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# Investment in Concealable Information\*

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*Abstract.* A sender who wants to influence a decision maker has no incentive to collect information if he has to reveal all evidence so obtained, because the expected value of posterior belief is equal to the prior. If he can conceal his evidence at a cost, he invests more in obtaining information when this cost is lower, and this dampens the incentive to conceal evidence as the decision maker would become skeptical upon hearing nothing. In equilibrium greater freedom to conceal information may lead to greater information revelation. A sender has less incentive to conceal evidence when there is another sender who can obtain conditionally independent information, regardless of whether the other sender has the same or the opposite bias.

*JEL classification.* D82, D83

*Keywords.* influence, investigation, competing senders, opposing biases, law of iterated expectations

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## 1. Introduction

A decision maker often has to rely on biased senders to provide information relevant to the decision. For example, a judge gets information from the defendant and the plaintiff, the Congress gets information from regulatory agencies that may be captured by interest groups, and the U.S. Food and Drug Administration gets information from clinical trials conducted by experts who are hired by companies that develop the drugs. Following Milgrom (1981), Milgrom and Roberts (1986) and Shin (1994), we model these situations as a “persuasion game,” with the following features. First, the sender’s payoff does not depend on the state, but only on the decision maker’s action. Second, the sender presents hard evidence that cannot be falsified but can be withheld. Third, the decision maker is passive in the sense that he only forms belief about the state and lacks the ability to offer contingent payments.

It is a well-known result that skepticism on the part of the decision maker imposes severe constraints on the extent of selective information disclosure by the sender in a persuasion game (Milgrom 2008). But when the sender cannot selectively use information to influence the decision making, his incentive to investigate to obtain information will be diminished. Consider a biased sender who does not care about the state and only wants to maximize the decision maker’s posterior belief. Before the sender obtains the information, he does not know whether it will be favorable or unfavorable to him. A fundamental property of Bayesian updating is that the expected value of the posterior is equal to the prior. If the sender has to fully reveal the results of his investigation to the decision maker, then he cannot expect to alter the decision maker’s belief on average. As a result he would have no incentive to engage in costly investigation. Kamenica and Gentzkow (2011) provide general conditions for the sender to benefit from persuasion when information has to be fully revealed. In our paper the sender’s payoff is linear in the decision maker’s belief, so the sender cannot benefit from persuasion—and hence from investigation—unless he can manipulate his evidence. Therefore if the decision maker cannot offer contingent payments, letting the sender to withhold evidence turns out to be important to motivate him to gather evidence in the first place. This interaction between disclosure and discovery is the subject of our paper.

We consider a model in which the sender invests in gathering information before he tries to persuade the decision maker. Investment is costly, and the level of investment is unobserved by the decision maker. Furthermore, even after paying the investment cost, the sender may still obtain no information. Because the decision maker does not

know whether the sender is hiding some evidence or simply possesses no information, the sender can feign ignorance and pool with those who are genuinely uninformed. However hiding unfavorable evidence can have costly consequences. The possibility of being found out (by auditors, whistleblowers, journalists, or by mere happenstance) and the negative consequences that follow in terms of legal punishment or reputation loss create a deterrence to concealment. Both the extent of information non-disclosure and the level of investment in information are determined endogenously in this model.

If the level of investment in information is fixed, then the sender discloses unfavorable information more often whenever the cost of hiding information is increased. When the investment level is endogenous, however, a higher cost of hiding information reduces the ability of the sender to influence the decision maker, and as a result dampens the incentive to invest in obtaining information. This will cause the decision maker to believe that the sender is not well informed. Because the decision maker will then attribute a lack of evidence more to the sender's ignorance than to his concealment of bad news, the inference that the decision maker draws from no evidence is less skeptical than in the case when the sender is believed to be well informed. This effect creates a larger benefit from hiding unfavorable evidence, and we show that the feedback effect can outweigh the direct effect to produce a larger probability of hiding information in equilibrium. In this case, the decision maker is hurt by an increase in the hiding cost not only because the sender is more ignorant, but also because he is more dishonest.

We also use this model to study the effect of competition in a persuasion game. Many discussions on this topic (e.g., Milgrom 2008; Gentzkow and Shapiro 2008; Kamenica and Gentzkow 2012) implicitly assume that competing information providers have access to the same information. In that setting what is hidden by one sender can be revealed by a competing sender, and it is not difficult to show that competition benefits the decision maker. We consider situations in which the signals received by competing senders are conditionally independent. In this case the comparison between persuasion by a single sender and persuasion by competing senders is more subtle. First, because an sender does not know the information possessed by his competitor, the incentive to strategically conceal information is affected. Second, because the decision maker's action depends on the messages sent by both senders, the influence that each sender can exert on the decision is changed, which in turn alters the incentive to investigate to obtain information. Taking into account these two effects, it is possible that a decision maker is better off with a single sender than with two senders.<sup>1</sup>

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<sup>1</sup>Shin (1998) show that having two biased senders in an adversarial setting is better than having one single unbiased sender. His result is partly driven by the fact that, other things equal, having access to two

