

Nonparametric Estimation of Entry Cost in First-Price Procurement Auctions

Pai Xu^{*†}

University of Hong Kong

September 2010

This Version: May 2011

Abstract

In this paper, I investigate the Samuelson (1985) low-price auction model with entry costs. The model's equilibrium implies that the distribution of bids is truncated at the threshold for participation. I use the model to estimate the cost of participation in Michigan highway procurement auctions. The null hypothesis of zero entry costs is rejected. Using my empirical results, I then construct an estimate of the optimal auction, which employs regular policy tools such as entry fees. Finally, I demonstrate the savings that the Michigan government could have made on payments if optimal auctions had been employed.

JEL classification: C14, D44, L40, L50

Keywords: Endogenous Entry, First-Price, Procurement Auctions, Nonparametric.

^{*}I would like to thank the Editor, Thierry Magnac, and three anonymous referees whose insightful comments have greatly improved the paper. I am also grateful to Shinichi Sakata, Michael Peters, Han Hong, Jacques Crémer, Thomas Lemieux, Vadim Marmer, Art Shneyerov, Okan Yilankaya, Brian Krauth and Unjy Song for their useful comments, as well as the participants at the 2006 Canadian Economics Association Annual Meeting, the 2007 Canadian Econometrics Study Group conference, and the empirical workshops at the University of British Columbia and the University of Hong Kong. The financial support of the RGC under grant HKU753709H is also greatly acknowledged.

[†]Contact: Room 908, K.K.Leung Building, Pokfulam, Hong Kong. Tel: +852 28578501. Fax: +852 25481152.
E-mail: paixu@hku.hk.

1 Introduction

A robust feature of highway procurement auctions is that some bidders choose not to submit a bid, even though they previously demonstrated potential interest in the project under auction. A plausible explanation is that auction entry is costly. Understanding the entry behavior of these bidders is of great research interest and is important if desirable economic policies are to be implemented. In particular, policy makers are often interested in whether costly entry induces insufficient competition in the market.

This paper considers the independent private-value, first-price auction model with endogenous entry proposed by Samuelson (1985) (hereafter, the S model). To participate in an auction, a bidder often bears various costs in preparing its bid, such as the costs of traveling to the auction site and the cognitive efforts the bidders exert in the bidding process. In the S model, these entry costs do not occur unless the bidder enters the auction. Thus, the bidder participates in the auction if and only if she expects the profits from the auction to be sufficiently large to compensate for the costs. An interesting policy implication of the S model is that discouraging actual participation can be optimal for the government. In particular, given a certain number of potential bidders, making entry costlier can screen less favorable bidders and increase competition, in turn helping to save procurement costs. To quantify such “screening” effects, or even to implement optimal auctions in the S model context, it is essential to know the entry cost level.

In the S model, the equilibrium entry condition implies that a marginal participant in the auction right at the cutoff point must have the same expected profit as the amount of entry costs. Therefore, the estimation of entry costs amounts to evaluating an expected profit function at this cutoff point. To this end, one needs to estimate the truncated (bid) density function at the truncation point, as it appears in the bidders’ expected profit function. In this paper, I suggest the one-sided nearest neighbor technique (hereafter, NN) to estimate the entry cost, because it follows a normal distribution asymptotically, which in turn delivers a normal distribution in the limit for the proposed estimator of entry costs. This knowledge of participation cost then enables me to investigate the implementation of an optimal auction. The data I consider show that the Michigan government could have saved up to 10-15% by setting up its highway procurement auctions optimally. Thus, this work provides the first empirical insights into the use of policy tools to attain optimal outcomes in such an auction environment.

Endogenous entry in highway procurement auctions has recently received a significant amount of empirical research attention. Krasnokutskaya and Seim (2010), for example, measure the effect

of bid preference programs in California highway procurement auctions. Also allowing bidder asymmetry in California procurement auctions, Bajari, Hong and Ryan (2010) investigate bidders' entry decisions in the context of discrete games with complete information. Their focus is on providing an identification and estimation strategy in the presence of multiple equilibria. Li and Zheng (2009) propose a semi-parametric Bayesian method to jointly estimate entry and bidding models. They use structural estimates to quantify the "entry effect" and "competition effect" in bidding. Most, if not all, of the related work in this arena employs various versions of the Levin and Smith (1994) (hereafter, LS) model to describe bidders' entry decisions, with bidders making costly entry decisions *before* knowing their own valuations. Here, in contrast, I assume the entry decisions are made *after* the bidders know their valuations, as suggested by the S model. To the best of my knowledge, this paper is the first structural work on the S model using a fully nonparametric approach.

With regard to the choice of competing entry models for the empirical framework, the evidence documented in the literature is rather mixed. Li and Zheng's (2009) empirical work is the first to make such an attempt. Their statistical test employing highway mowing auction data from Texas fails to support the S model. Therefore, they estimate a LS model instead. Focusing on the choice of entry models, Marmer, Shneyerov and Xu (2010) propose a nonparametric structural test and employ the same dataset as Li and Zheng (2009). They find no empirical support for the LS model, but cannot reject the S model. Their testing approach employs variation in the number of potential bidders, which is different from this work.¹

The relevance of competing models for empirical analysis thus depends largely on the context of the application and the empirical questions to be addressed. In the application in this paper, viz., Michigan highway procurement auctions, there is an important step in the letting process. A qualified firm needs to collect a bidding proposal from the Department of Transportation. By doing so, it not only becomes a potential bidder, but also acquires detailed information on the project under auction. A common observation from highway procurement practice is that not all potential bidders actually submit a bid. The motivation of this work is to examine entry of this kind. A plausible reason for the failure of some firms to submit a bid is that preparing and submitting a bid is a costly process, as in the S model. Hence, I implicitly assume that potential bidders figure out their valuations of the project through careful study of the bidding documents. "Costly entry" in this paper thus incorporates all the time and labor needed to compile a bidding plan, check for

¹Marmer et al. (2010) also propose a new model to study the entry behavior of bidders. The bidders in their model observe a noisy signal when deciding on entry. See also Robert and Sweeting (2010).

errors, and travel to bid meetings, as well as the monetary costs the firm may have to devote to the bid submission process.

The counterfactual experiments in this work deliver another message to the literature. One should take special care in interpreting the counterfactual experiments in models with data selection problems, as a certain range of population density is unobserved by econometricians. The counterfactual experiments involved in implementing optimal auctions in this paper, however, are not subject to this so-called extrapolation problem, as the optimal policy is to push the threshold from the observed boundary into the observed region. Through this empirical exercise, I document the possibility of reliable counterfactuals with nonparametric estimates in a selection model proposed in the literature.

It is fair to mention that the Rosenblatt-Parzen class of bandwidth estimators could be employed as an alternate method of estimating the density functions in this paper. When the bandwidth estimator is applied in boundary regions, however the estimate is not necessarily consistent. This inconsistency problem has elicited an extensive body of literature on the correction of the boundary effect. As they are not designed for estimation of a density function at the boundary point, most of the existing methods focus on how to modify the weighting scheme in such a way that the boundary region can be automatically detected and adjusted, while leaving the issue of the limiting distribution unsettled. Here, instead, the proposed NN estimator is shown to follow a normal distribution in the limits. In a supplement, I compare the one-sided NN estimator with several typical bias-corrected bandwidth methods in Monte Carlo experiments. In these experiments, the one-sided NN estimate always has a smaller mean squared error than other bandwidth estimators. The stable performance of the one-sided NN estimator may reflect the local adaptive nature of the NN method.

The remainder of the paper is organized as follows. First, I introduce the S model in Section 2 to develop a theoretical framework within which to investigate optimal auctions. In the third section, I then discuss the identification issues, with a focus on whether any additional assumptions are required to identify the model elements. I also establish large sample properties for the proposed estimator of entry costs. In Section 4, I apply the method to estimate the costs of participation in Michigan highway procurement auctions, assuming that the observed data are generated by an S model. This study rejects the null hypothesis that these participation costs are zero at any reasonable significance level. With my estimated participation cost levels, I then further investigate the implementation of an optimal auction by employing only regular policy tools, such as the reserve prices and/or entry fees. Section 5 concludes the paper, and the mathematical proofs are collected

in the appendix.

2 First-Price Auction Model with Entry Costs

2.1 Equilibrium entry and bidding

I consider a first-price, sealed-bid procurement auction for a single indivisible good or service. Within the symmetric independent private-cost (IPC) framework, each potential risk-neutral participant $i \in \{1, 2, \dots, N\}$ knows the number of potential bidders and her own cost, c_i , to supply the object, but only the distribution of costs to the other potential bidders. It is assumed that the costs to individuals are drawn independently from the absolutely continuous distribution $F(c)$, with support $[\underline{c}, \bar{c}] \subset \mathbb{R}_+$. Bidders submit their bids simultaneously, and the object is supplied by the lowest bidder. The winner receives her bid from the buyer, provided that it is no greater than the reserve price r .

