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Stability and Dissipativity Analysis of Static Neural Networks with Time Delay
Zheng-Guang Wu, James Lam, Senior Member, IEEE, Hongye Su, and Jian Chu

Abstract—This paper is concerned with the problems of stability and dissipativity analysis for static neural networks (NNs) with time delay. Some improved delay-dependent stability criteria are established for static NNs with time-varying or time-invariant delay using the delay partitioning technique. Based on these criteria, several delay-dependent sufficient conditions are given to guarantee the dissipativity of static NNs with time delay. All the given results in this paper are not only dependent upon the time delay but also upon the number of delay partitions. Some examples are given to illustrate the effectiveness and reduced conservatism of the proposed results.

Index Terms—Dissipativity, stability, static neural networks, time delay.

I. INTRODUCTION

In recent years, there has been increasing research interest on neural networks (NNs) for their successful application in different areas, such as pattern recognition, associate memory, and combinatorial optimization [1]. According to whether the neuron states or local field states of the neurons are chosen as basic variables to describe the evolution rule of an NN, NNs can be classified as static NNs or local field NNs [2]. The two models can be transferred equivalently from one to the other under some assumptions. However, in many applications, these assumptions cannot always be satisfied [3]. Thus, it is necessary and important to study them separately. The detailed relationship between static NNs and local field NNs can be found in [1], [2], and [4].

Recently, much attention has been paid to this paper on the stability problem of time-delay systems, because time delays are inherent features of many physical processes such as chemical reactions, nuclear reactors, and biological systems, and may lead to instability or significantly deteriorated performances for the corresponding closed-loop systems ([5]–[19] and references therein). It should be pointed out that time-delay NNs have been widely applied in many areas, such as signal and image processing, industrial automation, system identification, and artificial intelligence. Many important theoretical results have been obtained for the stability problem of all kinds of local field NNs with time delay ([20]–[32] and references therein). In contrast, the corresponding results for static NNs with time delay are relatively few.

It should be pointed out that the neurons of static NNs form layered network configurations through only feedforward interlayered synaptic connections in terms of the neural signal flow. In general, an individual neuron aggregates its weighted inputs and yields an output through a nonlinear activation function with a threshold. As a tool for scientific computing and engineering application, an obvious characteristic of static NNs is its capability for implementing a nonlinear mapping from many neural inputs to many neural outputs [1]. The static NN model plays an important role in many types of problems, for example, the linear variational inequality problem that contains linear and convex quadratic programming problems and linear complementary problems as special cases. The stability problem for static NNs with a constant delay has been investigated in [3], and a sufficient condition is proposed for ascertaining the global asymptotic stability of the unique equilibrium of the NNs based on the linear matrix inequality (LMI) approach. However, the condition proposed in [3] is delay-independent and thus appears to be somewhat conservative, especially when the time delay is comparatively small. Based on the delay partitioning approach and Finsler’s Lemma, some delay-dependent stability criteria have been established in [33] to guarantee the global asymptotic stability of static NNs with a constant delay. It should be pointed out that the idea of delay partitioning was independently proposed in [34] and [35], and has been widely applied to obtain the less conservative delay-dependent results for various kinds of time-delay systems, [24], [27], [36], [37]. For time-varying delay, the stability problem has been considered for static NNs in [38] and [39], where both delay-independent and delay-dependent criteria were obtained in terms of LMIs. However, the range of time-varying delays considered in [38] and [39] is from zero to an upper bound. In practice, a time-varying interval delay is often encountered, that is, the range of delay varies in an interval for which the lower bound is not restricted to zero. In this case, the stability criteria in [38] and [39] are conservative because they do not take into account the information of the lower bound of the delay. In [40], the stability problem for static NNs with time-varying interval delay has been studied, and a new type of delay-range-dependent condition was proposed using the free-weighting

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matrix technique to obtain a tighter upper bound on the derivative of the Lyapunov–Krasovskii functional, which has less conservatism than those of [38] and [39]. The condition of [40] can be viewed as an extension of existing result on local field NNs with time delay proposed in [6] to static NNs with time delay. It is worth pointing out that the free-weighting matrix technique was originally proposed in [41] and further developed in [6], [7], and [42]. However, there still exists room for further improvement because some useful terms are ignored in the Lyapunov–Krasovskii functional employed in [40], which may lead to conservatism to some extent. Thus, it is important and necessary to further study the stability of static NNs with time delay, which is the motivation for this paper.

On the other hand, the theory of dissipative systems plays an important role in system and control areas, and thus has been attracting a great deal of attention. It has been shown that the dissipative theory gives a framework for the design and analysis of control systems using an input–output description based on energy-related considerations [43]. It serves as a powerful or even indispensable tool in characterizing important system behaviors such as stability and passivity, and has close connections with passivity theorem, the bounded real lemma, and the circle criterion. With connections with passivity theorem, the bounded real lemma, it is worth pointing out that the free-weighting matrix method to obtain a tighter upper bound on the delay-dependent stability and dissipativity of the addressed NNs.

In this paper, the problems of stability and dissipativity analysis are investigated for static NNs with time delay using the delay partitioning technique. First, several improved delay-dependent stability criteria are established for static NNs with time-varying or time-invariant delay. Then, several delay-dependent sufficient conditions are given to guarantee the dissipativity of these networks. All the results given in this paper are delay-dependent as well as partition-dependent. The effectiveness as well as the reduced conservatism of the derived results is demonstrated by several illustrative examples.

**Notation:** The notations used throughout this paper are fairly standard. \( \mathbb{R}^n \) denotes the \( n \)-dimensional Euclidean space, and \( \mathbb{R}^{m \times n} \) is the set of all \( m \times n \) real matrices. The notation \( X > Y \) \((X \geq Y)\), where \( X \) and \( Y \) are symmetric matrices, means that \( X - Y \) is positive definite (positive semidefinite). \( I \) and \( 0 \) represent the identity matrix and a zero matrix, respectively. The superscript \( \cdot^T \) represents the transpose, diag\{\cdots\} stands for a block-diagonal matrix, and \( L_{2}[0, \infty) \) stands for the space of square integrable functions on \([0, \infty)\). For an arbitrary matrix \( B \) and two symmetric matrices \( A \) and \( C \)

\[
\begin{bmatrix}
A & B \\
* & C
\end{bmatrix}
\]

denotes a symmetric matrix, where \( \cdot^* \) denotes the term that is induced by symmetry. Matrix dimensions, if not explicitly stated, are assumed to be compatible for algebraic operations.