Consider first the incentive to withhold information when one is faced with a competing sender. This decision depends on the sender's expectation about what message his competitor will send. If the sender had the same information set as the decision maker's, the expected action of the decision maker induced by concealing information does not depend on the presence or absence of a competing sender. This follows from the law of iterated expectations. However strategic pooling with the ignorant types creates a divergence of information between the sender and the decision maker: the decision maker assigns *some probability* that her sender is concealing bad news, while the sender who is contemplating to hide information *knows* that the news is bad. Because the sender is more pessimistic about the underlying state than the decision maker is, he also attaches a greater probability that the competing sender will deliver bad news to the decision maker. As a result the expected action of the decision maker induced by his hiding of information is worse for the sender compared to the single sender case. Holding the level of information investment fixed, the sender would be less likely to hide information when facing competing senders.<sup>2</sup>

While introducing a competing sender makes the existing sender worse off if he has bad news and hides it, it makes the existing sender better off if he has no news. The intuition is the same as above. When receiving no evidence, the decision maker thinks that the sender has received unfavorable evidence with some probability, but the sender himself knows that he is truly ignorant, so he is more optimistic about the messages from the other sender than is the decision maker. Therefore, adding another sender has the effect of both lowering the payoff from receiving bad news and raising the payoff from getting no news (the payoff from receiving good news is not changed). This dampens the sender's incentive to acquire information, and leads to more ignorant senders in equilibrium. To put it simply, competition reduces the influence that each agent has on the decision maker's action, which lowers the senders' incentive to manipulate their information. But precisely because each has a smaller influence on the decision maker's action, the senders also have less incentive to collect information in the first place.<sup>3</sup> As in the single-sender case, a sender who is believed to be less informed has more to gain from hiding information

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signals is better than having one. In his model, the level of informativeness of the senders are exogenous. In our paper, the investigation level is endogenous. We show that having two biased senders can be worse than having just one biased sender—and by implication it must be worse than having one unbiased sender.

<sup>2</sup>We note that this conclusion does not depend on the competing senders having opposite biases. Introducing a competing sender with the same direction of bias will also reduce a sender's incentive to hide information.

<sup>3</sup>It should be emphasized that the smaller influence of an individual sender when there are multiple signals is not just a statistical property. We show that the smaller influence is a consequence of the strategic hiding of information in a pooling equilibrium.

than a more informed one. So the feedback effect from competition can overwhelm the direct effect to result in more information manipulation in equilibrium, once endogenous information acquisition is taken into account. In this case competition in the provision of advice can turn out to be bad for the decision maker.

## 2. Related Literature

This paper is related to studies that model an information acquisition stage before information transmission. Austen-Smith (1994) and Argenziano, Squintani and Severinov (2011) consider this issue in the context of cheap talk games. In a persuasion game, if the decision maker knows that the sender is informed, information will simply unravel (Grossman 1981; Milgrom 1981). This implies that there is no incentive for the sender to invest in information. Therefore we assume that the decision maker knows neither the level of the sender's investment in information nor whether he becomes informed or not. Che and Kartik (2009) studies information acquisition before a persuasion game, like we do. They show that a greater difference in the prior beliefs of the sender and the receiver causes the sender to acquire more information but discloses less of the information he acquires. We argue that the relationship between information acquisition and information disclosure may not be a trade-off with respect to certain parameters. In our setup, when the cost of hiding information falls, it can happen that the sender invests more in obtaining information and also hides less.

Our paper also contributes to the literature on multiple-sender information transmission game. Much of this literature focuses on the special case where two senders have identical information (Gilligan and Krehbiel 1989; Krishna and Morgan 2001; Battaglini 2002; Ambrus and Takahashi 2008; Chan and Suen 2009), and therefore any divergence in their messages would imply at least one sender is not reporting truthfully. The equilibrium construction in such models is very sensitive to assumptions about off-equilibrium beliefs. In our model, senders receive conditionally independent signals which do not perfectly reveal the state. Because any combination of messages is on the equilibrium path, our model is not sensitive to equilibrium refinement concepts.

Krishna and Morgan (2001) and Bhattacharya and Mukherjee (2011) study how competing senders with same biases or opposing biases affect information transmission. Both papers assume that the competing senders have access to the same data. Bourjade and Jullien (2011) show that introducing a competing sender reduces the incentive for the existing sender to conceal information. In their model the decision maker takes binary actions. When there is another sender, concealing information may not change the action

of the decision maker, so the value of hiding becomes zero with some probability. In our model, the decision maker takes a continuous decision, so concealing information always has some impact. We use the differential beliefs of the decision maker and the sender generated by a pooling equilibrium to show that competing senders has less incentive to conceal information given any fixed level of investment in information gathering. This effect is similar to that described in the multiple-sender, multiple-receiver model of Damiano, Li and Suen (2008).

One recurrent theme of this paper is that a sender's ability to hide information gives him the incentive to investigate. This result applies to many other settings. For example, Dahm, Gonzales and Porteiro (2009) show that forcing pharmaceutical companies to disclose the results of their clinical trials will deter them from conducting the trials in the first place. Shavell (1994) looks at a setting where buyers and sellers can acquire information about the value of the product before a transaction, and shows that a voluntary disclosure rule spurs more information acquisition than does a mandatory disclosure rule.

This paper does not take the mechanism design approach to the problem of eliciting information from senders. We assume that the decision maker does not make monetary payments contingent on the action she takes or on the messages sent by the senders, and she simply takes the optimal action based on her posterior belief. Wolinsky (2002) and Gerardi, McLean and Postlewaite (2009) study the optimal mechanism to extract hard information from senders with biases. Dewatripont and Tirole (1999) takes an intermediate approach, where they allow the decision maker to use monetary payments that are contingent on the decision, but not on the specific evidences that the senders present.

