>
<tr>
<td>Basmann</td>
<td>2.863</td>
<td>2.756</td>
</tr>
<tr>
<td>Durbin (score)</td>
<td>24.400***</td>
<td>12.282***</td>
</tr>
<tr>
<td>Wu-Hausman</td>
<td>24.742***</td>
<td>12.306***</td>
</tr>
<tr>
<td>Observations</td>
<td>1195</td>
<td>1195</td>
</tr>
<tr>
<td>Prob&gt; Wald Chi2</td>
<td>0.0738</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

**Remark 1.** We have also run regressions separately on the 4-day, 5-day, ..., till 60-day (-1 to 59) CARs with the same set of model specifications in Table 3. All our main results remain.

**Remark 2.** Up to March 2009, China’s two stock exchanges released total 134 announcements on second lockups committed by N-holders in 64 companies, with lengths ranging from 1 to 4 years. However, data from these second lockups in 64 companies is not enough to run well-specified regressions in order to provide further support for our theoretical model.

**Remark 3.** We have also run regressions on the lockup dummy, a third way to measure lockups as defined in Section 6.2, with the same set of model specifications in Table 2. All our main results remain.

**Remark 4.** The lockup effect may include the effects of signalling, moral hazards and risks. Unfortunately, our data is not sufficient to separate these effects from the lockup effect.
7. Concluding Remarks

The most recent and also the most thorough privatization of Chinese SOEs is a staged privatization with multistage market-oriented lockups and step-by-step unlocking of shares. The speed varies with the economic environment and depends on several crucial factors such as the lockup effect, demand elasticity, performance growth and business fluctuations.

We present a theory for this process and provides evidence in support of our theoretical model and its findings. Our theory is based on the lockup effect and our empirical analysis shows the existence of this lockup effect. This work contributes to the literature in three aspects. First, our theory is the first theory on a market-oriented multistage privatization process. Second, we contribute to the literature on how privatization should be carried out by providing both theoretical and empirical analyses of a market-based privatization program. Third, we contribute to the literature on IPO lockups by showing the existence of the lockup effect and a downward sloping demand curve for shares. The multistage nature of lockups in the recent Chinese reform allow us to unambiguously identify the lockup effect empirically.

We have only considered two stages of lockups. Data and reality do not provide us with evidence on further lockups. However, even if the government allows more stages of lockups, the influence of N-shares will be greatly diminished after two rounds of lockups. When most firms arrive at the end of their first lockups between 2008 and 2009, the market turned out to be very bad, due to the subprime crisis. Even in such a bad situation, we find that the sales of newly unlocked shares are about half of the original total volume of shares in the Chinese stock market, implying a 50% increase in tradable shares. Hence, we expect the privatization process to be largely complete, or most firms are largely in private hands, after two rounds of lockups.

8. Appendix

We now derive the solution in (9).

At \( t = t_0 \)

The first-order conditions (FOCs) for problem (5) are

\[
0 = \left( \frac{A_b}{\theta_0 + \theta_1} \right)^{\frac{1}{\alpha}} \left[ \frac{1}{\theta_0 + \theta_1} + \beta \left( \frac{1}{\theta_0 + \theta_1} - 1 \right) \left( 1 - e^{-\rho t} \right) \right] - E_b \left( \frac{A_b + q B_{3h_0}^{1/\alpha}}{e^{-\rho t}} \right) e^{-\rho t} \\
- \frac{\theta_t}{\alpha} \left( \frac{A_b}{\theta_0 + \theta_1} \right)^{\frac{1}{\alpha}} \left[ \frac{1}{\theta_0 + \theta_1} + \beta \left( \frac{1}{\theta_0 + \theta_1} - 1 \right) \left( 1 - e^{-\rho t} \right) \right]^{\frac{1}{\alpha} - 1} \frac{1}{(\theta_0 + \theta_1)^2} \left[ 1 + \beta \left( 1 - e^{-\rho t} \right) \right],
\]
0 = \frac{\theta_1}{\alpha (\theta_0 + \theta_1)} \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right]^{-1} \beta \rho (1 - \theta_0 - \theta_1)e^{-\rho (l_0 + l_1)} + \left( \frac{\gamma}{\alpha - \beta} \right) (1 - \theta_0 - \theta_1) E_b \left( \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right)^{\frac{1}{\alpha}} e^{\alpha \rho (l_0 + l_1)}.

These two equations determine \((\theta_1^*, l_1^*)\). From these two equations, we know that the solution \((\theta_1^*, l_1^*)\) does not depend on \(l_0\). We also observe that we need \(\alpha \rho > \gamma\), otherwise \(l_1^* = \infty\).

