# RESOLVING THE PHASE AMBIGUITIES FOR COHERENT DEMODULATION OF $\pi/4$ -DEQPSK

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In recovering the carrier signal for coherent demodulation of  $\pi/4$ -differentially encoded quaternary-phase-shift keying ( $\pi/4$ -DEQPSK), there is an inherent phase ambiguity which may significantly degrade the system performance. The paper proposes a simple and novel technique to resolve this problem, i.e., to rotate the received signal vectors alternately by  $-\pi/4$  before phase-error estimation in the carrier-recovery loop. The proposed technique incorporated with carrier-recovery loops is studied by computer simulation. A surprisingly good performance is achieved.

#### INTRODUCTION

 $\pi/4$ -differentially encoded quaternary-phase-shift keying ( $\pi/4$ -DEQPSK) is an attractive modulation scheme for digital satellite mobile communication systems because of its signal spectrum being the same as that of conventional QPSK and its reduced envelope fluctuation [1-3]. In  $\pi/4$ -DEQPSK, two signal constellations, each with four signal vectors as shown in Figs. 1a and 1b, are used alternately and form a resultant signal constellation with eight signal vectors as shown in Fig. 1c. To detect the data from the received signal at the receiver, the decision thresholds of the associated signal constellations are used.

In regenerating the carrier from the received signal, since  $\pi/4$ -DEQPSK has eight possible transmitted phases (Fig. 1c), there is an inherent phase ambiguity of eight possible angles that are multiples of  $\pi/4$ . The ambiguity occurs in the sense that the carrier-recovery loop may lock onto any one of the eight possible phases. If the phase is an even multiple of  $\pi/4$ , it can be eliminated by differential decoding. But, if the phase is an odd multiple of  $\pi/4$ , the wrong set of decision thresholds will be used for data detection, resulting in significant degradations in system performance. Of course, for non-coherent detection, as is used in the digital land mobile systems in the US, this is not a problem. However, the penalty for using non-coherent detection is about 2.3 dB degradation in tolerance to noise which is undesirable in satellite systems.

A simple and novel technique is proposed here to resolve the ambiguity, i.e. to rotate every other received signal vectors by an angle of  $-\pi/4$  before phase-error estimation in the carrier-recovery loop. The technique incorporated with a feedforward loop and a feedback

loop is studied by computer simulation. Results have shown that, when the technique is incorporated with a feedforward loop, coherent demodulation of  $\pi/4$ -DEOPSK can be achieved at the receiver.

#### MODEL OF SYSTEM

The model of the system considered here is shown in Fig. 2. The information to be transmitted is carried by the binary digits  $\{s_i\}$ , where  $s_i = 0$  or 1. The transmitted symbol, at the time instance t = iT seconds, is related to the phase change  $\theta_i$  by [1]

$$\begin{aligned} \mathbf{q}_{0,i} &= \mathbf{q}_{0,i-1} \cos \theta_i - \mathbf{q}_{1,i-1} \sin \theta_i \\ \mathbf{q}_{1,i} &= \mathbf{q}_{0,i-1} \sin \theta_i + \mathbf{q}_{1,i-1} \cos \theta_i \end{aligned} \qquad 1.a$$

The relationship between  $\theta_i$  and the input data is given

in Table 1 which indicates that the transmitted signal is, in fact, differentially encoded. Each premodulation filter, in Fig. 2a, has an impulse response p(t), so that the signals at the outputs of the inphase and quadrature lowpass filters are  $\Sigma q_{0,i}p(t-iT)$  and  $\Sigma q_{1,i}p(t-iT)$  respectively, with a symbol rate of 1/T bauds. These signals are used to form the corresponding  $\pi/4$ -DEQPSK signal element at the output of the modulator. At the receiver, the demodulated signal from the postdemodulation filters is sampled at the correct time instances {iT}, once per symbol. The sampled signals are then detected and decoded to give the

corresponding detected binary digits {s,}. In the

absence of errors,  $s_i = s_i$  for all  $\{i\}$ .