### 3. Collection and Concealment of Information in a Persuasion Game

#### 3.1. Model setup

A decision maker, whom we refer to as "DM," chooses an action  $a \in [0, 1]$ . Her loss function is  $(a - \omega)^2$ , where the state  $\omega \in \{0, 1\}$  is binary. The belief about the state is characterized by the probability that  $\omega = 1$ . The prior belief of DM is  $\pi \in [0, 1]$ . For simplicity we assume that  $\pi = 1/2$ .<sup>4</sup> Given that DM is an expected loss minimizer, her optimal action is  $a = P$ , where  $P$  denotes her posterior belief that  $\omega = 1$ .

DM has to rely on the sender to collect information about the state. The sender shares the same prior belief  $\pi$  as DM, but is biased in the sense that his preferences are different

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<sup>4</sup>None of the results in this section depends on this assumption. Proposition 5 that compares the incentive to conceal information in the single-sender case and in the two-sender case also holds for any  $\pi$ .

from DM's. The sender's utility from DM's action is  $a$ , which means that the sender prefers the action to be as high as possible. Given DM's decision rule to choose  $a = P$ , this also means that the sender wants to maximize DM's belief that  $\omega = 1$ .

The sender can potentially obtain hard evidence about the state by engaging in costly investigation. Let the level of investigation be denoted by  $x \in [0, 1]$ . If he chooses investigation level  $x$ , he obtains some hard verifiable evidence with probability  $x$ , and remains ignorant with probability  $1 - x$ . The verifiable evidence is either  $g$  or  $b$ . Denote the outcome of the investigation stage by  $s \in \{g, b, n\}$ , where  $n$  denotes the outcome of no evidence. The information production cost is given by  $C(x)$ , which is assumed to be increasing and strictly convex, with  $C(0) = 0$ ,  $C'(0) = 0$ ,  $C'' > 0$  and  $C''' \geq 0$ .<sup>5</sup> The investigation level  $x$  is unobserved by DM.

Let  $\gamma$  denote the precision of the evidence generating process, that is,

$$\Pr[s = g \mid \omega = 1, s \neq n] = \Pr[s = b \mid \omega = 0, s \neq n] = \gamma.$$

We assume the evidence is informative but not definitive in that  $\gamma \in (1/2, 1)$ . Since the sender prefers DM's belief to be high, the outcome  $g$  is "good" from the sender's perspective while the outcome  $b$  is "bad." We assume that the precision  $\gamma$  is exogenously given.<sup>6</sup>

The sender can send one of the three messages to the DM:  $M \in \{G, B, N\}$ . Sending message  $G$  means showing the verifiable evidence  $g$ ; sending the message  $B$  means showing the verifiable evidence  $b$ ; and sending the message  $N$  means showing nothing. In order to show  $G$  or  $B$ , the sender has to have obtained evidence  $g$  or  $b$  from his signal. Whatever the outcome of his signal, he can choose message  $N$ . If the sender's investigation generates a verifiable evidence, but he chooses to send message  $N$ , then we say the sender is hiding evidence. DM observes neither the level of investigation  $x$  nor the signal  $s$  obtained by the sender. She uses Bayes' rule to reach her posterior belief based on the message  $M$  sent by the sender.

The sender's payoff is  $a - C(x)$  if he does not hide information, and is  $a - C(x) - \theta$  if he hides information (i.e., if  $s \neq n$  and  $M = N$ ). We assume that the expected

<sup>5</sup>The assumption that  $C''' \geq 0$  is needed only to show the uniqueness of equilibrium.

<sup>6</sup>A sender may also want to spend resources on improving his signal precision. This incentive is well understood and straightforward. If an sender is optimistic about the state ( $\pi > 1/2$ ), then he wants to increase the precision of the signal to increase the chance of obtaining favorable evidence. If he is pessimistic about the state ( $\pi < 1/2$ ), then he wants to decrease the precision of the signal. In our model, since  $\pi = 1/2$ , even if we allow the sender to increase the signal precision at a cost, he would have no incentive to do so.



hiding cost  $\theta > 0$  is exogenous. For example,  $\theta$  can be interpreted as a utility cost of dishonesty. Alternatively, one can think of situations in which acts of hiding information has a probability of being found out, and the sender would suffer a reputation loss or other penalties when exposed. We do not give an explicit model of hiding costs; Morris (2001), Kartik (2009) and Bourjade and Jullien (2011) provide some examples. Our main focus is how variations in  $\theta$  affect equilibrium outcomes. Of course, if  $\theta$  is too large, the sender will always reveal any evidence he obtains, in which case he cannot expect to influence DM's action on average. We assume throughout this paper that  $\theta$  is small enough so there is an incentive to invest in information:

$$\theta < \gamma - \frac{1}{2}. \quad (1)$$

To summarize, the timing of the game is as follows. The sender chooses his investigation level  $x$ ; the evidence  $s$  realizes; the sender sends message  $M$  to DM; DM chooses her action  $a$ ; and the payoffs are realized. The equilibrium concept is perfect Bayesian equilibrium, though DM is essentially not a player here.

Here we develop some notation. Since DM does not observe the actual investigation level  $x$  chosen by the sender, her posterior belief depends on the investigation level she conjectures her sender to be choosing, which we denote by  $\hat{x}$ . In equilibrium, consistency of beliefs requires  $x = \hat{x}$ . Similarly, suppose the sender hides bad evidence with probability  $t$ . DM's posterior belief depends on the hiding probability that she conjectures her sender to be choosing, which we denote by  $\hat{t}$ . In equilibrium, consistency of beliefs requires  $t = \hat{t}$ .