Our analysis deviates from the standard IPC framework by allowing the presence of a common entry cost, κ , which each bidder has to pay to join the auction. Given her private cost, the bidder then decides whether or not to submit a bid (paying κ) and become an actual bidder. All potential bidders make this decision simultaneously. Therefore, they make their participation and bidding decisions without knowing the number of competitors they will face.

I focus on the unique symmetric Bayes-Nash equilibrium (Milgrom, 2004), in which each potential participant joins the auction if her value is no more than c_ρ , the cutoff point (which is common to all bidders); otherwise she chooses not to participate. The cutoff point is such that the participant with cost c_ρ is indifferent to entering the auction. Suppose a firm with cost c_ρ enters. It will only win the auction if no other firm enters. Consequently, this marginal type of firm will optimally choose to bid r . In such an event, the probability of facing no competing bidders is $[1 - F(c_\rho)]^{N-1}$.² Here, c_ρ should solve the marginal bidder's expected profit from bidding,

$$[r - c_\rho][1 - F(c_\rho)]^{N-1} - \kappa = 0. \quad (2.1)$$

Conditional on entering the auction, the expected profit for the i -th bidder is then given by

$$\Pi_i[c_i, y, (b_j)_{j \neq i}] = (y - c_i)[1 - F(b^{-1}(y))]^{N-1} - \kappa, \quad (2.2)$$

²Throughout the paper, I assume that the entry cost is moderate, such that $c_\rho \in (\underline{c}, r)$. This assumption effectively rules out the uninteresting case of an entry cost so large that there is no entry.

where y is bidder i 's bid, given c_i and all other parameters. b^{-1} is the inverse bidding strategy for bidders. Thus, imposing the initial condition $b(c_\rho) = r$, the maximization of Π_i with respect to y yields the equilibrium bidding strategy $b : [\underline{c}, c_\rho] \rightarrow [\underline{b}, r]$

$$b(c) = \frac{1}{[1 - F(c)]^{N-1}} \left(\int_c^{c_\rho} u(N-1)f(u)(1 - F(u))^{N-2} du + r[1 - F(c_\rho)]^{N-1} \right). \quad (2.3)$$

A potential bidder j will not participate if and only if $c_j > c_\rho$.

2.2 Optimal auctions

Because optimal auctions are of concern to policy makers, I next consider optimal auctions with entry costs. The optimality analyzed here is from the revenue perspective, i.e., it refers to minimizing the government's expected payment on the auctioned project. My goal is to examine the possible policy implications of implementing these optimal auctions.

I shall restrict my attention to the class of auctions with deterministic entry decisions. Each bidder will continue to use cutoff rules when deciding entry in the optimal auctions. If the bidders are allowed to have different cutoffs for their entry decisions, then the model can accommodate multiple equilibria. In the absence of the revenue equivalent theorem, I may be unable to reach any definitive conclusion on the improvement of optimal auctions.³ Therefore, I also focus on the symmetric optimal cutoff in this optimality analysis.

Let \tilde{c} denote the common cutoff used by all bidders. The government's expected profit can be expressed by the following function of the cutoff.

$$\pi(\tilde{c}) = N \int_{\underline{c}}^{\tilde{c}} [r - J(c)][1 - F(c)]^{N-1} f(c) dc - \kappa N F(\tilde{c}), \quad (2.4)$$

where $J(c) = c + \frac{F(c)}{f(c)}$ is the virtual cost function, following the terminology in the literature. Equation (2.4) reflects the fact that the entry costs incurred by bidders are not received by the government.⁴

³This does not rule out the possibility that the government may save more money with an asymmetric equilibrium in our symmetric environment. See, for example, Celik and Yilankaya (2009).

⁴This setup of the government's problem implicitly assumes that r is the government's maximum willingness-to-pay for the auctioned project, which can be understood as the amount for which the government could have procured the good from a third party. This assumption appears to be consistent with the application. Further discussion appears in the application section.

The cost cutoff for an optimal symmetric auction, c^* , should solve the first-order necessary condition of problem (2.4):

$$[r - J(c^*)][1 - F(c^*)]^{N-1} = \kappa. \quad (2.5)$$

Equation (2.5) can be interpreted as follows. Suppose that all bidders are at cutoff c^* . Introducing a slight decrease in the cutoff to one bidder will increase the gross gain to the buyer by $[r - J(c^*)][1 - F(c^*)]^{N-1}$, while paying the bidder κ the marginal cost of inducing participation. Following the literature, I further assume that $J(\cdot)$ is a monotonic function of c . It can easily be verified that such a regularity condition uniquely determines the symmetric cutoff point in (2.5).⁵

Restricting entry. The following proposition compares the equilibrium cutoff (c_ρ) with the optimal cutoff (c^*) in the endogenous entry model. Such comparison has policy implications, that is, it determines whether the government should encourage or restrict entry to achieve an optimal auction outcome.

Proposition 2.1 *In a first-price IPC procurement auction model with participation costs (Samuelson, 1985), for any given number of potential bidders, the auctioneer (buyer) minimizes the expected payment by discouraging entry in the symmetric equilibrium (i.e., $c^* < c_\rho$).*

Intuitively, a decrease in the equilibrium cutoff (c_ρ) makes the auctioneer better off by extracting from all bidders who have sufficiently favorable costs that they decide to enter and by further screening out the least favorable bidders.⁶ It is worth noting that restricting entry here refers to discouraging actual participation, as the number of potential bidders is fixed. It purely reflects the “screening” effect by making entry more costly.⁷ Interestingly, however, such a “screening” effect does not hold for the LS model. In that model, an increase in the entry cost reduces both the probability of entry and the expected number of actual bidders. In turn, the actual bidders face less competition and bid less aggressively.

⁵The regularity condition is generally used in the optimal auction literature. Under such a condition, the optimal selling mechanism is said to be regular. See, for example, Myerson (1981) and Krishna (2002).

⁶The result of proposition 2.1 relies on comparison of the first-order conditions of equilibrium bidding and the optimal auction. Therefore, the result applies only to a given r , which refers to the government’s maximum willingness-to-pay. I am grateful to one of the referees for pointing this out.

⁷Li and Zheng (2009) notably find that “increasing competition may not always be desirable for the government” in procurement auctions. In their work, the “entry effect” is defined as the bidders’ equilibrium bidding behavior across any variation in the number of potential bidders. As the number of potential bidders increases, the existing bidders bid less aggressively. Their result is driven by the dominance of the entry effect on the competition effect.

Such equilibrium and policy analysis motivates me to consider the following empirical questions in the remainder of the paper. (i) Are the entry costs in the S model statistically significant in empirical applications? (ii) With knowledge of entry costs, how can policy tools be optimally set? (iii) How much can a government save by implementing optimal auctions?

3 Identification and Estimation

My goal in this empirical exercise is to conduct counterfactual analysis of the optimal auctions proposed herein. To this end, I require an estimate of the entry costs in the S model. In this section, I first discuss the identification issues of κ . My focus is on determining whether the current assumptions and conditions imposed in the model equilibrium are sufficient to imply the identification of κ . I then propose a strategy to estimate κ and investigate its asymptotic properties.

3.1 Identification

Let b^* denote the observed equilibrium bid. G^* and g^* are the distribution and density functions of b^* over support $[\underline{b}, r]$, respectively. p denotes the probability of submitting a bid. Conditional on entry, a bidder with private cost c expects to gain the following profit by choosing to submit an equilibrium bid b^* .

$$\Pi^*(b^*, c) = (b^* - c)[1 - p + p(1 - G^*(b^*))]^{N-1} - \kappa. \quad (3.6)$$

The probability of winning by submitting bid b^* in (3.6) can be understood in the following way. Bid b^* beats a typical rival with probability $[1 - p + p(1 - G^*(b^*))]$, because the rival either does not bid or submits a bid higher than b^* . There are $(N - 1)$ such rivals. Rearranging the first-order condition of (3.6) with respect to b^* gives the following inverse bidding function.

$$\xi(b^*) = b^* - \frac{1 - p + p(1 - G^*(b^*))}{(N - 1)pg^*(b^*)}. \quad (3.7)$$

As in (2.3), the bidding strategy b is strictly increasing and continuous. Therefore, ξ is the inverse function of b , defined as the unique solution to the equation $b(\xi(b^*)) = b^* \forall b^* \in [\underline{b}, r]$.⁸ First, it is worth noting that $b(c_\rho) \equiv r$ at equilibrium bidding. Then, (3.7) implies that c_ρ is identified through $\xi(r)$.

⁸In the case of no endogenous entry, (3.7) becomes the counterpart of the inverse bidding strategy of Guerre, Perrigne, and Vuong (2000) in low-bid (procurement) auctions.

As g^* appears in the denominator of (3.7), this demands that g^* be nonzero over $[\underline{b}, r]$. Moreover, $G^*(\tilde{b}) = Pr[b(c) \leq \tilde{b} | c \leq c_\rho] = F(b^{-1}(\tilde{b}))/F(c_\rho)$. Differentiating with respect to \tilde{b} gives $g^* = \frac{f(c)}{b'(c)F(c_\rho)}$. To ensure that g^* is positive, $f(c)$ has to be nonzero on $[\underline{c}, c_\rho]$. Because I focus on the strictly increasing bidding strategy, f over $[\underline{c}, \bar{c}]$ is implicitly assumed to be positive for the strict single crossing differences property to hold in auction theory. (See, for example, Milgrom[2004].)