# CARRIER-RECOVERY LOOP

### Feedforward Loop

The  $\pi/4$ -DEQSPK receiver employing a feedforward loop is shown in Fig. 3a. The received signal vector  $\mathbf{x}_1$  +  $\mathbf{j}\mathbf{y}_1$ , at the time instance  $\mathbf{t} = \mathbf{i}\mathbf{T}$  seconds, is fed to a phase-error estimator [4] which estimates the phase error through three steps. The first step extracts the signal phase

$$\phi_{i} = \tan^{-1} y_{i} / x_{i}$$
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The second step multiplies the phase value by 8 to remove the modulation and then converts it back to a rectangular form

$$x_{i}' + jy_{i}' = \cos 8\phi_{i} + j\sin 8\phi_{i}$$

The final step averages the received signal vectors over 2N+1 symbol intervals, converts the resulting average to a phase angle, and then divides it by 8 to produce an unbiased estimate of the phase error [4]

$$\varepsilon_{i} = \frac{1}{8} \tan^{-1} \left[ \sum_{i=-N}^{N} y_{i-1}^{i} / \sum_{i=-N}^{N} x_{i-1}^{i} \right]$$

For  $\pi/4$ -DEQPSK, since the value  $\varepsilon_i$  should lie in the range  $-\pi/8 \le \varepsilon_i \le \pi/8$ , if  $|\varepsilon_i| > \pi/8$ , the phase error  $\varepsilon_i$  is estimated as  $\varepsilon_i \pm \pi/8$ . Thus the phase-error estimator produces the phase error

$$\dot{\varepsilon_{i}} = \begin{cases}
\varepsilon_{i}, & \text{for } |\varepsilon_{i}| < \frac{\pi}{8} \\
\varepsilon_{i} + \frac{\pi}{8}, & \text{for } \varepsilon_{i} < -\frac{\pi}{8} \\
\varepsilon_{i} - \frac{\pi}{8}, & \text{for } \varepsilon_{i} > \frac{\pi}{8}
\end{cases}$$

which always lies within the range  $-\pi/8 \le \varepsilon_1^{'} \le \pi/8$ . It is then used to correct the phase of the signal vector received at the time instance t = -NT seconds (Fig. 3a).

# Feedback Loop

The receiver employing a feedback loop is shown in Fig. 3b, which is similar to the one in the feedforward loop. The only difference is that the phase error  $\varepsilon_i$  here is used to control the phase of the carrier signal from the digital-control oscillator (DCO). This is a First-Order loop and the multiplication constant  $\beta$  determines the loop bandwidth.

# RESOLVING THE AMBIGUITY

From Eqns. 1 and Table 1, it can be seen that the transmitted  $\pi/4$ -DEQPSK signal phase is shifted by an odd multiple of  $\pi/4$  every symbol time. Thus the signal has eight possible transmitted phases, depending upon the present phase shift  $\theta$  and the previous transmitted phase. The constellation of the signal has eight signal vectors as shown in Fig. 1c which is formed by two signal constellations, as shown in Figs. 1a and 1b, being used alternately.

For coherent demodulation of the signal at the receiver, the carrier is regenerated from the received signal. Since the signal has eight possible transmitted phases, a conventional carrier-recovery loop such as the Costas loop or 8th power loop, etc., will have 8 ambiguities in the regenerated carrier signal [5], i.e. a constant phase error of  $0, \pm \pi/2, \pm 3\pi/4, \pm \pi/2$ , or  $\pi$  is introduced to the demodulated signal. If the phase error is  $\pm \pi/2$  or  $\pi$ , it is not a problem because the correct signal constellation (hence the associated decision thresholds) will still be used to detect the received signal. The ambiguity is resolved by differential decoding. However, if the phase error is  $\pm \pi/4$  or  $\pm 3\pi/4$ , then the wrong signal constellation (hence the wrong decision thresholds) will be used for data detection. When these happen, the system performance is significantly degraded.