We let  $P_M (M = G, B, N)$  denote the DM's posterior after receiving the message  $M$ . We let  $U_s (s = g, b, n)$  denote the sender's expected utility (before paying the investigation cost) upon receiving evidence  $s$ . That is,  $U_s$  is the sender's expectation about DM's action given his signal  $s$ .

### 3.2. Incentive to conceal evidence

Since the sender wants to increase the posterior of DM, there is no benefit from hiding good evidence. Therefore when  $s = g$ , he sends message  $M = G$ . Given the good evidence and the prior  $\pi = 1/2$ , DM's posterior belief is  $\gamma$ . The sender's expected utility is  $U_g = P_G = \gamma$ .

Suppose the evidence is  $s = b$ . The sender can either present this evidence, in which case DM's posterior becomes  $P_B = 1 - \gamma$ , or hide the evidence, in which case DM's pos-

terior would be  $P_N$ . The value of  $P_N$  depends on DM's belief about the sender's investigation level  $\dot{x}$  and about the probability that the sender hides his evidence  $\dot{t}$ . By Bayes' rule,

$$P_N(\dot{x}, \dot{t}) = \frac{\frac{1}{2}(1 - \dot{x} + \dot{x}(1 - \gamma)\dot{t})}{\frac{1}{2}(1 - \dot{x} + \dot{x}(1 - \gamma)\dot{t}) + \frac{1}{2}(1 - \dot{x} + \dot{x}\gamma\dot{t})} = \frac{1 - \dot{x} + \dot{x}(1 - \gamma)\dot{t}}{2(1 - \dot{x}) + \dot{x}\dot{t}}. \quad (2)$$

We have  $U_b = P_B$  if he sends message  $M = B$ , or  $U_b = P_N - \theta$  if he sends message  $M = N$ .

Finally, if the evidence is  $s = n$ , we have  $U_n = P_N$  because the sender cannot manufacture fake evidence.

For  $\dot{x} \in (0, 1)$ , define  $h(\dot{x}, \dot{t})$  to be the benefit of hiding bad evidence:

$$h(\dot{x}, \dot{t}) \equiv P_N(\dot{x}, \dot{t}) - P_B.$$

It is straightforward to verify that  $h(\dot{x}, \dot{t})$  is strictly decreasing in  $\dot{t}$  and strictly decreasing in  $\dot{x}$ . The more often the sender is believed to hide bad evidence, the less favorable is the message  $N$  interpreted by DM. Hence  $h(\dot{x}, \dot{t})$  is decreasing in  $\dot{t}$ . The higher the chance the sender is believed to have obtained evidence, the less favorable is the message  $N$  interpreted by DM. Hence  $h(\dot{x}, \dot{t})$  is decreasing in  $\dot{x}$ .<sup>7</sup>

**Lemma 1.** *Let  $\tilde{x}_1(\theta)$  be the value of  $\dot{x}$  that solves  $h(\dot{x}, 1) = \theta$ . Let  $\tau(\dot{x}, \theta)$  be the value of  $\dot{t}$  that solves  $h(\dot{x}, \dot{t}) = \theta$  for  $\dot{x} > \tilde{x}_1(\theta)$ . If there exists an equilibrium with investigation level  $x \in (0, 1)$  and hiding probability  $t$ , then*

$$t = \tau(x, \theta) \equiv \begin{cases} 1 & \text{if } x \in (0, \tilde{x}_1(\theta)], \\ \tau(x, \theta) & \text{if } x \in (\tilde{x}_1(\theta), 1). \end{cases}$$

The proof of Lemma 1 is in the Appendix. The function  $\tau(\dot{x}, \theta)$  is given explicitly by:

$$\tau(\dot{x}, \theta) = \frac{(1 - \dot{x})(2\gamma - 1 - 2\theta)}{\dot{x}\theta}.$$

When  $x = \dot{x} \leq \tilde{x}_1(\theta)$ , for any belief  $\dot{t}$  the benefit of hiding evidence  $h(\dot{x}, \dot{t})$  is higher than the cost  $\theta$ , so the sender must hide with probability one. When  $x \in (\tilde{x}_1(\theta), 1)$ , the benefit of hiding evidence is strictly greater than the cost if  $\dot{t} = 0$ , and the benefit of hiding evidence is strictly less than the cost if  $\dot{t} = 1$ . Therefore, the only possibility is that the sender

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<sup>7</sup>Note that  $h(\dot{x}, \dot{t}) \leq \gamma - 1/2$  for any  $\dot{x}$  and  $\dot{t}$ . If assumption (1) does not hold, then the sender will not hide evidence for any  $\dot{x}$  and consequently he will have no incentive to investigate.

randomizes between hiding and not hiding evidence, with a probability  $t = \hat{t}$  of concealing evidence that satisfies  $h(\hat{x}, \hat{t}) = \theta$ . We also note that the equilibrium probability of concealing information  $t = \tau(x, \theta)$  is decreasing in the equilibrium probability of obtaining information  $x$ , with  $\lim_{x \rightarrow 1} \tau(x, \theta) = 0$ . In other words, the higher is the probability of obtaining evidence, the less likely it is that an sender would hide his evidence once it is obtained; and if he obtains evidence almost for sure, he reveal it almost for sure.