Further note that $p = \mathbf{E}[n/N]$. Therefore, observing the number of both actual bidders and potential bidders helps to identify p . Moreover, the observed equilibrium bids identify the distributions (G^* and g^*) on region $[\underline{b}, r]$. Together with (2.3), (3.7) suggests that $F(c)$ is nonparametrically identified up to $[\underline{c}, c_\rho]$.

By the construction of the model, the equilibrium entry condition requires the cutoff value entrant (c_ρ) to make a positive expected gain from entry to cover the entry costs, κ , exactly.⁹ That is,

$$\kappa = \Pi(c_\rho) \equiv (r - c_\rho)[1 - F(c_\rho)]^{N-1} = [r - \xi(r)][1 - p]^{N-1}, \quad (3.8)$$

where the last equality employs the fact of $p = F(c_\rho)$. Therefore, the identification of κ is achieved, as both p and ξ at r are identified. The identification of κ calls only for (3.7) at c_ρ . Therefore, the local positiveness of f at c_ρ suffices to identify κ .

The foregoing identification results are collected in the following proposition.

Proposition 3.1 *Assume a Samuelson (1985) entry model. Further assume an econometrician observes the reserve price (r), the number of potential bidders (N), and the number of actual bidders (n), as well as all of the submitted bids. Then, $F(c)$ is identified on region $[\underline{c}, c_\rho]$, and the entry cost (κ) is also identified.*

Two remarks are in order. First, the inverse bidding function (3.7) is derived conditional on N , which is part of the information set of the common knowledge at bidding. This observation also motivates my empirical analysis conditional on N . Second, there is another complication in the application. r is known by the bidders, but is not observed by the econometrician. However, the identification results in proposition 3.1 remain valid as long as a consistent estimator of r is available.

⁹It is the positive expected gain that implies $b'(c_\rho) \neq 0$, which further implies $\lim_{b \uparrow r} g^*(b) \neq +\infty$ in the S model. It is different from the result in Theorem 4 (C2) in Guerre, Perrigne, and Vuong (2000). The subtle difference is due to the boundary conditions at equilibrium bidding. If endogenous entry is incurred by the reserve price, as in Guerre, Perrigne, and Vuong (2000), then the marginal bidder with cost r will bid just r to make a zero expected gain. I am grateful to one of the referees for pointing this out.

3.2 Estimation method

Consider L homogeneous auctions with the same number of potential bidders N , from which the data are observed. n_l is the number of bidders who actually submit a bid in the l th auction, where $l = 1, 2, \dots, L$. I define the sample of observed bids as B , where $B := \{b_{il} : i \in \{1, \dots, n_l\}, l \in \{1, \dots, L\}\}$. The total sample size is denoted by T . $B_{(t)}^T$ denotes the t -th largest order statistic in sample B ; therefore, the order statistics are in a sequence of decreasing order in this paper.

The following assumption on the data-generating process is imposed in accordance with the estimation situation.

Assumption 3.1 (i) *The observed bids are a realization of an independently and identically distributed (i.i.d.) univariate stochastic process, $\{B_t\}_{t \in \mathbb{N}}$.*

(ii) *The probability distribution of B_1 has a support bounded from above and a probability density function (pdf) g^* that is positive and left-continuous at r , the upper bound of the support of B_1 .*

Equation (3.8) in the identification results provides an expression for entry cost κ , which leads to a natural and convenient estimation method. To this end, we first evaluate equation (3.7) at r ,

$$\xi(r) = r - \frac{1-p}{(N-1)pg^*(r)}, \quad (3.9)$$

where the fact of $G^*(r) = 1$ has been used. After substituting equation (3.9) into equation (3.8), some elementary algebra enables me to write κ as

$$\kappa = \frac{M}{g^*(r)}, \quad \text{where} \quad M := \frac{(1-p)^N}{p(N-1)}. \quad (3.10)$$

The rate of participation, p , can be consistently estimated by the sample analogue $\hat{p} := \frac{1}{L} \sum_{l=1}^L (\frac{n_l}{N})$, which implies that M can be estimated by $\hat{M} = \frac{(1-\hat{p})^{N-1}}{\hat{p}(N-1)}$. Then, a plug-in estimator for participation costs κ can be defined as

$$\hat{\kappa} = \frac{\hat{M}}{\hat{g}^*(\hat{r})} \quad (3.11)$$

if $\hat{g}^*(\hat{r})$, a consistent estimator of $g^*(r)$, is available. As the reserve price is not observed by the econometrician, $\hat{g}^*(\hat{r})$ entails estimating the truncated density function at its unknown truncation point. Two separate issues are involved: (i) estimation of the pdf at a given point of truncation and (ii) estimation of the truncation point. These two issues are considered separately.

First, to estimate $g^*(r)$, I propose using the nearest neighbor estimation technique, which is usually referred to as the k -NN estimator in the density estimation literature. k is the number of closest observations taken from the sample to estimate the density at the point of interest. Due to the boundary issue, the data observations are only from one side of the estimation point. To accommodate this feature of our estimation problem, I slightly modify the NN estimator. The resulting estimator is the one-sided NN estimator \hat{g}^* at r :

$$\hat{g}^*(r) = \frac{k_T/T}{r - B_{(k_T)}^T}, \quad (3.12)$$

where subscript T is added to k to stress its dependence on sample size T . The NN estimators are simply the empirical measure divided by the Lebesgue measure over the interval $[B_{(k_T)}^T, r]$. The one-sided NN estimator differs from the regular NN estimator in the denominator, where the regular NN estimator implicitly requires symmetry on the defining interval for the distance measure.

Several restrictions are imposed on k_T to ensure the asymptotics of our proposed estimator.

Assumption 3.2 *Let $\{k_T\}_{T \in \mathbb{N}}$ be a sequence of positive integers satisfying $k_T/T \rightarrow 0$ and $k_T \rightarrow \infty$, as $T \rightarrow \infty$.*

Assumption 3.3 *g^* is left differentiable at r . Moreover, $k_T = o(T^{2/3})$.*

Assumption 3.2 imposes restrictions on the divergence rate of k_T . The weak consistency of $\hat{g}^*(r)$ requires only Assumption 3.2, which is satisfied, for instance, if $k_T = T^\alpha$ for some $0 < \alpha < 1$. The consistency property is intuitive. Recall that the NN estimator is defined as the ratio of an empirical measure over a Lebesgue measure on a set. When such a set shrinks towards zero, the ratio converges to the first-order derivative of a cumulative distribution function at the point, which the set shrinks to. Therefore, the consistency of the NN estimator follows. The conditions in Assumption 3.3 together imply a connection between the local behavior of g^* and k_T , which is necessary for the $\hat{g}^*(r)$ to have the asymptotic normality property. Recall that $g^* = \frac{f(c)}{b'(c)F(c_\rho)}$. Then, the differentiability of g^* at r implicitly asks for the local smoothness of f .¹⁰ The NN estimator per se employs two sample quantiles, which in turn define an event in terms of order statistics. By using the standard device of restating such an event in terms of a binomial random variable, the asymptotic normality result can be established.

¹⁰Note that the smoothness of the bidding equilibrium strategy also relies on the smooth feature of f .

I next consider the issue of estimating the unknown truncation point. A natural candidate to estimate the bound is the extreme order statistic in the sample.¹¹ I denote the estimator of r as $\hat{r} = B_{(1)}^T$. One may readily show that \hat{r} converges to r with rate T under Assumption 3.1. (See, for example, Hong [1998].) The fast convergence rate implies that the asymptotic behavior of $\hat{g}^*(\hat{r})$ is dominated by $\hat{g}^*(r)$.

The one-sided NN estimator and \hat{r} can now be combined to estimate $g^*(r)$, i.e., $\hat{g}^*(\hat{r}) = \frac{k_T/T}{B_{(1)}^T - B_{(k_T)}^T}$. The following propositions demonstrate that the combined estimator is consistent and has a normal distribution in the limit. Let \rightarrow_d denote the convergence in distribution.

Proposition 3.2 *Suppose that Assumptions 3.1 and 3.2 hold. Then, $\hat{g}^*(\hat{r}) \rightarrow g^*(r)$, as $T \rightarrow \infty$ in probability. If, in addition, Assumption 3.3 holds, then*

$$(k_T)^{\frac{1}{2}} \{\hat{g}^*(\hat{r}) - g^*(r)\} \rightarrow_d N[0, g^*(r)^2].$$

Recall that the plug-in estimator for participation costs κ is defined in (3.11). \hat{p} is an empirical cumulative distribution function evaluated at cutoff point c_ρ . This estimator converges at rate $T^{-1/2}$, which is faster than any other nonparametric estimators. Moreover, κ is a continuous function with respect to both p and $g^*(r)$. The following asymptotic properties of $\hat{\kappa}$ are derived by applying the Slutsky theorem and the delta method to the previous propositions.