**II. Preliminaries**

Consider the following static NN with time delay:

\[
\begin{cases}
\dot{z}(t) = -Az(t) + f(Wz(t - d(t)) + J) \\
z(t) = \phi(t), t \in [-\max(d(t)), 0]
\end{cases}
\]  

(1)

where \( A = \text{diag}(a_1, a_2, \ldots, a_n) \geq 0 \), \( W = [\hat{W}_1^T \hat{W}_2^T \cdots \hat{W}_n^T] \) is the delayed connection weight matrix, \( J = [J_1 J_2 \cdots J_n]^T \) represents the external inputs, \( z(t) = [z_1(t) \ z_2(t) \cdots z_n(t)]^T \) is the state vector associated with the \( n \) neurons, \( f(Wz(t)) = [f_1(W_1z(t)) f_2(W_2z(t)) \cdots f_n(W_nz(t))]^T \) is the activation function of neurons, \( \phi(t) \) is the initial condition, and \( d(t) \) is the time delay and satisfies the following three cases:

C1) time-varying delay: \( 0 < d_1 \leq d(t) \leq d_2, \ d(t) \leq \mu \); 
C2) time-varying delay: \( 0 < d_1 \leq d(t) \leq d_2 \); 
C3) time-invariant delay: \( d(t) \equiv d > 0 \); 

where \( d_1, d_2, \mu, \) and \( d \) are known real constants, and \( d_2 > d_1 \).

It is assumed that each activation function \( f_i(\cdot) \) in (1) is bounded and satisfies

\[
0 \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq l_i, \ s_1 \neq s_2 \in \mathbb{R}
\]  

(2)

where \( l_i > 0 \) are known real constants. This assumption guarantees that there is an equilibrium point \( u^* \) of the NN (1). Let \( x(t) = z(t) - u^* \), \( g(Wx(t)) = f(W(x(t) + u^*) + J) -
\[ f(Wu^* + J), \text{ and } \varphi(t) = \varphi(t) - u^*. \text{ Then NN (1) can be expressed as} \]
\[ \begin{align*}
\dot{x}(t) &= -Ax(t) + g(Wx(t - d(t))) \\
x(t) &= \varphi(t), \quad t \in [-\max[d(t)], 0].
\end{align*} \tag{3} \]

It can be shown that the activation function \( g_i(\cdot) \) is bounded and satisfies
\[ 0 \leq \frac{g_i(s)}{s} \leq l_i, \quad g_i(0) = 0. \tag{4} \]

**Remark 1:** The NN (3) is called a static NN. It is interesting that, when the delayed connection weight matrix \( W \) is nonsingular and \( WA = AW \), by defining \( \hat{y}(t) = Wx(t) \), NN (3) can be transformed into
\[ \dot{\hat{y}}(t) = -A\hat{y}(t) + Wg(\hat{y}(t - d(t))) \]
which is called a local field NN, which has been extensively studied in the literature. However, most static NNs do not satisfy the transformation condition. It is thus necessary and important to study such NNs.

One aim in this paper is to investigate the stability of the static NN (3) and establish some new delay-dependent stability criteria that are better than the existing ones.

When an external disturbance appears in the NN (3), we have the following NN:
\[ \begin{align*}
\dot{x}(t) &= -Ax(t) + g(Wx(t - d(t))) + \omega(t) \\
y(t) &= g(Wx(t)) \\
x(t) &= \varphi(t), \quad t \in [-\max[d(t)], 0]
\end{align*} \tag{5} \]

where \( y(t) \) is the output of the NN, and \( \omega(t) \) is the disturbance input which cannot be fully measured and is not completely known beforehand. In this paper, it is assumed that \( \omega(t) \in L_2[0, \infty) \), which implies that it is a function of finite energy.

We are now in a position to introduce the concept of dissipativity. Let the energy supply function of the NN (5) be defined by
\[ G(\omega, y, \tau) = \langle y, Qy \rangle_t + 2\langle y, S\omega \rangle_t + \langle \omega, R\omega \rangle_t, \quad \forall \tau \geq 0 \tag{6} \]
where \( Q, S, \text{ and } R \) are real matrices with \( Q, R \) symmetric, and \( \langle (a, b) \rangle_t = \int_0^t a^Tb \, dt \). Without loss of generality, it is assumed that \( Q \leq 0 \) and that \( -Q = Q^T \). For some \( Q \).

**Definition 1:** NN (5) is said to be strictly \((Q, S, R)\)-\( \gamma \)-dissipative if, for some scalar \( \gamma > 0 \), the inequality
\[ G(\omega, y, \tau) \geq \gamma \langle \omega, \omega \rangle_t, \quad \forall \tau \geq 0 \tag{7} \]
holds under zero initial condition for any nonzero disturbance \( \omega \in L_2[0, \infty) \).

Another aim of this paper is to establish delay-dependent condition such that the NN (5) is globally asymptotically stable and strictly \((Q, S, R)\)-\( \gamma \)-dissipative.

Before moving on, the following results are required.