### 3.3. Incentive to collect evidence

At the information gathering stage, the sender expects that if he successfully finds any evidence the evidence is equally likely to be  $g$  or  $b$  (because his prior is  $\pi = 1/2$ ). Therefore, the payoff from choosing investigation level  $x$  is

$$\frac{1}{2}xU_g + \frac{1}{2}xU_b + (1-x)U_n - C(x). \quad (3)$$

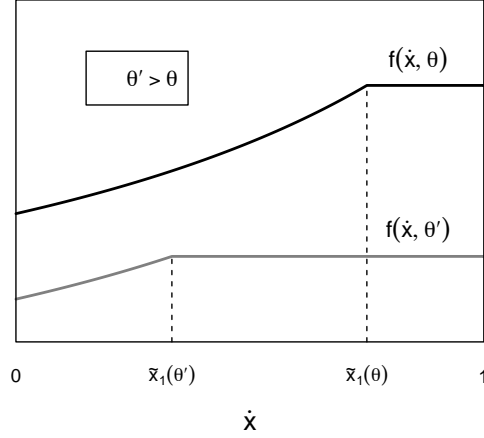
Note that  $U_g, U_b$  and  $U_n$  only depend on DM's conjecture  $\hat{x}$  and  $\hat{t}$  but not the actual choice of  $x$ . Therefore, the marginal benefit of increasing  $x$  given DM's beliefs is

$$MB(\hat{x}, \hat{t}) = \frac{1}{2}U_g + \frac{1}{2}U_b - U_n.$$

There are two types of equilibria to be considered: a "corner equilibrium" with  $x = 1$ , and an "interior equilibrium" with  $x \in (0, 1)$ . For  $\hat{x} \in (0, 1)$ , define

$$f(\hat{x}, \theta) \equiv MB(\hat{x}, \tau(\hat{x}, \theta)).$$

Essentially  $f(\hat{x}, \theta)$  is the marginal benefit from investigation to the sender given that DM believes that the investigation level is  $\hat{x}$  and that she believes the sender will hide bad information with probability  $\tau(\hat{x}, \theta)$  as implied by Lemma 1. Figure 1 plots  $f(\hat{x}, \theta)$  as a function of  $\hat{x}$ . The upward sloping section corresponds to the case  $\hat{x} \leq \tilde{x}_1(\theta)$ . In this case, the sender is believed to hide bad evidence with probability one. So  $U_n = P_N(\hat{x}, 1) = (1 - \hat{x}\gamma)/(2 - \hat{x})$  from equation (2), and  $U_b = P_N(\hat{x}, 1) - \theta$ . The curve is upward sloping because  $\partial MB(\hat{x}, 1)/\partial \hat{x} = -(1/2)(\partial P_N/\partial \hat{x}) > 0$ . The horizontal section of  $f(\cdot, \theta)$  corresponds to the case  $\hat{x} > \tilde{x}(\theta)$ . In this case, the sender is indifferent between hiding and not hiding. Therefore,  $U_b = P_B = 1 - \gamma$  and  $U_n = P_N(\hat{x}, \tau(\hat{x}, \theta)) = P_B + \theta$ , and as a result



**Figure 1.** Marginal benefit of investigation as a function of DM's belief

$f(\dot{x}, \theta)$  does not depend on  $\dot{x}$  in this region. More explicitly, we have

$$f(\dot{x}, \theta) = \begin{cases} \frac{1}{2} \frac{2\gamma-1}{2-\dot{x}} - \frac{1}{2}\theta & \text{if } \dot{x} \leq \tilde{x}_1(\theta), \\ \gamma - \frac{1}{2} - \theta & \text{if } \dot{x} > \tilde{x}_1(\theta). \end{cases}$$

**Proposition 1.** For any  $\theta$  that satisfies assumption (1), there is a unique equilibrium with  $x^* > 0$ .

- (i) If  $C'(1) \leq \gamma - 1/2 - \theta$ , then  $x^* = 1$  and  $t^* = 0$ .
- (ii) If  $C'(1) > \gamma - 1/2 - \theta \geq C'(\tilde{x}_1(\theta))$ , then  $x^* \in [\tilde{x}_1(\theta), 1)$  and satisfies  $f(x^*, \theta) = C'(x^*)$ , and  $t^* = \tau(x^*, \theta) \in (0, 1)$ .
- (iii) If  $C'(\tilde{x}_1(\theta)) > \gamma - 1/2 - \theta$ , then  $x^* < \tilde{x}_1(\theta)$  and satisfies  $f(x^*, \theta) = C'(x^*)$ , and  $t^* = 1$ .