Proposition 3.3 *Suppose that Assumptions 3.1 and 3.2 hold. $\hat{\kappa}$ converges to κ in probability, as $T \rightarrow \infty$. Furthermore, if Assumption 3.3 also holds, then*

$$(k_T)^{\frac{1}{2}}(\hat{\kappa} - \kappa) \rightarrow_d \mathbf{N}[0, \frac{M^2}{(g^*(r))^2}]. \quad (3.13)$$

3.3 On the nearest neighbor estimation method

The NN method can be extended to a more general form by admitting weighting functions, i.e., kernel functions in the bandwidth estimators can also be applied to the NN estimates. The asymptotic properties of the NN estimator are unaffected even if the kernel weighting functions are employed. (See Moore and Yackel [1976, 1977] and Stone [1977] among others.) For ease of exposition, this paper sticks to the uniform kernel weighting functions, as in (3.12). However, all of the results established herein remain valid when using other kernel weighting functions.

¹¹Theoretically, there is an alternative way to estimate the upper bound (r). For auctions with only one potential bidder, the equilibrium bidding strategy is to bid r . However, I do not observe such auctions in the data used here.

The NN method can also be extended to estimate a conditional density function. For example, suppose the econometrician observes an *i.i.d.* bivariate stochastic process $\{B_t, Z_t\}_{t \in \mathbb{N}}$ and, conditional on Z_t , B_t is continuously distributed with g^* . First consider that Z_t follows a discrete distribution and that z is one of the values. Let the observations corresponding to $Z_t = z$ be denoted as T^* . Then, an estimator of conditional density g^* at boundary r can be defined as

$$\hat{g}^*(r|Z = z) := \frac{k_{T^*}/T^*}{r - \widetilde{B}_{(k_{T^*})}^{T^*}},$$

where $\widetilde{B} = \{B_t : Z_t = z \forall t\}$. This argument reveals that estimating the conditional density requires forming the subsample such that the condition is satisfied. Moving to the case where Z_t is a continuous random variable, the exact value z will occur in the data with probability zero. Then, a kernel smoothing technique can be applied to redefine the estimator. Here, however, I do not consider conditional density out of data concerns. Practically speaking, the gain from using the kernel smoothing technique in the estimation is rather small due to the data's limited sample size.

There is an extensive body of literature in the statistics field on density estimation at the boundaries. Most of them focus on how to manipulate the weighting scheme to improve the performance of the bandwidth estimators *near* the boundaries. Of the existing proposals, the boundary-kernel methods (Jones, 1993; Müller, 1991) involve only kernel modifications and make no attempt to estimate the population density value in correcting the bias. Therefore, these methods are always associated with a large degree of variance. Pseudodata methods (Cowling and Hall, 1996) require knowledge of the boundary location. Moreover, implementing these proposed methods is generally not an easy task. Although some of the existing literature focuses intensively on the convergence property, the asymptotic distribution issue is left relatively unexplored. To the best of my knowledge, the only exception is Hall and Park (2002), who propose using the translation bootstrap method to correct the bias at the boundaries. The limiting distribution of their method is found only to be of zero mean. The NN method, in contrast, attempts to adapt the amount of smoothing to the local density of the data observations. The adaptive nature of the NN method enables the proposed NN estimator to outperform other methods in terms of the mean squared error in my simulation study. The proposed estimator is easy to implement and has been shown to follow a normal distribution in the limit, which allows the possibility of constructing confidence intervals and carrying out testing in applications. These features motivate my choice of the NN method over other candidates.

I conduct a simulation experiment to study the finite sample properties of the proposed one-

sided NN estimator. The details of my simulation methods and the results are reported in the Supplement. There, I first examine how the coverage of the estimator responds to changes in the smoothing parameter. I then compare the one-sided NN estimator with other boundary correction methods proposed in the literature. An important question here is how well they compare to my suggested one-sided NN estimator. An ideal approach to this issue would be careful analysis of all earlier proposals and a detailed comparison of their properties. However, this would be a tedious task, as Cheng et al. (1997) noted. They encountered a similar situation when proposing their local polynomial fitting estimator. I select some well-known, representative methods to compare with the one-sided NN estimator: the pseudodata method, the boundary kernel method, and the local linear fitting method. The one-sided estimator is found to outperform the other methods in terms of MSE.

It is worth emphasizing here that the proposed one-sided NN estimation method is just one of the solutions to the problem of density estimation at boundaries. It is its normal limiting distribution and reasonably good finite sample performance that prompted me to make this choice. Comparison with some of the kernel-type bias correction estimators is made because of the strong attention they have elicited in the literature. However, the issues of whether the one-sided NN estimator is always the best choice and whether incorporating other kernel smoothing techniques could further improve the MSE of the one-sided NN estimator are beyond the scope of this work and are therefore left open for future research.

4 Empirical Application

In this section, I apply the estimator of κ defined in (3.11) to the Michigan highway procurement auction data. I first introduce the dataset. The discussion of data issues focuses on the application's relevance to the S model, which in turn provides a grounding to form a subsample for estimation. I then estimate the participation costs for the auctions and present the estimates of the costs in terms of both absolute value (in dollars) and relative value (to project size). Finally, I approximate the levels of the optimal cutoffs in the construction costs, with which I empirically address the questions regarding implementation of the optimal auctions.

4.1 Data

This paper employs data on the highway procurement auctions held by the Michigan Department of Transportation (MDOT) between January 2001 and December 2002. This dataset comprises 1,538

projects. For each project, the dataset includes the letting date, the expected completion date, the location, the tasks involved, the identities of all of the bidders, all bids, the engineer’s estimate of the construction costs of completing the project, and a list of the plan-holders for all projects in the dataset.

The letting proceeds as follows. The MDoT announces a project to be let and invites the submission of bids. The length of this advertising period ranges from four to ten weeks. The announcement of a project comes with a brief description of the project, including the location, completion time and the engineer’s estimate. These estimates are based on the engineer’s assessment of the work needed to fulfill the task and information extracted from similar projects let previously. Qualified firms that are interested in the project may collect a detailed bid proposal from the MDoT, and then become plan-holders. Based on its proposal, a plan-holder may submit a bid in a sealed envelope, which has to be 48 hours prior to the letting date. For each bid, the MDoT checks that the bidding firm is among those qualified to do business with it. Then, on the letting date, the project is awarded to the lowest bidder.

The S model requires that the number of potential bidders be common knowledge among the participants in the bidding. In practice, the MDoT maintains an industry-specific list of qualified firms. It grants qualifications to firms annually based on their specialty, experience, equipment data, and financial statements. Then, at the time bids are solicited for each project, the MDoT also publicly announces the list of plan-holders, i.e., the bidders who acquired bidding proposals before the letting date. In the application in this paper, the plan-holder list appears the more relevant. Therefore, a potential bidder is defined as a qualified firm that requested an official bidding proposal before bidding begins.¹²

It is worth emphasizing again that when deciding to submit a bid the bidders already know their own private construction costs for the project in the S model. Potential bidders may have to exert some effort to learn their costs upon collection of the bid proposals. This paper, in contrast, explores the impact of the cognitive efforts a firm devotes to the bid preparation and submission processes, which include the time costs involved in compiling bids and checking for errors and the monetary costs involved in labor and traveling to the letting meetings.

In this study, I consider only a symmetric bidding environment. In the application, the main

¹²This way of defining potential bidders can also be found in other empirical works on highway procurement auctions. See, for example, Li and Zheng (2009), Krasnokutskaya and Seim (2010), and Shneyerov, Marmer, and Xu (2010).

sources of heterogeneity among the bidders in the market are size and location.¹³ Location reflects the bidder’s costs to move equipment, materials, and labor to the work site. Firm size entails the scale of the economy. In the data analysis, I consider projects involving only non-fringe bidders and those from either Michigan or neighboring states. A qualified firm is considered to be non-fringe if it participated more than once in the sample period.

The next concern is the private-cost bidding framework, which motivates my choice of project types to work with. Intuitively, the nature of contracts and worktypes may or may not support the private-cost assumption. Using a more general framework, Hong and Shum (2002) find that they cannot reject the assumption of private values for paving- and grading-type jobs in New Jersey construction procurement auctions. Therefore, I form a subsample of auctions that involve only paving or grading jobs.¹⁴ Consequently, a controlled subsample is composed of observations only from auctions for paving- and grading-type projects and involving only non-fringe firms from Michigan or neighboring states.

There is another complication in the empirical analysis, that is, the absence of publicly announced reserve prices in the application. However, the *finiteness* of the equilibrium bidding strategy relies on the existence of such a bound as common knowledge in first-price procurement auctions. To see this more clearly, consider any bidder. If she knows that there is no binding reserve price, and that there is a nonzero probability of being the single actual bidder, no matter how small this probability is, her optimal strategy is always to bid infinity. To rationalize bidders’ behavior for a finite equilibrium bidding strategy, I assume that all auction participants have a common belief that the government has maximum-willingness-to-pay, which can be understood as the cost for which the government could have procured the good from a third party. A winning bid above this value will be rejected. This line of justification effectively amounts to assuming a reserve price. Moreover, this maximum-willingness-to-pay is not observed by econometricians.¹⁵

¹³Firm specialization may also matter in terms of bidder heterogeneity. However, there is not enough information in the dataset to handle this issue.