Then, the above FOCs can be simplified to

\[
\frac{1}{\theta_0 + \theta_1} + \beta \left( \frac{1}{\theta_0 + \theta_1} - 1 \right) (1 - e^{-\rho l_1}) \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right]^{-1} \frac{1}{\alpha} \left[ \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right]^{\frac{1}{\alpha}}
\]

\[
= \frac{\theta_1}{\alpha (\theta_0 + \theta_1)} + \beta \left( \frac{1}{\theta_0 + \theta_1} - 1 \right) (1 - e^{-\rho l_1}) \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right]^{-1} \frac{1}{\alpha} \left[ \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right]^{\frac{1}{\alpha}} [1 + \beta(1 - e^{-\rho l_1})],
\]

\[
\frac{\theta_1}{\alpha} (\theta_0 + \theta_1)^{\frac{1}{\alpha}} \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right]^{\frac{1}{\alpha} - 1} \beta \rho = \left( \rho - \frac{\gamma}{\alpha} \right) e^{\alpha \rho} E_b \left( \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right)^{\frac{1}{\alpha}},
\]

implying

\[
1 = \frac{1 + \beta(1 - e^{-\rho l_1})}{1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \alpha (\theta_0 + \theta_1)} \frac{\theta_1}{\alpha} + \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right]^{-\frac{1}{\alpha}} \frac{1}{\alpha} \left[ \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right]^{\frac{1}{\alpha}} e^{\alpha \rho} E_b \left( \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right)^{\frac{1}{\alpha}},
\]

\[
= \left[ 1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \right] (\alpha \rho - \gamma) e^{\alpha \rho} E_b \left( \frac{\tilde{A}_{b+i}^{L_0}}{A_b} \right)^{\frac{1}{\alpha}}.
\]

Then,

\[
1 - \frac{1 + \beta(1 - e^{-\rho l_1})}{1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) \alpha (\theta_0 + \theta_1)} \frac{\theta_1}{\alpha} \frac{\theta_1 \beta \rho}{\alpha \rho - \gamma} = e^{-\rho l_1},
\]

implying

\[
1 + \beta(1 - \theta_0 - \theta_1)(1 - e^{-\rho l_1}) - \frac{\theta_1}{\alpha (\theta_0 + \theta_1)} [1 + \beta(1 - e^{-\rho l_1})] = \frac{\theta_1 \beta \rho}{\alpha \rho - \gamma} e^{-\rho l_1},
\]
implying
\[ 1 + \beta (1 - \theta_0 - \theta_i) - \beta (1 - \theta_0 - \theta_i) e^{-\rho t_i} = \frac{\theta_i (1 + \beta)}{\alpha (\theta_0 + \theta_i)} + \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} \beta e^{-\rho t_i} = \frac{\theta_i \beta \rho}{\alpha \rho - \gamma} e^{-\rho t_i}, \]
implying
\[ 1 + \beta (1 - \theta_0 - \theta_i) - \theta_i (1 + \beta) = \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right] \beta e^{-\rho t_i}, \]
which gives us an explicit solution of \( \tilde{I}_i \) in (9). Substituting this \( \tilde{I}_i \) into the second equation in (11) yields
\[
\left[ 1 + \beta (1 - \theta_0 - \theta_i) \right] \left[ 1 - \frac{1 + \beta (1 - \theta_0 - \theta_i) - \theta_i (1 + \beta)}{\alpha (\theta_0 + \theta_i)} \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= (\alpha \rho - \gamma) e^{\alpha t_i} E_{\alpha} \left\{ \frac{A_{\alpha + \beta}}{A_\alpha} \right\},
\]
implying
\[
\left[ 1 + (1 - \theta_0 - \theta_i) \right] \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
\[
= \beta \left[ \frac{\theta_i \rho}{\alpha \rho - \gamma} - \frac{\theta_i}{\alpha (\theta_0 + \theta_i)} + 1 - \theta_0 - \theta_i \right]^{\frac{1}{\alpha - 1}} (\theta_0 + \theta_i) = \frac{1}{\alpha} \theta_0 \beta \rho \]
implying the first equation in (9).

At $t = 0$

We have

$$\pi_N(l_0) = \theta_t \frac{1}{\theta_0 + \theta_t} + \beta \left[ \frac{1}{\theta_0 + \theta_t} - 1 \right] \left( 1 - e^{-\rho_0} \right)^{\frac{1}{\alpha}} e^{\frac{2 - \rho}{\alpha}} \theta + (1 - \theta_0 - \theta_t) e^{\frac{2 - \rho}{\alpha}} \theta_0 \left( A_{l_0}^{1/\alpha} \right)$$

$$= \left[ \theta_t \left[ \frac{1}{\theta_0 + \theta_t} + \beta \left[ \frac{1}{\theta_0 + \theta_t} - 1 \right] \left( 1 - e^{-\rho_0} \right) \right] \right]^{1/\alpha} A_{l_0}^{1/\alpha} + \left( 1 - \theta_0 - \theta_t \right) e^{\frac{2 - \rho}{\alpha}} \theta_0 \left( A_{l_0}^{1/\alpha} \right) e^{\frac{2 - \rho}{\alpha}} \theta_0.$$
References