Part (i) of Proposition 1 says that there is full information gathering and full information revelation when the marginal cost of investigation is sufficiently low. Note that the message  $N$  does not appear in such a full information equilibrium. The equilibrium is

supported by the off-equilibrium belief:<sup>8</sup>

$$P_N = \lim_{\dot{x} \rightarrow 1} P_N(\dot{x}, \tau(\dot{x}, \theta)) = 1 - \gamma + \theta.$$

Parts (ii) and (iii) of Proposition 1 state that when the marginal cost of investigation is high, the equilibrium level of investigation is given by the condition that the marginal benefit of investigation is equal to marginal cost. In case (ii), the equilibrium  $x^*$  exceeds  $\tilde{x}_1(\theta)$ , and if the sender receives bad evidence he randomizes between revealing it and concealing it. In case (iii), the equilibrium  $x^*$  is below  $\tilde{x}_1(\theta)$ , and the sender always hides bad evidence. For any fixed  $\dot{x}$ , the sender's objective function is concave in  $x$ . Therefore the point at which marginal benefit equals marginal cost corresponds to the optimal choice of  $x$ . Since  $\dot{x} = x$  in equilibrium, we have

$$f(x^*, \theta) = C'(x^*).$$

As shown in Figure 1, the marginal benefit  $f(\dot{x}, \theta)$  is upward sloping in the conjecture  $\dot{x}$ . Nevertheless Proposition 1 establishes that there exists a unique intersection between marginal benefit and marginal cost, provided that the marginal cost curve is convex (i.e.,  $C''' \geq 0$ ). Moreover, since  $f(0, \theta) > C'(0) = 0$ ,  $x = 0$  is never an equilibrium. The sender will always invest some effort into obtaining evidence.

One interesting implication of Proposition 1 is that information gathering and information revelation move together as the cost of information gathering changes. As the marginal cost of investigation  $C'$  goes up, equilibrium investigation decreases from  $x^* = 1$  in case (i) to  $x^* \in [\tilde{x}_1(\theta), 1)$  in case (ii) to  $x^* \in (0, \tilde{x}_1)$  in case (iii). A lower level of  $x^*$  makes the message  $N$  more believable in the sense that DM would tend to interpret it as the result of no evidence rather than bad evidence being concealed. This in turn raises the gain from concealing bad evidence. As a result, the equilibrium probability of truthful revelation decreases ( $t^*$  increases from 0 in case (i) to  $\tau(x^*, \theta) \in [0, 1)$  in case (ii) to 1 in case (iii)).

Our next result relates to the comparative statics with respect to the exogenous cost of

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<sup>8</sup>In this paper, we restrict the off-equilibrium belief  $P_N$  when  $x = 1$  to be the limit  $\lim_{\dot{x} \rightarrow 1} P_N(\dot{x}, \tau(\dot{x}, \theta))$ . This can be justified in several ways. First, if there is a positive but arbitrarily small probability that a sender's  $B$  message does not reach the decision maker (i.e.,  $B$  message is turned into  $N$  message with a small probability exogenously), then the belief  $P_N$  will be the limit as this probability of mistake goes to zero. Second, one can construct a game where the sender's choice of investigation level has to be in the interval  $[0, 1 - \epsilon]$ , perhaps because getting a sure probability of evidence is impossible for some reason. Then as  $\epsilon$  goes to zero, the belief  $P_N$  at  $x = 1 - \epsilon$  will converge to the limit.

concealing evidence,  $\theta$ .

**Proposition 2.** *An increase in the cost  $\theta$  of concealing evidence reduces the equilibrium level of investigation  $x^*$ . Furthermore, if the cost function of investigation  $C$  is quadratic, a higher cost of concealing evidence raises the equilibrium probability  $t^*$  that the sender conceals bad evidence, and lowers the payoff to DM.*

When investigation yields hard evidence, there are two cases: either the outcome is  $g$  or it is  $b$ . If the evidence turns out to be  $g$ , the sender can show it to his advantage. If the evidence turns out to be  $b$ , the sender can hide it by incurring a cost  $\theta$ . If the cost of hiding bad evidence is so high that the sender always shows both good and bad news, then the event of obtaining an evidence ( $s \neq n$ ) brings no benefit to the sender due to the linearity of his payoff in DM's posterior. When  $\theta$  is not too high, the sender conceals evidence with some probability. Because DM makes a negative inference from the message  $N$ , a higher chance of obtaining evidence is beneficial to the sender. The higher is the hiding cost, the lower is the probability of hiding *given fixed  $\dot{x}$* , and hence the better is the DM's interpretation of the message  $N$ . This decreases the marginal benefit from collecting information. Figure 1 above shows that a higher  $\theta$  shifts down the marginal benefit curve  $f(\dot{x}, \theta)$ . Since Proposition 1 shows that an interior equilibrium  $x^*$  is characterized by  $f(x^*, \theta) = C'(x^*)$ , and since  $C'$  is increasing, equilibrium  $x^*$  must fall as  $\theta$  increases.

Recall that by Lemma 1 the equilibrium probability of hiding evidence is given by  $t^* = \tau(x^*, \theta)$ . An increase in  $\theta$  has a direct negative effect on  $t^*$  because  $\partial\tau/\partial\theta \leq 0$ . However since a higher  $\theta$  lowers  $x^*$ , and since  $\partial\tau/\partial\dot{x} \leq 0$ , the indirect effect tends to raise  $t^*$ . Intuitively, when DM believes that the sender does not have a high chance of obtaining bad evidence, she does not interpret the message  $N$  so unfavorably, which increases the sender's incentive to report  $N$  when he observes  $b$ . Whether the direct effect or the indirect effect is stronger depends on the cost function  $C$ . Proposition 1 establishes that the indirect effect is stronger when the cost function is quadratic. In this case, a higher cost of hiding evidence paradoxically causes the sender to hide evidence more often. Since Proposition 1 also shows that a higher cost of evidence always lowers the equilibrium level of investigation, the payoff to DM necessarily falls.