¹⁴The entire sample has up to 30 different type codes. The specific classification codes used in the subsample are “B, Ba, E, and Ea.” See MDOT document 1313(09/05), which is available on the MDoT website.

¹⁵An alternative way to address this concern is to consider a model with a random unobserved reservation price. See, for example, Li and Perrigne (2003). This would greatly complicate the identification issue and the derivation of the equilibrium bidding strategy, however, and I therefore leave it for future research.

4.2 Estimates of participation costs

The identification and estimation methods are conditional on the number of potential bidders, as it is common knowledge in the equilibrium entry and bidding strategies. Therefore, I group the auctions by the number of potential bidders in implementing my estimator. For example, Group 5 represents the auctions with five potential bidders. In the dataset, I observe that the number of potential bidders ranges from two to 22. There is no auction in the dataset with 19 potential bidders. When there are only two potential bidders, the participation rate is one, and thus the entry cost cannot be estimated with my approach. I also eliminate auctions with 13 to 22 potential bidders from my analysis, because they offer either fewer than 100 observed bids or fewer than 40 auctions.¹⁶ Based on these observations, I choose Groups 3-12 for the next steps. Column 3 in Table 2 reports the rates of participation, which decrease monotonically from 0.8 to 0.5 for the chosen groups.

In assumption (3.1), the observed bids are assumed to be generated from homogeneous auctions within each group. Of all of the covariates, the engineer's estimate represents the size of the project, which is clearly the most important variable. By itself, it explains 98.38% of the bid variations. (See Column 2 in Table 1.) Therefore, in the spirit of Bajari, Hong, and Ryan (2010), I normalize all of the observed bids by dividing the engineer's estimates. In a structural framework, this way of normalizing bids is equivalent to normalizing the private costs. To see this more clearly, I denote the observed (raw) bids as \tilde{b} . Moreover, \tilde{G} and \tilde{g} are their cumulative distribution and density functions, respectively. The inverse bidding function (3.7) implies that

$$\tilde{c} = \tilde{b} - \frac{1 - p + p(1 - \tilde{G}(\tilde{b}))}{(N - 1)p\tilde{g}(\tilde{b})}. \quad (4.14)$$

Normalizing both sides of (4.14) by Eng , the engineer's estimates, we obtain

$$\frac{\tilde{c}}{Eng} = \frac{\tilde{b}}{Eng} - \frac{1 - p + p(1 - \tilde{G}(\tilde{b}))}{(N - 1)p\tilde{g}(\tilde{b}) \cdot Eng}. \quad (4.15)$$

At the risk of abusing the notations, let B^* denote the normalized bids, and G^* , g^* their cumulative distribution and density functions. Then, (4.15) can be rewritten as

$$c^* \equiv \frac{\tilde{c}}{Eng} = B^* - \frac{1 - p + p(1 - G^*(B^*))}{(N - 1)pg^*(B^*)}, \quad (4.16)$$

where I apply the change of variable to the denominator and the fact that $G^*(B^*) = Pr(B \leq B^*) = Pr(b \leq B^* \cdot Eng) = Pr(b \leq \tilde{b}) = \tilde{G}(\tilde{b})$.

¹⁶The number of auctions used in the sample matters because of the estimated participation rate for each group.

I next consider whether other covariates on the auction heterogeneities may have significant impacts on the normalized bids. In particular, I regress the normalized bids on the only two observed auction characteristics, that is, whether the job is grading (dummy) and the number of days it takes to complete the project. Regressions are conducted for each group. Columns 3-6 of Table 1 report the regression outputs for the selected groups, from which it can be seen that the number of days plays no significant role. However, there are mixed messages concerning the dummy variable of job type. For most of the groups, the coefficients for this variable are statistically insignificant. Among the groups in which the job type matters, the impact can be positive or negative. Most importantly, the observed auction heterogeneities do not account for significant variation in the normalized bids. Therefore, I decide to keep the analysis simple and use *Eng* alone to normalize the bids.¹⁷ This practice is also consistent with recent empirical works on highway procurement auctions. For example, the structural estimates in Li and Zheng (2009) suggest that the engineer's estimate is the only significant variable.

The estimation results on the participation costs are listed in the fourth column of Table 2. These estimated participation costs vary from 2% to .001% of the engineer's estimates. The fifth and sixth columns of Table 2 report the inference bounds at the 1% significance level of the estimates. The results show that the participation costs are significantly different from zero at any reasonable significance level for all groups. In absolute terms, I investigate the levels of participation costs for the average-sized project in each group. The numbers are listed in the last column of Table 2, from which it can be seen that the costs in dollar value decrease dramatically over the range. The costs can be as high as \$12,225 when $N = 3$, then drop down to \$3,143 when $N = 5$. For the groups that $N > 8$, the costs fluctuate between a few hundred.

The declining pattern of entry costs in the groups demands further discussion. One may suspect that firm heterogeneity largely drives the observed pattern. For example, large firms with lower entry costs may tend to participate in auctions with large groups. As it is more costly for small firms to enter auctions, they instead tend to remain only in small groups. In other words, bidder symmetry is less likely to hold in large group auctions.

To address this concern, I take a closer look at bidders' participation choices. For each group, I examine the other auctions in which the bidders participated. More specifically, I focus on the variation in the group number (the number of potential bidders) in these other auctions. I compute the average and the range of the group of auctions the bidders also participated in. The results

¹⁷This does not imply that the unobserved auction heterogeneity may not be present in these data.

are reported in Table 3, which shows that the bidders get involved only in the neighboring groups. There is no evidence in the data of participation in a large range of groups.

Another issue of concern is that arising from the economies of scale, that is, whether the firms joining the large groups tend to participate in more auctions. The last column of Table 3 shows that the number of auctions participated by bidders is roughly the same across the groups.

4.3 Counterfactual experiments: implementing optimal auctions

Is there anything that a government can do to improve its situation when it knows the level of participation costs? Following the previously discussed theoretical framework, I address this question empirically by calculating the optimal level of cutoffs in private costs. This, in turn, suggests the policy tools (entry fee/subsidy or reserve price) a government should adopt for optimal auction outcomes. As a counterfactual experiment, I compute the savings a government could have made from optimal auctions of the given project.

Recall that the optimal solution to the government's problem (2.4) entails the choice of cutoff in the cost for an optimal symmetric auction, c^* , which solves equation (2.5):

$$[r - J(c^*)][1 - F(c^*)]^{N-1} = \kappa.$$

The value of c , recovered through (3.7), is used to create a pseudo sample of costs. I then estimate F by an empirical cumulative distribution function and f by a kernel-bandwidth estimator, respectively. Following the empirical auction literature, I employ a tri-weight function for the kernel and $2.978 \times 1.06 \times T^{-1/5} \times \sigma$ for the bandwidth, where T is the sample size and σ is the standard deviation; 2.978 is required because of the tri-weight kernel.¹⁸ As with the estimation section, I employ the maximum bids in the sample as the estimates for r . With these estimates, I then numerically solve (2.5) from the pseudo sample and derive an estimate for c^* . The estimates are reported in the second column of Table 4. For all of the auctions in the sample, the optimal cutoff values vary between 0.9 and 1.1.

Proposition 2.1 suggests that, in the S model, entry discouragement should be in place for optimal auction outcomes. Effectively, this optimal policy requires that the cutoff in the symmetric bidding equilibrium, c_ρ , be greater than that in an optimal auction, c^* . This theoretical implication provides a device to check the reliability of the computed cutoffs. I report the maximum recovered costs (after trimming the near boundaries) in the third column of Table 4, which clearly shows that

¹⁸Examples can be found in Guerre, Perrigne, and Vuong (2000) and Li, Perrigne, and Vuong (2000).

$c^* < c_\rho$. This result may be more striking than it appears. The observed data are drawn from the truncated density rather than the full range of the distribution, which causes one reasonably to expect that, in a nonparametric framework, the predictive power of the counterfactuals that econometricians may conduct is limited. This work thus demonstrates that, even in the case of truncated density, such reliable counterfactuals are possible. This is because the optimal policy of restricting entry uses information from the observed range on the truncated distribution that enables me to do so.

I next investigate how to implement optimal auctions by using only the entry fee (δ^*). In such a context, participation in optimal auctions becomes costly due to two components: the implicit cost of entry (κ), and the explicit payment of an entry fee. In the literature, $\kappa + \delta^*$ is called the effective participation costs.

To implement an optimal auction, the policy parameter, δ^* , should be chosen such that the following equilibrium entry condition holds.

$$(r - c^*)[1 - F(c^*)]^{N-1} = \kappa + \delta^*. \quad (4.17)$$

The optimal entry fee δ^* that should be charged is listed in the fourth column of Table 4. The δ^* 's are all positive, which entails further charges for participating in the auctions in the sample. The amount of the fee decreases with an increase in the number of potential bidders.