**Lemma 1 (Jensen Inequality) [13], [48]:** For any matrix \( W > 0 \), scalars \( \gamma_1 \) and \( \gamma_2 \) satisfying \( \gamma_2 > \gamma_1 \), a vector function \( \omega: [\gamma_1, \gamma_2] \to \mathbb{R}^n \) such that the integrations \[ \int_{\gamma_1}^{\gamma_2} \omega(\alpha)^T W \omega(\alpha) \, d\alpha \text{ and } \int_{\gamma_1}^{\gamma_2} \omega(\alpha) \, d\alpha \text{ are well defined, then} \]
\[ \gamma_2 - \gamma_1 \int_{\gamma_1}^{\gamma_2} \omega(\alpha)^T W \omega(\alpha) \, d\alpha \leq \int_{\gamma_1}^{\gamma_2} \omega(\alpha)^T W \omega(\alpha) \, d\alpha \tag{8} \]

\[ (y_2 - y_1) \int_{\gamma_1}^{\gamma_2} \omega(\alpha)^T W \omega(\alpha) \, d\alpha \]

**Remark 2:** Lemma 2 is a special case of [49, Th. 1], which is presented in a form more convenient for the present application.

### III. Stability Analysis

In this section, the delay partitioning technique will be developed to investigate the stability problem for the NN (3). For convenience of presentation, we denote

\[ \eta_1(t) = \begin{bmatrix} x(t) \\ x(t - \frac{1}{m} d_1) \\ \vdots \\ x(t - \frac{m-1}{m} d_1) \end{bmatrix}, \quad \eta_2(t) = \begin{bmatrix} g(Wx(t)) \\ g(Wx(t - \frac{1}{m} d_1)) \\ \vdots \\ g(Wx(t - \frac{m-1}{m} d_1)) \end{bmatrix} \]

\[ \theta_1(t) = \begin{bmatrix} \eta_1(t) \\ x(t - d_1) \end{bmatrix}, \quad \theta_2(t) = \begin{bmatrix} \eta_2(t) \\ g(Wx(t - d_1)) \end{bmatrix} \]

\[ L = \text{diag}(1, 1, \ldots, 1), \quad W_1 = \begin{bmatrix} I_{nm} & 0 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0_{mn \times n} & I_{nm} \end{bmatrix} \]

\[ e_i = I_{n \times (m+i-1)n} I_{n} 0_{n \times (m+i-2)n}, \quad i = 1, 2, \ldots, m + 1 \]

and \[ \hat{\eta}_1(t) = \eta_1(t)|_{d_1 = \hat{d}}, \quad \hat{\eta}_2(t) = \eta_2(t)|_{d_1 = \hat{d}}, \quad \hat{\theta}_1(t) = \theta_1(t)|_{d_1 = \hat{d}}, \quad \hat{\theta}_2(t) = \theta_2(t)|_{d_1 = \hat{d}}, \text{ and } \hat{d} = d_2 - d_1. \]

**Theorem 1:** Given an integer \( m > 0 \), the NN (3) with C1 is globally asymptotically stable if there exist matrices \( P > 0 \), \( Z_i > 0 \) (\( i = 1, 2, \ldots, m \)), \( \begin{bmatrix} Q_1 & Z_1 \end{bmatrix} > 0, \quad \begin{bmatrix} Q_2 & Z_2 \end{bmatrix} > 0, \)
\[ \begin{bmatrix} Y_1 U_1 & Y_2 \\ * & Y_3 \end{bmatrix} > 0, \quad \begin{bmatrix} Z_{m+1} & U_3 \end{bmatrix} \geq 0, \text{ and diagonal matrices} \]
where

\[
\Xi = \begin{bmatrix}
\Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & \Xi_{15} & 0 \\
* & \Xi_{22} & \Xi_{23} & 0 & \Xi_{25} & 0 \\
* & * & \Xi_{33} & 0 & 0 & \Xi_{36} \\
* & * & * & \Xi_{44} & \Xi_{45} & 0 \\
* & * & * & * & \Xi_{55} & 0 \\
* & * & * & * & * & \Xi_{66}
\end{bmatrix} < 0 \quad (10)
\]

Proof: Construct the following Lyapunov–Krasovskii functional candidate for NN (3):

\[
V(x(t)) = \sum_{i=1}^{6} V_i(x(t))
\]

where

\[
V_1(x(t)) = x^T(t)Px(t) + 2 \sum_{i=1}^{n} s_i \int_{0}^{t} \tilde{w}_i(x(s)) g_i(s) \, ds \\
+ 2 \sum_{i=1}^{n} r_i \int_{0}^{t} \tilde{w}_i(x(s)) (l_i s - g_i(s)) \, ds
\]

\[
V_2(x(t)) = \int_{t - \frac{d_1}{m}}^{t} \left[ \eta_1(s) \right]^T \begin{bmatrix} Q_1 & V \end{bmatrix} \eta_2(s) \, ds
\]

\[
V_3(x(t)) = \frac{d_1}{m} \sum_{i=1}^{m} \int_{t - \frac{i}{m}d_1}^{t} \dot{x}(s)^T Z_i \dot{x}(s) \, ds \, da
\]

\[
V_4(x(t)) = \int_{t - d_1}^{t} \left[ x(s) \right]^T \begin{bmatrix} Y_1 U_1 & * \end{bmatrix} \begin{bmatrix} g(W(x(s))) \end{bmatrix} \, ds
\]

\[
V_5(x(t)) = \int_{t - d_2}^{t} \left[ x(s) \right]^T \begin{bmatrix} Y_3 U_2 & * \end{bmatrix} \begin{bmatrix} g(W(x(s))) \end{bmatrix} \, ds
\]

\[
V_6(x(t)) = \tilde{d} \int_{t - d_1}^{t} \dot{x}(s)^T Z_{m+1} \dot{x}(s) ds \, da.
\]