The technique proposed here to resolve this problem is to rotate every other received signal vectors by a fixed phase of  $-\pi/4$ . Consider the signal constellations shown in Figs. 1a and 1b. If one is rotated by a phase of  $-\pi/4$ , it becomes the other one. Since these two signal constellations are used alternately for transmission, at the receiver, if every other received signal vectors are rotated by a fixed phase of  $-\pi/4$ , then effectively one of the two transmitted constellations is rotated by  $-\pi/4$ , resulting in a single constellation with 4 signal vectors. The resultant signal constellation could be either one of them shown in Figs. 1a and 1b, depending upon which one of the transmitted signal constellation is rotated by  $-\pi/4$ . The carrier-recovery loop can make use of this phase-error resultant signal constellation for estimation. To produce the phase error  $\varepsilon$ , now, the phase estimator uses the similar operations described in Eqns. 2, 3, 4 and 5, but takes into account that the received signal vector has only 4 possible phase values. The phase error  $\varepsilon_i$  lies within the range  $-\pi/4 \le \varepsilon_1 \le \pi/4$  and so the loop will lock onto only four possible phase positions which can be resolved by differential decoding.

## RESULTS AND DISCUSSIONS

Computer simulation tests have been carried out on the system of Fig. 2, using the feedforward and feedback loops of Fig. 3, with and without the phase rotation of  $\tau/4$ . In all tests, the premodulation filter and postdemodulation filter in cascade has a square-root Nyquist rolloff shape of 50% and introduces a delay of 2T in data detection at the receiver [6]. The value of N = 4 has been used for phase-error estimation through out the tests..

The performance of the feedforward loop, with different constant phase errors is shown in Fig. 4. The  $-\pi/4$  phase rotation is not used here. It can be seen that the symbol error rate (SER) performance degrades with the magnitude of the added phase error. This is because as

the phase error increases, the estimated phase error  $\varepsilon_{\rm i}$  is closer to the decision thresholds of  $\pm \pi/8$  and so, in the presence of noise, the accuracy of correct estimation reduces.

Tests on the feedback loop, also without the  $-\pi/4$  phase rotation, have been carried. The results have shown that the loop locks onto one of the eight possible phase positions, 0,  $\pm \pi/4$ ,  $\pm 3\pi/4$ ,  $\pm 5\pi/4$ , and  $\pm \pi$ . This is expected because  $\pi/4$ -DEQPSK has the same signal constellation as that of an 8PSK signal and so the conventional carrier-recovery loops always lock onto one of the 8 possible phase positions [5]. Of course, 4 of the 8 phase ambiguities can be resolved by differential encoding/decoding, but the other 4 would cause severe degradation to the system performance.

When a phase rotation of  $-\pi/4$  is used with the feedforward loop of Fig. 3a, the results are shown in Fig. 5. Comparing these results with those shown in Fig. 4, it can be seen that, for a given phase error less than  $\pm \pi/4$ , a significant improvement can be obtained by the  $-\pi/4$ -phase rotation. This is because now a QPSK signal constellation is used and so the distance between the thresholds has been increased from  $\pm \pi/4$  to  $\pm \pi/2$ , resulting in a better phase error estimation. Unfortunately, as the phase error approaches  $\pm \pi/4$ , the performance is still degraded severely.

Finally, the performance of the feedback loop (Fig. 3b), with the use of the  $-\pi/4$ -phase rotation technique, is shown in Fig. 6. A comparison of these results with those of Fig. 5 shows that significant improvements have been obtained by the  $-\pi/4$ -phase rotation. With  $\beta$  = 0.07, the degradation in performance is less than 0.5 dB at the bit-error rate of  $10^{-3}$ . Since the loop tracks the carrier of the received signal and removes the phase offset, the performance is independent of the value of the phase error. The transient response of the loop to a phase error with different values of  $\beta$  and  $E_b/N_0$  = 8 dB

is shown in Fig. 7. It can be seen that the loop acquires lock at one of 4 possible phase positions, as is expected. With  $\beta=0.07,$  the loop acquires lock within 20-symbol intervals.