At the end, I calculate the expected payment (EP) made by the government using the following equation.

$$EP(c^*) = N \int_{\underline{c}}^{c^*} J(c)[1 - F(c)]^{N-1} f(c) dv + \kappa NF(c^*). \quad (4.18)$$

Both the calculated expected payment and the actual payment in current auctions are provided in Table 4. The difference between the two payments indicates the amount the government would have saved if it had implemented optimal auctions. The results show that, for all of the auctions in the sample, the government would have paid less if optimal auctions had been implemented. For some of the auctions, the improvement could have been up to 10-15%.

It is worth noting that implementing optimal auctions in the application effectively requires the work of the optimal cutoff c^* to be in action. Recall that r in the government's problem (2.4) can only be interpreted as maximum-willingness-to-pay. Therefore, the c^* that solves the first-order condition (2.5) is fixed. Suppose that, together with the entry fee δ^* , a reserve price r^* can also be used as a policy tool. Any pair of policy tools can entail a way of implementing the optimal

auction, as long as that pair meets the following equilibrium entry condition.

$$[r^* - c^*][1 - F(c^*)]^{N-1} = \kappa + \delta^*.$$

Given any reserve price at consideration, the foregoing equation pins down the optimal entry fee to charge for optimal auction outcomes.

U.S. federal law requires that the winning bid be no greater than 110% of the engineer’s estimates. However, a state is still allowed to let a project with a price higher than this threshold if it can justify its actions in writing. In practice, a reserve price based on the engineer’s estimates can be set. However, consultations with professionals in the procurement business reveal the infeasibility of a government implementing a binding reserve price. Bidders may use a group of factors to argue for inaccuracies in the engineer’s estimates. Therefore, considering a reserve price seems to lack practical sense in procurement auctions. I thus leave this empirical exercise for practitioners out of necessity.

5 Conclusion

In this paper, I propose using the combination of the one-sided nearest neighbor estimator with the extreme order statistics in the sample to estimate the truncated univariate probability density functions at their unknown truncation points. In practice, the proposed method is applied to estimate the entry costs incurred by bidders in joining Michigan highway procurement auctions. The empirical results suggest that the entry costs are statistically significant from zero. Based on this estimate, I then infer how optimal auction outcomes can be implemented using regular policy tools. It shows explicitly the degree of improvement the Michigan government could have achieved on payments if optimal entry fees had been enforced.

The proposed estimation method is general and easy to implement, although there are also important limitations that could be addressed in future research. In auction datasets, one typically finds that the variation in bids can be only partially explained by the variation within auctions. Between-auction variation may also be present. In the framework of independent values, this pattern can be explained by unobserved heterogeneity (c.f., Krasnokutskaya, 2003). Extending the proposed method to allow for unobserved heterogeneity would be an interesting direction for future research work.

In terms of its theoretical framework, this paper employs the S model to capture the bidders’ decision-making process on entry. However, I believe that full observability of construction costs

prior to bidding is a strong assumption. Conversations with contractors in the industry indicate that, depending on the situation, negotiations with subcontractors and suppliers can be settled late. Therefore, the S model may be unable to describe contractors' entry decisions precisely. Recently, a new direction in the literature has been to consider the partial observability of construction costs at entry, which may be closer to the reality of procurement auctions. See, for example, Marmer, Shneyerov, and Xu (2010) and Roberts and Sweeting (2010). Whether the optimal policy of restricting entry explored in this paper still applies to the more general framework is left for future research.

Another possible extension would be to allow bidder asymmetries. The obvious difficulty here would be the necessity of dealing with multiple equilibria. However, with regard to optimal auctions, Celik and Yilankaya (2009) recently investigated the existence in theory of equilibria with asymmetric entry and bidding strategies. More rigorous empirical examination of the implementability of such equilibria would be interesting to pursue. Finally, incorporating dynamic features, as in Jofre-Bonet and Pesendorfer (2003), may also be worth exploring in future.

References

- Bajari P, Hong H, Ryan S. 2010. Identification and Estimation of a Discrete Game of Complete Information. *Econometrica* **78**(5): 1529-1568.
- Celik G, Yilankaya O. 2009. Optimal Auctions with Simultaneous and Costly Participation. *The B.E. Journal of Theoretical Economics* **9**(1): Article 24.
- Cheng M, Fan J, Marron J. 1997. On Automatic Boundary Corrections. *Annals of Statistics* **25**: 1691-1708.
- Cowling A, Hall P. 1996. On Pseudodata Methods for Removing Boundary Effects in Kernel Density Estimation. *Journal of the Royal Statistical Society. Series B.* **58**: 551-563.
- Guerre E, Perrigne I, Vuong Q. 2000. Optimal Nonparametric Estimation of First-Price Auctions. *Econometrica* **68**(3): 525-574.
- Hall P, Park B. 2002. New Methods for Bias Correction at Endpoints and Boundaries. *Annals of Statistics* **30**: 1460-1479.

- Hong H. 1998. Econometric Models of Asymmetric Ascending Auctions. Stanford University Dissertation.
- Hong H, Shum M. 2002. Increasing Competition and the Winner's Curse: Evidence from Procurement. *Review of Economic Studies* **69**(4): 871-898.
- Jofre-Bonet M, Pesendorfer M. 2003. Estimation of a Dynamic Auction Game. *Econometrica* **71**(5): 1443-1489.
- Jones M. 1993. Simple Boundary Correction for Kernel Density Estimation. *Statistics and Computing* **3**: 135-146.
- Krasnokutskaya E. (forthcoming). Identification and Estimation in Procurement Auctions under Unobserved Auction Heterogeneity. *Review of Economic Studies*.
- Krasnokutskaya E, Seim K. (forthcoming). Bid Preference Programs and Participation in Highway Procurement Auctions. *American Economic Review*.
- Krishna V. 2002. *Auction Theory*. Academic Press.
- Levin D, Smith JL. 1994. Equilibrium in Auctions with Entry. *American Economic Review* **84**(3): 585-599.
- Li T, Perrigne I. 2003. Timber Sale Auctions With Random Reserve Prices. *Review of Economics and Statistics* **85**(1): 189-200.
- Li T, Perrigne I, Vuong Q. 2000. Conditionally Independent Private Information in OCS Wildcat Auctions. *Journal of Econometrics* **98**: 129-161.
- Li T, Zheng X. 2009. Entry and Competition Effects in First-Price Auctions: Theory and Evidence from Procurement Auctions. *Review of Economic Studies* **76**(4): 1397-1429.
- Marmer V, Shneyerov A, Xu P. 2010. What Model for Entry in First-Price Auctions? A Non-parametric Approach. University of British Columbia Working Paper.
- Milgrom P. 2004. *Putting Auction Theory to Work*. Cambridge University Press.
- Moore D, Yackel J. 1976. Large Sample Properties of Nearest Neighbor Density Function Estimators. *Statistical Decision Theory and Related Topics*: 269-279.

- Moore D, Yackel J. 1977. Consistency Properties of Nearest Neighbor Density Function Estimators. *Annals of Statistics* **5**: 143-154.
- Müller H. 1991. Smooth Optimum Kernel Estimators Near Endpoints. *Biometrika* **78**: 521-530.
- Myerson R. 1981. Optimal Auction Design. *Mathematics of Operations Research* **6**: 58-73.
- Roberts JW, Sweeting A. 2010. Entry and Selection in Auctions. Duke University Working Paper.
- Samuelson W. 1985. Competitive Bidding with Entry Costs. *Economics Letters* **17**(1-2): 53-57.
- Silverman B. 1986. *Density Estimation for Statistics and Data Analysis*. Chapman and Hall.
- Stone C. 1977. Consistent Nonparametric Regression. *Annals of Statistics* **5**: 595-620.
- Wilks S. 1962. *Mathematical Statistics*. Wiley.
- Zhang S, Karunamuni R, Jones M. 1999. An Improved Estimator of the Density Function at the Boundary. *Journal of the American Statistical Association* **94**: 1231-1241.