Evaluating the derivative of \( V(x(t)) \) along the solution of NN (3), we obtain

\[
\dot{V}_1(x(t)) = 2x(t)^T P \dot{x}(t) + 2g(W(x(t)))^T SWg(x(t))
+ 2(LWx(t) - g(W(x(t))))^T RWx(t)
= \left( \theta(t)^T e_1^T P(-Ae_1 \theta(t)) + g(W(x(t) - d(t))) \right)
+ 2 \theta(t)^T e_1^T SW(-Ae_1 \theta(t)) + g(W(x(t) - d(t)))
+ 2(LWe_1 \theta(t) - e_1 \theta_2(t))^T RW
\]

\[
\times (-Ae_1 \theta(t) + g(W(x(t) - d(t))))
= -2 \theta(t)^T e_1^T P Ae_1 \theta_1(t) - 2 \theta_2(t)^T e_1^T SW Ae_1 \theta_1(t)
+ 2 \theta_1(t)^T e_1^T P g(W(x(t) - d(t)))
+ 2 \theta_2(t)^T e_1^T SW g(W(x(t) - d(t)))
- 2 \theta_1(t)^T e_1^T SW g(W(x(t) - d(t)))
- 2 \theta_2(t)^T e_1^T SW g(W(x(t) - d(t)))
\]

\[
\dot{V}_2(x(t)) = \left[ \eta_1(t) \right]^T \begin{bmatrix} Q_1 & V \end{bmatrix} \eta_2(t)
- \left[ \eta_1 \left( t - \frac{d_1}{m} \right) \right]^T \begin{bmatrix} Q_1 & V \end{bmatrix} \eta_2 \left( t - \frac{d_1}{m} \right)
- \left[ \eta_1 \left( t - \frac{d_2}{m} \right) \right]^T \begin{bmatrix} Q_1 & V \end{bmatrix} \eta_2 \left( t - \frac{d_2}{m} \right)
\]

\[
= \begin{bmatrix} W_1 \dot{\theta}_1(t) & \dot{W}_1 \dot{\theta}_1(t) \\
W_1 \dot{\theta}_2(t) & \dot{W}_1 \dot{\theta}_2(t) \end{bmatrix}^T \begin{bmatrix} Q_1 & V \end{bmatrix} \begin{bmatrix} \dot{W}_1 \dot{\theta}_1(t) \\
W_1 \dot{\theta}_2(t) \end{bmatrix}
- \begin{bmatrix} W_2 \dot{\theta}_1(t) & \dot{W}_2 \dot{\theta}_1(t) \\
W_2 \dot{\theta}_2(t) & \dot{W}_2 \dot{\theta}_2(t) \end{bmatrix}^T \begin{bmatrix} Q_1 & V \end{bmatrix} \begin{bmatrix} \dot{W}_2 \dot{\theta}_1(t) \\
W_2 \dot{\theta}_2(t) \end{bmatrix}
\times \begin{bmatrix} \theta_1(t) \ \\
\theta_2(t) \end{bmatrix}
\]

\[
\dot{V}_3(x(t)) = \left( \frac{d_1}{m} \right)^2 \dot{x}(t)^T \left( \sum_{i=1}^{m} Z_i \right) \dot{x}(t)
\]

\[
\leq \left( \frac{d_1}{m} \right)^2 \theta_1(t)^T e_1^T A \left( \sum_{i=1}^{m} Z_i \right) A_1 \theta_1(t)
\]
\[-2\left(\frac{d}{m}\right)^2 \theta(t) e^T_1 A \left(\sum_{i=1}^m Z_i\right) g(Wx(t - d(t))) + \left(\frac{d}{m}\right)^2 g(Wx(t - d(t)))^T \left(\sum_{i=1}^m Z_i\right) g(Wx(t - d(t))) \\
- \sum_{i=1}^m \int_{t-d_i}^{t-d_i} x(s)^T d(s) Z_i \int_{t-d_i}^{t-d_i} x(s) ds \]
\[= \left(\frac{d}{m}\right)^2 \theta(t) e^T_1 A \left(\sum_{i=1}^m Z_i\right) A e_1 \theta(t) + \left(\frac{d}{m}\right)^2 g(Wx(t - d(t)))^T \left(\sum_{i=1}^m Z_i\right) g(Wx(t - d(t))) \\
- \sum_{i=1}^m \theta(t) e^T_{i-1} e_i^T Z_i (e_i - e_{i+1}) \theta(t)\]

where Lemma 1 is applied

\[\hat{V}_4(x(t)) \leq \left[\begin{array}{c} x(t) \\ g(Wx(t)) \end{array}\right]^T \left[\begin{array}{cc} Y_1 & U_1 \\ * & Y_2 \end{array}\right] \left[\begin{array}{c} x(t) \\ g(Wx(t)) \end{array}\right] - (1 - \mu) \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right]^T \left[\begin{array}{cc} Y_1 & U_1 \\ * & Y_2 \end{array}\right] \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right] \\
- (1 - \mu) \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right]^T \left[\begin{array}{cc} Y_1 & U_1 \\ * & Y_2 \end{array}\right] \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right] \]
\[= \left[\theta_1(t)^T e^T_1 Y_1 e_1 \theta_1(t) \right] \left[\begin{array}{cc} Y_1 & U_1 \\ * & Y_2 \end{array}\right] \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right] \]
\[= \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right]^T \left[\begin{array}{cc} Y_1 & U_1 \\ * & Y_2 \end{array}\right] \left[\begin{array}{c} x(t - d(t)) \\ g(Wx(t - d(t)) \end{array}\right] \]

That is

\[\gamma_1 = 2 \sum_{i=1}^{m+1} \theta_2(t)^T e^T_i D_1 (LW e_i \theta_1(t) - e_i \theta_2(t)) \geq 0. \]

We can also get from (4)

\[\gamma_2 = 2g(Wx(t - d(t)))^T D_{m+2} \]
\[\times (LW x(t - d(t)) - g(Wx(t - d(t)))) \geq 0 \]

and

\[\gamma_3 = 2g(Wx(t - d(t)))^T D_{m+3} \]
\[\times (LW x(t - d(t)) - g(Wx(t - d(t)))) \geq 0. \]

Thus, we have from (12)–(17) and (19)–(21)

\[\hat{V}(x(t)) \leq \hat{V}_1(x(t)) + \hat{V}_2(x(t)) + \hat{V}_3(x(t)) + \hat{V}_4(x(t)) + \hat{V}_5(x(t)) \]
\[+ \hat{V}_3(x(t)) + \hat{V}_6(x(t)) \leq \rho(t)^T \tilde{Z} \rho(t) \]
where
\[
\rho(t) = \begin{bmatrix} \rho_1(t) & \rho_2(t) \end{bmatrix}^T,
\]
\[
\rho_1(t) = \begin{bmatrix} \theta_1(t) & 0 \end{bmatrix}^T x(t - d(t)) \begin{bmatrix} x(t - d_2) \end{bmatrix},
\]
\[
\rho_2(t) = \begin{bmatrix} \theta_2(t) & 0 \end{bmatrix}^T g(W x(t - d(t))) \begin{bmatrix} x(t - d_2) \end{bmatrix}.
\]
Clearly, if \( \Sigma < 0 \), \( \dot{V}(x(t)) < 0 \) holds, which implies that NN (3) is globally asymptotically stable. This completes the proof.