## 4. Competitive Persuasion

### 4.1. Persuasion model with two senders

In this section we study the persuasion model with two senders who can potentially observe conditionally independent signals. The decision maker's payoff and information

structure is the same as in section 3. She can receive advice from two senders, 1 and 2. Sender 1 is also the same as the sender in section 3. He prefers DM to choose high actions. Sender 2, on the other hand, prefers DM to choose low actions. In particular, his utility from the decision  $a$  is  $1 - a$ . Throughout this section we maintain the assumption that the two senders have “opposite biases,” in the sense that one sender wants to maximize DM’s posterior while the other sender wants to minimize it. In section 6 we discuss the case when the two senders have “same biases,” in the sense that they both want to move DM’s posterior in the same direction.

If sender 2 pays a cost  $C(y)$  to collect information, he has a probability  $y$  of obtaining hard information ( $s_2 \in \{g, b\}$ ) about the state, and a probability  $1 - y$  of obtaining no information ( $s_2 = n$ ). Conditional of receiving hard information, the signal precision is given by  $\gamma$ . The signal  $s_2$  is independent of the signal of sender 1 (denoted by  $s_1$ ) conditional on the state  $\omega$ , and each sender does not observe the other sender’s signal.

Since sender 2 wants to minimize DM’s posterior belief about the state, evidence  $g$  is bad news to him and he may want to conceal it. Let  $u$  denote the probability that sender 2 conceals information when he observes  $g$ . If sender 2 conceals information, he has to pay an expected penalty  $\theta$ . DM and sender 1 observes neither the investment level  $y$  nor the hiding probability  $u$  chosen by sender 2. We let  $\hat{y}$  and  $\hat{u}$  represent their beliefs about these respective quantities.

The timing of events is as follows. Senders 1 and 2 choose their respective investigation levels  $x$  and  $y$  simultaneously. The evidence of each sender, if any, is observed privately. Then each sender sends a message to DM simultaneously. DM chooses an action based on her posterior belief, and the payoffs are realized. The equilibrium concept is perfect Bayesian equilibrium. That is, each sender’s investment level and concealment decision are optimal given the other sender’s choices and DM’s beliefs. Also, DM’s beliefs are consistent with the senders’ choices.

#### 4.2. Information concealment game

Let DM’s posterior after receiving a pair of messages  $M_1, M_2 \in \{G, B, N\}$  be denoted by  $P_{M_1 M_2}$ . When sender 1 sends his message, he has some beliefs about what the other sender’s message  $M_2$  is. Let  $P_{M_1 | s_1}$  denote his expected payoff from sending message  $M_1$  given his private information  $s_1$ . Notice that, unlike in the single sender case, the expected payoff from sending a particular message depends on the privately observed evidence because it updates the sender about the probability that the other sender will receive a certain evidence, and thus the message combination that DM will observe.

Suppose sender 1 receives bad evidence  $b$ . If he reveals it (sends message  $B$ ), his expected payoff is

$$\begin{aligned}
P_{B|b} &= \sum_{M \in \{G, B, N\}} \Pr[M_2 = M \mid s_1 = b] \Pr[\omega = 1 \mid M_1 = B, M_2 = M] \\
&= \sum_{M \in \{G, B, N\}} \Pr[M_2 = M \mid M_1 = B] \Pr[\omega = 1 \mid M_1 = B, M_2 = M] \\
&= \Pr[\omega = 1 \mid M_1 = B],
\end{aligned}$$

where the last equality follows from the law of iterated expectations. Thus, we have  $P_{B|b} = 1 - \gamma$ . If sender 1 receives bad evidence but hides it (sends message  $N$ ), his expected payoff is

$$\begin{aligned}
P_{N|b} &= [(1 - \gamma)\dot{y}\gamma(1 - \dot{u}) + \gamma\dot{y}(1 - \gamma)(1 - \dot{u})]P_{NG} \\
&\quad + [(1 - \gamma)(\dot{y}\gamma\dot{u} + 1 - \dot{y}) + \gamma(\dot{y}(1 - \gamma)\dot{u} + 1 - \dot{y})]P_{NN} \\
&\quad + [(1 - \gamma)\dot{y}(1 - \gamma) + \gamma\dot{y}\gamma]P_{NB}.
\end{aligned} \tag{4}$$

Given evidence  $b$ , sender 1 attaches probability  $1 - \gamma$  to the state being  $\omega = 1$ . In this case, he expects that sender 2 would send message  $G$  with probability  $\dot{y}\gamma(1 - \dot{u})$ . With probability  $\gamma$ , the state is  $\omega = 0$ , and he expects sender 2 would send message  $G$  with probability  $\dot{y}(1 - \gamma)(1 - \dot{u})$ . This explains the coefficients associated with the term  $P_{NG}$ . Similarly, the coefficients associated with  $P_{NN}$  and  $P_{NB}$  are sender 1's subjective probabilities that sender 2 would send messages  $N$  and  $B$ , respectively. In equation (4),

$$P_{NG} = \frac{P_N\gamma}{P_N\gamma + (1 - P_N)(1 - \gamma)}, \tag{5}$$

$$P_{NN} = \frac{P_N(\dot{y}\gamma\dot{u} + 1 - \dot{y})}{P_N(\dot{y}\gamma\dot{u} + 1 - \dot{y}) + (1 - P_N)(\dot{y}(1 - \gamma)\dot{u} + 1 - \dot{y})}, \tag{6}$$

$$P_{NB} = \frac{P_N(1 - \gamma)}{P_N(1 - \gamma) + (1 - P_N)\gamma}, \tag{7}$$

where  $P_N$  is given by the equation (2) of the single-sender case. Note that  $P_{NG}$ ,  $P_{NN}$ , and  $P_{NB}$  are all increasing in  $P_N$ . Since  $P_N$  is decreasing in  $\dot{t}$  and decreasing in  $\dot{x}$ ,  $P_{N|b}$  must also be decreasing in  $\dot{t}$  and decreasing in  $\dot{x}$ .