A Proofs

A.1 Proof of Proposition 2.1

For any symmetric optimal auction, the optimal cutoff c^* should solve the following.

$$\left[r - c^* - \frac{F(c^*)}{f(c^*)} \right] [1 - F(c^*)]^{N-1} = \kappa. \quad (\text{A.19})$$

As shown in the text, equilibrium bidding should entail the cut-off, c_ρ , to solve the following.

$$(r - c_\rho)[1 - F(c_\rho)]^{N-1} = \kappa. \quad (\text{A.20})$$

For the desired result, I need to show that $c^* < c_\rho$. In negation, I suppose that $c^* > c_\rho$. Then, it must be the case that $[1 - F(c^*)]^{N-1} < [1 - F(c_\rho)]^{N-1}$. (A.19) and (A.20) together suggest that $c^* + \frac{F(c^*)}{f(c^*)} < c_\rho$. This further implies that $c^* < c_\rho$, which contradicts the supposition. ■

A.2 Proof of Proposition 3.2

I first establish the result on weak consistency. It is noted that

$$|\hat{g}^*(\hat{r}) - g^*(r)| \leq |\hat{g}^*(\hat{r}) - \hat{g}^*(r)| + |\hat{g}^*(r) - g^*(r)|. \quad (\text{A.21})$$

For the desired result to hold, it suffices to show that both terms on the right-hand side of (A.21) converge to zero in probability under Assumptions 3.1 and 3.2. To this end, I rewrite the first term on the right-hand side of (A.21):

$$\hat{g}^*(\hat{r}) - \hat{g}^*(r) = \frac{k_T/T}{B_{(1)}^T - B_{(k_T)}^T} - \frac{k_T/T}{r - B_{(k_T)}^T} = \frac{k_T/T}{(\ddot{B} - B_{(k_T)}^T)^2} (r - B_{(1)}^T),$$

where $B_{(1)}^T < \ddot{B} < r$, and the second equality follows from the mean value theorem. Furthermore, it can be shown that

$$\hat{g}^*(\hat{r}) - \hat{g}^*(r) \leq \frac{k_T/T}{(B_{(1)}^T - B_{(k_T)}^T)^2} (r - B_{(1)}^T) = \hat{g}^*(\hat{r})^2 \frac{r - B_{(1)}^T}{k_T/T} = O_p\left(\frac{T^{-1}}{k_T/T}\right) = O_p(k_T^{-1}),$$

where the second-to-last equality follows from the fact that the sample's extreme order statistic converges to the bound of support at rate T . That is, $B_{(1)}^T - r = o_{a.s.}(T^{-1})$. See, for example, Hong (1998).

For the consistency result, it remains to show that $|\hat{g}^*(r) - g^*(r)|$ converges to zero in probability.

I follow the line of proof for the weak consistency of a regular NN estimate in the literature, but with two modifications: (i) this is a univariate case, and (ii) the proposed estimator is one-sided. I denote $S_e := \{B | 0 \leq r - B \leq e\}$. The measure of S_e is denoted d_e , which simply equals e in the univariate case. Therefore, the convergence of $\hat{g}^*(r)$ entails $g^*(r) = \lim_{e \rightarrow 0} P(S_e)/d_e$, i.e., for any arbitrary $\epsilon > 0$, there exists an E , such that if $e < E$, then

$$|P(S_e)/d_e - g^*(r)| < \epsilon. \quad (\text{A.22})$$

I further denote $e_{k_T} := r - B_{(k_T)}^T$. It is easy to show that $P(S_{e_{k_T}}) = U_{k_T}$ has a beta distribution with parameters k_T and $T - k_T + 1$ (Theorem 8.7.1, Wilks, 1962, p.236).

I first show that $U_{k_T}/d_{e_{k_T}} \rightarrow g^*(r)$ in probability. Application of the Tchebychev inequality yields $U_{k_T} \rightarrow 0$ in probability. However, this can only happen when $d_{e_{k_T}} \rightarrow 0$ in probability, i.e., $e_{k_T} \rightarrow 0$ in probability. Let E be as defined in (A.22). There exists a t such that for $T > t$ and any arbitrary $\eta > 0$,

$$P\{e_{k_T} < E\} > 1 - \eta.$$

This suffices to show that $U_{k_T}/d_{e_{k_T}} \rightarrow g^*(r)$ in probability.

For the desired result, it remains to show that $\{T/k_T\}U_{k_T} \rightarrow 1$ in probability, which can be done by using the fact that U_{k_T} has a beta distribution along with application of the Tchebychev inequality.

I next prove the asymptotic normality claim in the proposition. To this end, I first establish the result that $k_T^{1/2}[\hat{g}^*(r) - g^*(r)]$ follows a normal distribution with zero mean and variance $g^*(r)^2$ in the limit. For this purpose, note that the left differentiability of g^* at r allows one to write

$$k_T^{1/2}[g^*(B_t) - g^*(r)] = k_T^{1/2}[g^{*'}(r)(B_t - r) + o(|B_t - r|)] \text{ when } B_t - r = o(k_T/T).$$

Then, $k_T = o(T^{2/3})$ in Assumption 3.3 further implies that

$$k_T^{1/2}[g^*(B_t) - g^*(r)] \rightarrow 0. \quad (\text{A.23})$$

By the continuity of g^* at r , it can be written that

$$\hat{g}^*(r) = \frac{k_T/T}{U_{k_T}} g^*(B_t), \quad \text{where } r - B_t < e_{k_T}.$$

Moreover, it can be shown that $e_{k_T} \rightarrow 0$ and $\frac{k_T/T}{U_{k_T}} \rightarrow 1$ in probability. Hence,

$$\frac{k_T^{1/2}}{g^*(r)}(\hat{g}^*(r) - g^*(r)) = k_T^{1/2}\left(\frac{k_T/T}{U_{k_T}} - 1\right) + k_T^{1/2}\left(\frac{g^*(B_t)}{g^*(r)} - 1\right)\frac{k_T/T}{U_{k_T}}.$$

Therefore, (A.23) implies the desired result and we are required only to show that

$$k_T^{1/2} \left(\frac{k_T/T}{U_{k_T}} - 1 \right) \rightarrow \mathbf{N}(0, 1).$$

Because U_{k_T} is the k_T -th order statistic of T *i.i.d.* uniform random variables U_1, U_2, \dots, U_T on $[0, 1]$,

$$\begin{aligned} P_n(a) &= P[k_T^{1/2} \left(\frac{k_T/T}{U_{k_T}} - 1 \right) \leq a] \\ &= P[U_{k_T} \geq \frac{k_T/T}{1 + ak_T^{-1/2}}] \\ &= P[Q_T < k_T], \end{aligned}$$

where Q_T is the number of U_1, \dots, U_T falling below $\pi_T = \frac{U_{k_T}}{1 + ak_T^{-1/2}}$ and has a binomial distribution with (T, π_T) . By assumption, $\pi_T \rightarrow 0$ and $T\pi_T \rightarrow \infty$, such that Q_T is asymptotically normal.

Writing

$$P_T(a) = P\left[\frac{Q_T - T\pi_T}{\sigma_T} < \frac{k_T - T\pi_T}{\sigma_T}\right],$$

where $\sigma_T = [T\pi_T(1 - \pi_T)]^{1/2}$, and noting that $\frac{k_T - T\pi_T}{\sigma_T} \rightarrow a$, we obtain $P_T(a) \rightarrow g^*(a)$, with g^* being the standard normal distribution:

$$\hat{g}^*(\hat{r}) - g^*(r) = [\hat{g}^*(\hat{r}) - \hat{g}^*(r)] + [\hat{g}^*(r) - g^*(r)]. \quad (\text{A.24})$$

In the limit, the behavior of the terms in the first bracket on the right-hand side of (A.24) is dominated by the terms in the second bracket due to their different rates of convergence. The desired result therefore follows from the asymptotic equivalence lemma. This completes the proof. ■

B Tables

Table 1: OLS Regressions of Bids^a

Variable	Bid	Bid/Eng	Bid/Eng	Bid/Eng	Bid/Eng
Eng	1.024*				
	(0.002)				
Grading (Dummy)		-0.034	0.067*	-0.070*	-0.001
		(0.04)	(0.03)	(0.02)	(0.03)
Number of Days		-0.00009	-0.00008	-0.00006	0.00007
		(0.00005)	(0.00011)	(0.00005)	(0.00008)
Constant	-23,422*	1.04*	1.04*	1.05*	1.05*
	(10,191)	(0.01)	(0.02)	(0.01)	(0.02)
Group	All	N=5	N=7	N=9	N=12
R^2	0.9838	0.0085	0.0104	0.0366	0.0029

^a The numbers in parentheses are standard errors, and the asterisks indicate that the corresponding coefficients are statistically significant at the 5% level.

Table 2: Estimation Results on Entry Costs

Group	Number of Observations	Rate of Participation	κ (proportional to Eng)	Lower Bound 99% Interval	Upper Bound 99% Interval	κ (in dollars)
3	257	.79	0.0235	0.01395	0.03312	12225
4	354	.77	0.0153	0.00974	0.02090	12274
5	473	.73	0.0039	0.00264	0.00524	3143
6	503	.72	0.0015	0.00101	0.00198	2277
7	468	.68	0.0020	0.00136	0.00271	3189
8	410	.64	0.0007	0.00046	0.00095	1296
9	356	.62	0.0002	0.00010	0.00021	379
10	149	.51	0.0007	0.00034	0.00097	966
11	228	.53	0.0002	0.00011	0.00026	679
12	249	.52	0.0001	0.00006	0.00014	262

Table 3: Bidder Participation Choices

Group	Average Number of Groups Participating	Range of Groups Participating	Number of Auctions Participated in
3	3.97	3.61	5.67
4	5.12	4.11	4.82
5	5.61	3.13	4.16
6	6.43	3.07	3.90
7	7.12	3.24	3.76
8	8.13	4.31	4.04
9	8.91	3.67	3.49
10	9.51	3.66	3.26
11	10.46	4.06	3.29
12	11.61	3.53	3.02

Table 4: Implementing Optimal Auctions

Group	Optimal Cutoff	$\max(\hat{c})$	Optimal Entry Fee	Expected Payment	Actual Payment
3	0.931	0.951	0.0304	0.85	0.94
4	1.011	1.047	0.0133	0.89	0.95
5	1.022	1.069	0.0065	0.86	0.92
6	1.079	1.131	0.0021	0.87	0.93
7	1.026	1.088	0.0019	0.89	0.93
8	0.987	1.023	0.0012	0.81	0.88
9	1.035	1.088	0.0006	0.89	0.91
10	1.061	1.094	0.0019	0.82	0.95
11	1.071	1.143	0.0007	0.83	0.94
12	1.086	1.102	0.0003	0.91	0.93

Supplement to “Nonparametric Estimation of Entry Cost in First-Price Procurement Auctions”

Pai Xu^{*†}

University of Hong Kong

September 2010

This Version: May 2011

^{*}I would like to thank the Editor and three anonymous referees whose insightful comments have greatly improved the paper. I am also grateful to Shinichi Sakata, Michael Peters, Han Hong, Jacque Cremer, Thomas Lemieux, Vadim Marmer, Art Shneyerov, Okan Yilankaya, Brian Krauth, Unjy Song for useful comments, as well as participants at the 2006 Canadian Economics Association Annual Meeting, the 2007 Canadian Econometrics Study Group conference, and the empirical workshops at the University of British Columbia and the University of Hong Kong.