**Remark 3:** In terms of the delay partitioning technique, a new delay-range-dependent and delay-derivative-dependent sufficient condition is proposed in Theorem 1 for the global asymptotical stability of NN (3) with time-varying interval delay. When \( r_i = 0, V = Q_2 = 0, Q_1 = I_m \otimes Q_1, Z_i = (m/d_i) Z_1, U_i = Y_4 = U_2 = 0, \) and \( Z_{m+1} = d_{i-1}^{-1}(Z_1 + Z_2) \), the Lyapunov–Krasovskii functional candidate (11) reduces to that of [40]. Thus, our Lyapunov–Krasovskii functional candidate is much more general than that of [40], and Theorem 1 in this paper has less conservatism than [40, Th. 2]. Moreover, the conservatism reduction of Theorem 1 in this paper becomes more obvious with the partitioning getting thinner (i.e., \( m \) becoming larger), which will be demonstrated in Section V.

**Remark 4:** It is worth pointing out that, if we utilize the free-weighting matrix method together with the delay partitioning technique to deal with the delay-dependent stability problem of NN (3), a great many free-weighting matrices will be introduced with increasing number of partitions, which will make the obtained result rather complicated and consequently lead to computational burden [50]. But in this paper, we make use of integral inequalities (8) and (9) instead of the free-weighting matrix method. An obvious and important merit of such an approach is that only one matrix is introduced no matter how large the partitioning is, and thus we can achieve a condition with fewer decision variables.

As for C2, we can get the following stability criterion for NN (3) via the following Lyapunov–Krasovskii functional candidate:

\[
\dot{V}(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) + V_5(x(t)) + V_6(x(t))
\]

where \( V_1(x(t)), V_2(x(t)), V_3(x(t)), V_5(x(t)), \) and \( V_6(x(t)) \) follow the same definitions as those in (11).

**Theorem 2:** Given an integer \( m > 0 \), NN (3) with C2 is globally asymptotically stable if there exist matrices \( P > 0, Z_i > 0 (i = 1, 2, \ldots, m) \), \( Q_1 \otimes Q_2 > 0 \), \( Y_5 \otimes Y_4 > 0 \), \( Z_{m+1} \otimes Z_{m+1} > 0 \), and diagonal matrices \( S = \text{diag}\{s_1, s_2, \ldots, s_n\} > 0, R = \text{diag}\{r_1, r_2, \ldots, r_n\} > 0, D_i > 0 (i = 1, 2, \ldots, m + 3) \) such that (10) holds.

**Proof:** By using a similar method as employed in Theorem 1, we can easily obtain Theorem 2. This completes the proof.

**Remark 5:** Theorem 2 proposes a delay-range-dependent and delay-derivative-independent sufficient condition for the global asymptotical stability of NN (3) with time-varying interval delay. It should be pointed out that, when \( \mu \geq 1, \begin{bmatrix} Y_1 & U_1 \end{bmatrix} \) will no longer be helpful to improve the conservatism of Theorem 1, since \(- (1 - \mu) \begin{bmatrix} Y_1 & U_1 \end{bmatrix} \geq 0\). Therefore, Theorem 1 with \( \mu \geq 1 \) is equivalent to Theorem 2.

As for C3, we can get the stability criterion for NN (3) via the following Lyapunov–Krasovskii functional candidate:

\[
\dot{V}(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t))
\]

where \( V_1(x(t)) \) follows the same definition as that in (11) and

\[
\dot{V}_2(x(t)) = \int_{t - \alpha}^{t} \begin{bmatrix} \tilde{\eta}_1(s) \tilde{\eta}_2(s) \end{bmatrix}^T \begin{bmatrix} Q_1 & V \\ Q_2 & \hat{\eta}_2(s) \end{bmatrix} ds
\]

\[
\dot{V}_3(x(t)) = \frac{d}{m} \sum_{i = 1}^{m+1} \int_{t - \frac{1}{d} i}^{t} \dot{\chi}(s)^T Z_i \dot{\chi}(s) ds da.
\]

**Theorem 3:** Given an integer \( m > 0 \), NN (3) with C3 is globally asymptotically stable if there exist matrices \( P > 0, Z_i > 0 (i = 1, 2, \ldots, m) \), \( Q_1 \otimes Q_2 > 0 \), and diagonal matrices \( S = \text{diag}\{s_1, s_2, \ldots, s_n\} > 0, R = \text{diag}\{r_1, r_2, \ldots, r_n\} > 0, D_i > 0 (i = 1, 2, \ldots, m + 1) \) such that

\[
\begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \ast & \Psi_{22} \end{bmatrix} < 0
\]

where

\[
\Psi_{11} = -e_1^T PAe_1 - e_1^T PAe_1 + W_1^T Q_1 W_1 - W_2^T Q_1 W_2 + \left( \frac{d}{m} \right)^2 e_1^T A \left( \sum_{i = 1}^{m} Z_i \right) A e_1
\]

\[
- \sum_{i = 1}^{m} (e_i - e_{i+1})^T Z_i (e_i - e_{i+1})
\]

\[
- e_1^T W_1^T LRW e_1 - e_1^T A W_1^T R W e_1
\]

\[
\Psi_{12} = -e_1^T AW_1^T S e_1 + W_1^T V W_1 - W_2^T V W_2
\]

\[
+ \sum_{i = 1}^{m+1} e_i^T W_i^T LD_i e_i + e_1^T A W_1^T R e_1
\]

\[
- \left( \frac{d}{m} \right)^2 e_1^T A \left( \sum_{i = 1}^{m} Z_i \right) e_{m+1}
\]

\[
+ e_1^T P e_{m+1} + e_1^T W_1^T R W e_{m+1}
\]

\[
\Psi_{22} = W_1^T Q_2 W_1 - W_2^T Q_2 W_2 + e_1^T W_1^T S e_1
\]

\[
- 2 \sum_{i = 1}^{m+1} e_i^T D_i e_i + e_1^T S W e_{m+1} - e_1^T W_1^T R e_1
\]

\[
+ \left( \frac{d}{m} \right)^2 e_1^T e_{m+1} \left( \sum_{i = 1}^{m} Z_i \right) e_{m+1} - e_1^T R W e_{m+1}.
\]