# CONCLUSIONS

A simple and novel technique has been proposed to resolve the phase ambiguities in recovering the carrier signal for coherent demodulation of  $\pi/4$ -DEQPSK. The results of the studies have shown that when the proposed technique is incorporated with the feedback loop, coherent demodulation of  $\pi/4$ -DEQPSK can be achieved at the receiver.

#### REFERENCES

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TABLE 1 The phase shift as a function of input data

Data	θ
11	$-3\pi/4$
01	$3\pi/4$
00	π/4
10	-π/4

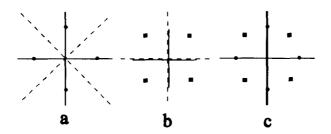
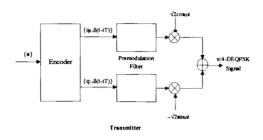


Fig. 1. Signal Constellation. - - -: Threshold decision boundaries



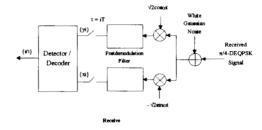


Fig.2. Model of System

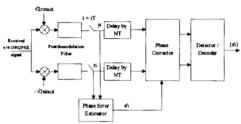


Fig. 3a. Receiver employing the feedforward loop

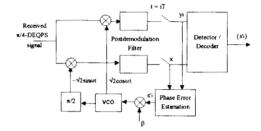


Fig. 3b. Receiver employing the feedback loop

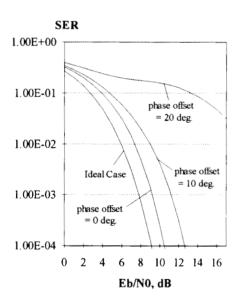


Fig. 4. SER performance of the feedforward system without phase rotation

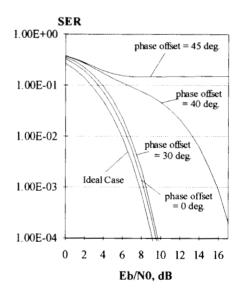


Fig. 5. SER performance of the feedforward system with phase rotation of  $-\pi/4$ 

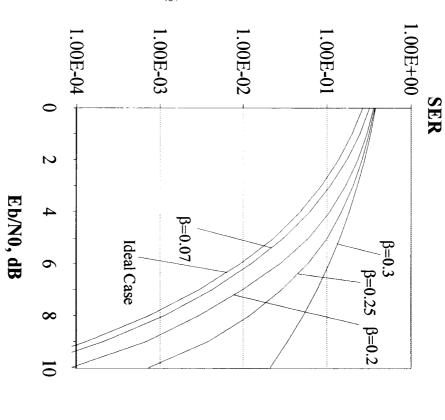
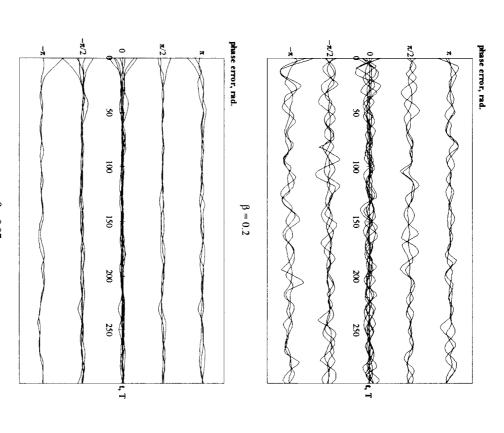


Fig. 6. SER performance of the feedback system with the phase rotation of  $-\pi/4$ 



 $\beta = 0.07 \label{eq:beta}$  Fig. 7. Transient response of the feedback loop