[†]Contact: Room 908, K.K.Leung Building, Pokfulam, Hong Kong. Tel: +852 28578501. Fax: +852 25481152. Email: paixu@hku.hk.

1 Simulation

I have conducted Monte Carlo simulations to assess the finite sample behavior of the proposed one-sided NN estimator in this supplement material. First, I illustrate the simulation method, and then remark on the simulation results. The results are reported in the tables followed.

Simulation Method In each experiment, I randomly drew n numbers from the $LogNormal(4, 1)$ distribution. Then, I chose q -th percentile as the cutoff value for the experiment, where q is 5, 10, or 25. The draws with their values greater than the cutoff are selected to form a sample for estimation. Each simulation was repeated 10,000 times.

Coverage I first check whether the constructed 90% and 95% confidence intervals provide right coverage. The results are reported in Table 1 followed. I experimented with three different choices of the smoothing parameter k , $T^{4/5}$, $T^{3/5}$ and $T^{1/2}$. $T^{4/5}$ is chosen because it is the MSE-best rate for regular (two-sided) NN estimators. $T^{3/5}$ and $T^{1/2}$ are chosen with a purpose to see how the coverage responds to the changes in the rates. I considered three sample sizes 200, 500, 1000 for the coverage study.

The results in Table 1 indicate that

- $T^{4/5}$, which violates Assumption 3.3, does not have right coverage.
- The smaller the smoothing parameter, as comparing $T^{3/5}$ with $T^{1/2}$, the better the coverage that the estimator provides.
- The slow rate of convergence, as the built-in feature of the non-parametric estimates, requires quite a large sample to provide better coverage. In the simulations, I need a sample size, at least, of 1000 for reasonable coverage.
- Under-rejection may be a concern in applications.

Other Boundary Correction Methods For the purpose of comparison, I selected some well-known and representative methods to compare with our one-sided NN estimators:

- The pseudodata method (hereafter, Pseudodata; see Cowling and Hall 1996);
- The boundary kernel method (hereafter, Kernel; see, for example, Jones 1993, Müller 1991 among others);

- The local linear fitting (hereafter, LLF; see Cheng et al. 1997).

In fairness, I chose the smoothing parameters for these boundary correction methods in the way such that they either are MSE-best, or make the resulting bias at similar level as one-sided NN estimates. Specifically, the rates of $T^{-1/5}$ are used for LLF and Pseudodata methods, and $T^{-1/3}$ is used for the boundary kernel method. Following the rule of thumb (Silverman (1986)), I use 1.06 for the constant in bandwidths estimators. The number of random draws (n) I take are 100, 200, 500. To satisfy the Assumption 3.3, I set smoothing parameters $k = T^{3/5}$ for the NN estimators throughout this subsection of the simulation study.

Simulation for density estimation Table 2 reports the results when DGP is assumed as $LogNormal(4, 1)$.

- (i) For NN estimates, consistent with the intuitions, the signs of bias are positive in any case. Moreover, MSE is decreasing in the sample size and increasing in the cutoff point. Over all, NN outperforms the other methods in the simulated cases.
- (ii) The bias of Pseudodata methods increase with sample size, which further indicates that it is an inconsistent estimator. The pseudodata method is regarded as an improvement from reflection methods, as it is claimed to be “considerably more adaptive” (Cowling and Hall, 1996). However, the generic pseudodata method requires the knowledge of truncation point, so that the sample order statistics can be used to interpolate the pseudodata. When the boundary point is not known, the method essentially extrapolates the pseudodata, which causes the inconsistency problem as I have just seen from the simulation.
- (iii) The kernel used in simulation is a one-sided kernel since I know exactly how to correct bias at the boundary. The simulation suggests that even though I can control bias to a desired level, the resulting variance is way too high compared to the NN estimator. This result is consistent with the findings in the literature – “Approaches involving only kernel modifications without regard to true density are always associated with larger variance.” (Zhang et. al., 1999)
The result reveals a real and practical phenomenon that the boundary kernel-related methods usually focus on getting the bias as one wants it with paying the price of increasing variance. It has gradually been realized by researchers that this variance inflation is important.
- (iv) LLF, as a special case of the boundary kernel method, is often thought of by some as a simple, hard-to-beat default approach. However, the simulation suggests that, it is not as good as

one-sided NN in terms of MSE.

References.

- Cheng M, Fan J, Marron J. 1997. On Automatic Boundary Corrections. *Annals of Statistics* **25**: 1691-1708.
- Cowling A, Hall P. 1996. On Pseudodata Methods for Removing Boundary Effects in Kernel Density Estimation. *Journal of the Royal Statistical Society. Series B.* **58**: 551-563.
- Jones M. 1993. Simple Boundary Correction for Kernel Density Estimation. *Statistics and Computing* **3**: 135-146.
- Müller H. 1991. Smooth Optimum kernel Estimators Near Endpoints. *Biometrika* **78**: 521-530.
- Silverman B. 1986. *Density Estimation for Statistics and Data Analysis*. Chapman and Hall.
- Zhang S, Karunamuni R, Jones M. 1999. An Improved Estimator of the Density Function at the Boundary. *Journal of the American Statistical Association* **94**: 1231-1241.

A Tables.

Table 1: Simulation Results on Coverage

		5%			10%			25%		
		200	500	1000	200	500	1000	200	500	1000
$T^{4/5}$	90% Interval	0.9027	0.8224	0.6964	0.9142	0.8664	0.8031	0.9205	0.8318	0.6954
	95% Interval	0.9906	0.9442	0.8617	0.9921	0.9616	0.911	0.9976	0.9554	0.8546
$T^{3/5}$	90% Interval	0.9995	0.9862	0.9311	0.9976	0.982	0.9247	0.9975	0.9821	0.8964
	95% Interval	0.9999	0.9998	0.9912	0.999	0.9979	0.9912	0.9985	0.9993	0.9902
$T^{1/2}$	90% Interval	0.9872	0.9514	0.9142	0.9852	0.9499	0.9195	0.9866	0.9276	0.8675
	95% Interval	0.9999	0.9998	0.9821	0.999	0.9983	0.9847	0.9998	0.999	0.9735

Table 2: Simulation Results, Lognormal(4,1), $k=T^{3/5}$

		5%			10%			25%		
		100	200	500	100	200	500	100	200	500
NN	RMSE	0.004693	0.003877	0.003034	0.004360	0.003447	0.002744	0.005496	0.004641	0.004038
	Bias	0.003705	0.003241	0.002660	0.002847	0.002427	0.002143	0.003690	0.00354	0.003429
	STD	0.002880	0.002128	0.001459	0.003302	0.002449	0.001714	0.004074	0.003003	0.002133
LLF	RMSE	0.031893	0.020386	0.011775	0.034401	0.022065	0.01329	0.041942	0.027865	0.017046
	Bias	0.027133	0.015935	0.008007	0.028858	0.016948	0.008892	0.035646	0.022264	0.012554
	STD	0.016761	0.012715	0.008634	0.018726	0.014128	0.009876	0.022101	0.016756	0.011531
Kernel	RMSE	0.053803	0.035848	0.021449	0.057082	0.038254	0.023641	0.067057	0.045892	0.0286
	Bias	0.048792	0.031056	0.017108	0.051156	0.032618	0.018302	0.060352	0.03946	0.023008
	STD	0.022674	0.017906	0.012938	0.025327	0.019985	0.014964	0.029228	0.023431	0.016989
Pseudodata	RMSE	0.012061	0.007062	0.004407	0.01274	0.007557	0.005117	0.016394	0.009978	0.005793
	Bias	0.008673	0.003074	-0.000889	0.008639	0.002684	-0.001343	0.01211	0.005419	0.000564
	STD	0.008381	0.006358	0.004317	0.009363	0.007064	0.004938	0.011050	0.008378	0.005766