**Proof:** Evaluating the derivative of \( V(x(t)) \) along the solution of NN (3), we obtain

\[
V_1(x(t)) = 2 x(t)^T P \dot{x}(t) + 2 g(W x(t))^T S W \dot{x}(t)
\]

\[
+ 2(L W x(t) - g(W x(t)))^T R W \dot{x}(t)
\]

\[
= - 2 \dot{\theta}_1^T e_1^T P A e_1 \dot{\theta}_1(t) - 2 \theta_2^T e_1^T S W A e_1 \dot{\theta}_1(t)
\]

\[
+ 2 \dot{\theta}_1^T e_1^T P e_{m+1} \dot{\theta}_2(t)
\]

\[
+ 2 \dot{\theta}_2^T e_1^T S W e_{m+1} \dot{\theta}_2(t)
\]

\[
- 2 \dot{\theta}_1^T e_1^T W^T L R W A e_1 \dot{\theta}_1(t)
\]

\[
+ 2 \dot{\theta}_2^T e_1^T R W A e_1 \dot{\theta}_1(t)
\]
\[
\dot{V}_5(x(t)) = \left(\frac{d}{m}\right)^2 \dot{x}(t)\left(\sum_{i=1}^{m} Z_i\right) \dot{x}(t) - d \sum_{i=1}^{m} \int_{t-rac{\tau}{m}}^{t} \dot{x}(s)^T Z_i \dot{x}(s) \, ds
\]

where Lemma 1 is applied. Thus, we have from (26)–(28) and (19)\(|d_1=d\)

\[
\dot{V}(x(t)) \leq \left[\hat{\beta}_1(t) \hat{\beta}_2(t)\right]^T \left[\Psi_{11} \Psi_{12} \Psi_{21} \Psi_{22}\right] \left[\hat{\beta}_1(t) \hat{\beta}_2(t)\right].
\]

Clearly, if (25) holds, \(\dot{V}(x(t)) < 0\) holds, which implies NN (3) is globally asymptotically stable. This completes the proof.

Remark 6: Theorem 3 presents a new delay-dependent stability criterion for NN (3) with constant delay by employing the Lyapunov–Krasovskii functional candidate (24). When \(r_i = 0, \dot{Q}_1 = \text{diag}(\dot{Q}_1, \dot{Q}_2, \ldots, \dot{Q}_m)\), \(\dot{Q}_2 = I_m \otimes \dot{Q}_{m+1}\), and \(Z_i = (m/d) \dot{Z}_i\), the Lyapunov–Krasovskii functional candidate (24) reduces to that applied in [33, Corollary 4]. Thus, our Lyapunov–Krasovskii functional candidate is much more general than that of [33], and Theorem 3 in this paper improves [33, Corollary 4].

IV. DISSIPATIVITY ANALYSIS

In this section, we will give the results on dissipativity analysis for NN (5) based on the stability conditions that were proposed in Section III.

Theorem 4: Given a scalar \(\gamma > 0\) and an integer \(m > 0\), NN (5) with C1 is globally asymptotically stable and strictly \((Q, S, R)\)-\(\gamma\)-dissipative if there exist matrices \(P > 0\), \(Z_i > 0\ (i = 1, 2, \ldots, m)\), \(\left[Q_i \, V\right] > 0\), \(\left[U_i \, Y_i\right] > 0\), and diagonal matrices \(S = \text{diag}(s_1, s_2, \ldots, s_p) > 0\), \(R = \text{diag}(r_1, r_2, \ldots, r_n) > 0\), \(D_i > 0\ (i = 1, 2, \ldots, m + 3)\) such that

\[
\begin{align*}
\Xi_{11} &\geq \Xi_{12} &\Xi_{13} &\Xi_{14} &\Xi_{15} &0 &\Xi_{17} \\
\ast &\ast &\ast &\ast &0 &0 &0 \\
\ast &\ast &\ast &\ast &0 &0 &0 \\
\ast &\ast &\ast &\ast &0 &0 &0 \\
\ast &\ast &\ast &\ast &0 &0 &0 \\
\ast &\ast &\ast &\ast &0 &0 &0 \\
\ast &\ast &\ast &\ast &0 &0 &0
\end{align*}
\]

where \(\Xi_{11}, \Xi_{12}, \Xi_{13}, \Xi_{14}, \Xi_{15}, \Xi_{22}, \Xi_{23}, \Xi_{25}, \Xi_{33}, \Xi_{36},
\Xi_{45}, \Xi_{55}, \text{and } \Xi_{66}\) follow the same definitions as those in Theorem 1, and

\[
\dot{\Xi}_{44} = W_1^T Q_2 W_1 - W_2^T Q_2 W_2 + e_1^T Y_2 e_1 + e_1^T Y_4 e_1
\]

\[
\Xi_{17} = e_1^T P + e_1^T W^T L R W - \left(\frac{d_1}{m}\right)^2 e_1^T A \left(\sum_{i=1}^{m} Z_i\right)
\]

\[
\Xi_{47} = e_1^T S W - e_1^T W^T R W - e_1^T S
\]

\[
\Xi_{57} = \left(\frac{d_1}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right) + \dot{d}^2 Z_{m+1}
\]

\[
\Xi_{77} = \left(\frac{d_1}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right) + \dot{d}^2 Z_{m+1} - R + \gamma I.
\]

Proof: It is clear that (30) holds implies (10) holds. Therefore, NN (5) with \(\omega(t) = 0\) is stable according to Theorem 1. To prove the dissipativity performance, we consider the Lyapunov–Krasovskii functional candidate (11) and the following index for NN (5):

\[
J_{r, \gamma} = \int_0^t \left(2\gamma \dot{V}(x(t)) + \dot{\gamma}\right)dt.
\]

Applying a similar analysis method employed in the proof of Theorem 1, we have

\[
\int_0^t \dot{V}(x(t)) dt = \int_0^t \dot{\gamma}(t) \dot{\Xi}(t) dt
\]

where

\[
\dot{\gamma}(t) = \left[\frac{\rho(t)}{\omega(t)}\right]
\]

and \(\rho(t)\) follows the same definition as that in (22). We can get from (30)

\[
\int_0^t \dot{V}(x(t)) dt \leq J_{r, \gamma}
\]

which implies

\[
V(x(t)) - V(x(0)) \leq J_{r, \gamma}.
\]
Thus, (7) holds under zero initial condition. Therefore, NN (5) is strictly \((Q,S,R)-\gamma\)-dissipative. This completes the proof.

Remark 7: Based on Theorem 1, a delay-dependent sufficient condition is proposed in Theorem 4 to ensure NN (5) with time-varying interval delay to be globally asymptotically stable and strictly \((Q,S,R)-\gamma\)-dissipative. It should be pointed out that, for given \(m, d_1, d_2,\) and \(\mu,\) by setting \(\delta = -\gamma\) and minimizing \(\delta\) subject to (30), we can obtain the optimal dissipativity performance \(\gamma^*\) (by \(\gamma^* = -\delta\)).

By using a similar analysis method employed in Theorem 4, we have the following results on dissipativity analysis of NN (5) with C2 or C3, respectively.

Theorem 5: Given a scalar \(\gamma > 0\) and an integer \(m > 0,\) NN (5) with C2 is globally asymptotically stable and strictly \((Q,S,R)-\gamma\)-dissipative if there exist matrices \(P > 0, Z_i > 0 (i = 1, 2, \ldots, m),\) \[Q_1 V \begin{bmatrix} Y_1 U_2 \end{bmatrix} > 0, \quad Y_3 U_3 > 0, \quad \begin{bmatrix} Z_m & U_3 \end{bmatrix} \geq 0,
\]
and diagonal matrices \(S = \text{diag}\{s_1, s_2, \ldots, s_n\} > 0,\) \(R = \text{diag}\{r_1, r_2, \ldots, r_n\} > 0,\) \(D_i > 0 (i = 1, 2, \ldots, m + 1)\) such that (30) holds.

Theorem 6: Given a scalar \(\gamma > 0\) and an integer \(m > 0,\) NN (5) with C3 is globally asymptotically stable and strictly \((Q,S,R)-\gamma\)-dissipative if there exist matrices \(P > 0, Z_i > 0 (i = 1, 2, \ldots, m),\) \[Q_1 V \begin{bmatrix} Y_1 U_2 \end{bmatrix} > 0, \quad S = \text{diag}\{s_1, s_2, \ldots, s_n\} > 0,\] \(R = \text{diag}\{r_1, r_2, \ldots, r_n\} > 0,\) \(D_i > 0 (i = 1, 2, \ldots, m + 1)\) such that (30) holds.

\[
\begin{bmatrix} \Psi_{11} & \Psi_{12} & \Psi_{13} \\ \Psi_{22} & \Psi_{23} & \end{bmatrix} < 0
\] (35)

where \(\Psi_{11}\) and \(\Psi_{12}\) follow the same definitions as those in Theorem 3, and

\[
\hat{\Psi}_{22} = W^T Q_2 W_1 - W^T Q_2 W_2 + e^T_{m+1} W^T S e_1 - e^T_{m+1} Q e_1
\]
\[-2 \sum_{i=1}^{m+1} e^T_i D_i e_i + e^T_{m+1} S W e_{m+1} - e^T_{m+1} W^T R e_1 + \left(\frac{d}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right)e_{m+1} - e^T_{m+1} R W e_{m+1}
\]

\[
\Psi_{13} = e^T_1 P + e^T_1 W^T L R W - \left(\frac{d}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right)e_{m+1} - e^T_{m+1} R W e_{m+1}
\]

\[
\Psi_{23} = e^T_1 S W - e^T_1 R W - e^T_1 S + \left(\frac{d}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right)e_{m+1} - e^T_{m+1} R W e_{m+1}
\]

\[
\Psi_{33} = \left(\frac{d}{m}\right)^2 \left(\sum_{i=1}^{m} Z_i\right) - R + \gamma I.
\]

V. NUMERICAL EXAMPLES

In this section, we will make use of some numerical examples to illustrate the advantages of the proposed results in this paper.

Example 1: Consider the NN (3) with

\[
A = \begin{bmatrix} 7.3458 & 0 & 0 \\ 0 & 6.9987 & 0 \\ 0 & 0 & 5.5949 \end{bmatrix}
\]

\[
W = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 21.6810 & 3.2100 & -2.6334 & -20.1300 \end{bmatrix}
\]

\[
L = \lambda \begin{bmatrix} 0.3680 & 0 & 0 \\ 0 & 0.1795 & 0 \\ 0 & 0 & 0.2876 \end{bmatrix}
\]

where the parameter \(\lambda > 0\) in matrix \(L\) can take different values for comparison purpose.

1) We assume that the involved time-delay in the underlying NN satisfies C1, and choose \(\lambda = 1.1229\) and \(d_1 = 0.2.\) The admissible delay upper bound \(d_2\) for various \(\mu\) values computed by [40, Th. 1] proposed in this paper can be found in Table I. It is obvious that our method gives a less conservative result than that in [40]. It can also be seen from Table I that, with the partitioning becoming thinner, the conservativeness of our method decreases.

2) We assume that the involved time delay in the underlying NN satisfies C2, and choose \(\lambda = 0.86\) and \(d_1 = 0.3.\) The admissible delay upper bound \(d_2\) computed by [40] is 0.5142, which can be found in Table II. Table II also gives the admissible delay upper bound \(d_2\) computed by Theorem 2 proposed in this paper. It is clear that our approach can obtain a larger \(d_2\) than the approach of [40].

3) We assume that the involved time delay in the underlying NN satisfies C3, and choose \(\lambda = 1.\) Table III gives the admissible delay upper bound \(d\) for different \(m\) values via the approach introduced in [33, Th. 3] proposed in this paper. It is obvious that our method improves that in [33].
Example 2: Consider the NN (5) with
\[
A = \begin{bmatrix} 7.0214 & 0 \\ 0 & 7.4367 \end{bmatrix}, \quad W = \begin{bmatrix} -6.4993 & -12.0275 \\ -0.6867 & 5.6614 \end{bmatrix},
\]
\[
L = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}.
\]

In this example, we choose
\[
Q = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad S = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}.
\]

We first assume that the involved time delay in the underlying NN satisfies C1. Choosing \(d_1 = 0.1\) and \(d_2 = 0.3\), we can get the optimal dissipativity performance \(\gamma^*\) for different \(m\) and \(\mu\), which is listed in Table IV. It is obvious from Table IV that the optimal dissipativity performance \(\gamma^*\) depends on \(m\) and \(\mu\). Specifically, when \(m\) is fixed, the larger \(\mu (\leq 1)\) corresponds to the smaller \(\gamma^*\), when \(\mu(\leq 1)\) is fixed, the larger \(m\) corresponds to the larger \(\gamma^*\). Furthermore, when \(\mu \geq 1\), the conservatism of Theorem 4 is dependent on \(m\) and independent of \(\mu\).

Next, we assume that the involved time delay in the underlying NN satisfies C2, and choose \(d_1 = 0.1\). Table V gives the optimal dissipativity performance \(\gamma^*\) for different \(m\) and \(d_2\) values. It can be found from Table V that the optimal dissipativity performance \(\gamma^*\) depends on \(m\) and \(d_2\). Specifically, when \(m\) is fixed, a larger \(d_2\) corresponds to a smaller \(\gamma^*\), when \(d_2\) is fixed, a larger \(m\) corresponds to a larger \(\gamma^*\).

From Tables IV and V, we can see that, when \(d_1 = 0.1\) and \(d_2 = 0.3\), Theorem 4 with \(\mu \geq 1.0\) and Theorem 5 give the same optimal dissipativity performance \(\gamma^*\), that is, for the same delay upper bound \(d_2\) and lower bound \(d_1\), the conservatism of Theorem 4 with \(\mu \geq 1.0\) is the same as that of Theorem 5.

Example 3: In this example, we will show the application of the proposed result to a biological network, which has been presented as the mathematical model of the repressor and experimentally studied in *Escherichia coli* [51]. Here, we take the time-varying delay into account and consider the following genetic regulatory network [52], [53]:

\[
\begin{align*}
\dot{m}(t) &= -\hat{A}m(t) + Bf(p(t-d(t))) + \bar{l} \\
\dot{p}(t) &= -Cp(t) + Dm(t-d(t))
\end{align*}
\]  

(36)

where \(m(t) = [m_1(t) \, m_2(t) \, \cdots \, m_n(t)]^T\), \(p(t) = [p_1(t) \, p_2(t) \, \cdots \, p_n(t)]^T\), \(\hat{A} = \text{diag}(\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_n) > 0\), \(C = \text{diag}(c_1, c_2, \ldots, c_n) > 0\), \(D = \text{diag}(d_1, d_2, \ldots, d_n) > 0\), \(\bar{l} = [l_1 \, l_2 \, \cdots \, l_n]^T\), \(f(p(t)) = [f_1(p_1(t)) \, f_2(p_2(t)) \cdots \, f_n(p_n(t))]^T\), and satisfies

\[
0 \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq l_i, \quad s_1 \neq s_2 \in \mathbb{R}.
\]  

(37)

In this example, we consider a three-gene network, that is, \(n = 3\). Choose \(f_i(s) = s^2/(1 + s^2)\) for any \(i\), which implies
It is assumed that \( d(t) = 0.4 + 0.2 \sin(t) \). A straightforward calculation gives \( d_1 = 0.2, d_2 = 0.6, \) and \( \mu = 0.2 \). Applying Theorem 1 with \( m = 2 \), it can be checked that the NN (3) is globally asymptotically stable, which implies the steady-state \((m^* = [0.738, 0.501, 0.576]^T, \ p^* = [0.443, 0.200, 0.288] \)^T of genetic regulatory network (36) is a stable and unique equilibrium.

Figs. 1 and 2 show the the trajectories of the system states \( m(t) \) and \( p(t) \), respectively, with the initial states chosen as \( m(t) = [1.3 + 0.5 \sin(t) \ 1.2 + 0.2 \cos(t) \ 1 + 0.6 \sin(t)]^T \) and \( p(t) = [1.5 + 0.5 \cos(t) \ 1.4 - 0.2 \sin(t) \ 0.8 + 0.4 \cos(t)]^T \), \( t \in [-0.6 \ 0] \), from which we find that the corresponding state responses converge to \((m^*, \ p^*) \). The corresponding phase diagrams of \( m(t) \) and \( p(t) \) are given in Figs. 3 and 4, respectively.

VI. C ONCLUSION

The problems of stability and dissipativity analysis were investigated In this paper for static NNs with time-delay. By taking advantage of the delay partitioning technique, some novel Lyapunov–Krasovskii functional candidates were introduced to arrive at the delay-dependent sufficient conditions that warrant the global stability and dissipativity of NNs. The obtained delay-dependent results also rely upon the partitioning size. Finally, three numerical examples were given to demonstrate the reduction of conservatism and effectiveness of the developed approaches.

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