For  $\dot{x} \in (0, 1)$ , define  $h(\dot{x}, \dot{y}, \dot{t}, \dot{u})$  to be the benefit to sender 1 of hiding bad evidence given the beliefs of DM:

$$h(\dot{x}, \dot{y}, \dot{t}, \dot{u}) \equiv P_{N|b}(\dot{x}, \dot{y}, \dot{t}, \dot{u}) - P_{B|b}.$$



Let  $t^{BR}(\dot{u}, \dot{x}, \dot{y}, \theta) = 1$  if  $h(\dot{x}, \dot{y}, 1, \dot{u}) - \theta > 0$ , and  $t^{BR}(\dot{u}, \dot{x}, \dot{y}, \theta) = 0$  if  $h(\dot{x}, \dot{y}, 0, \dot{u}) - \theta < 0$ . If neither of these two conditions is true, let  $t^{BR}(\dot{u}, \dot{x}, \dot{y}, \theta)$  be the value of  $\dot{t}$  that solves  $h(\dot{x}, \dot{y}, \dot{t}, \dot{u}) - \theta = 0$ . Similarly, let  $u^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta)$  be the value of  $\dot{u}$  such that sender 2 is indifferent between hiding good evidence and revealing it (with  $u^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta) = 1$  if sender 2 strictly prefers hiding good evidence, and  $u^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta) = 0$  if he strictly prefers revealing it).

**Lemma 2.** *If there is an equilibrium with investigation levels  $x \in (0, 1)$  and  $y \in (0, 1)$ , then the equilibrium probabilities of concealing evidence are given by  $t$  and  $u$ , where  $t = t^{BR}(u, x, y, \theta)$  and  $u = u^{BR}(t, x, y, \theta)$ .*

Lemma 2 states that the equilibrium probabilities of hiding unfavorable information are the fixed point of the best-response functions  $t^{BR}$  and  $u^{BR}$ , which is a standard Nash equilibrium requirement. In what follows, we focus on symmetric equilibria in which  $x = y$  and  $t = u$ . We use  $\tau(\dot{x}, \theta)$  to denote the symmetric equilibrium value of  $t$  (and  $u$ ) when the investigation levels are  $x = y = \dot{x}$ .

**Lemma 3.** *Define  $\tilde{x}_1(\theta)$  to be the value of  $\dot{x}$  such that  $h(\dot{x}, \dot{x}, 1, 1) = \theta$ . Define  $\tilde{x}_0(\theta)$  to be the value of  $\dot{x}$  such that  $h(\dot{x}, \dot{x}, 0, 0) = \theta$ . Let  $\tau(\dot{x}, \theta)$  be the value of  $\dot{t}$  such that  $h(\dot{x}, \dot{x}, \dot{t}, \dot{t}) = \theta$  for  $\dot{x} \in (\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\})$ . If there exists a symmetric equilibrium with investigation levels  $x = y \in (0, 1)$  and hiding probability  $t = u$ , then*

$$t = \tau(x, \theta) \equiv \begin{cases} 1 & \text{if } x \in (0, \tilde{x}_1(\theta)], \\ \tau(x, \theta) & \text{if } x \in (\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\}), \\ 0 & \text{if } x \in [\tilde{x}_0(\theta), 1). \end{cases}$$

Moreover  $\tau(x, \theta)$  is continuous and decreasing in  $x$  with  $\lim_{x \rightarrow 1} \tau(x, \theta) = 0$ .

In the Appendix, we show that  $h(\dot{x}, \dot{x}, \dot{t}, \dot{t})$  is strictly decreasing in  $\dot{x}$  and strictly decreasing in  $\dot{t}$ . It then follows that  $\tilde{x}_0(\theta) > \tilde{x}_1(\theta)$  and that  $\tau(\dot{x}, \theta)$  is strictly decreasing in  $\dot{x}$  and strictly decreasing in  $\theta$ . One way to understand this result is to consider the best-response function  $t^{BR}(\dot{u}, \dot{x}, \dot{y}, \theta)$ . It can be shown that  $t^{BR}$  is increasing in  $\dot{u}$  (i.e., information concealment by the two senders are strategic complements) and decreasing in  $\dot{x}$  and  $\dot{y}$ . Similarly,  $u^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta)$  is increasing in  $\dot{t}$  and decreasing in  $\dot{x}$  and  $\dot{y}$ . In a symmetric equilibrium, we have  $\dot{x} = \dot{y}$ ; and when both are raised by the same amount, monotone comparative statics of a supermodular game implies that the equilibrium  $\dot{t}$  and  $\dot{u}$  must fall.

### 4.3. Equilibrium investigation under competitive persuasion

Let  $U_{s_1}(\dot{x}, \dot{y}, \dot{t}, \dot{z})$  denote the expected payoff of sender 1 given an evidence  $s_1$ . For notational convenience we omit the arguments when there is no ambiguity. The payoff to sender 1 from choosing investigation level  $x$  is given by the same equation (3) as in the one-sender case. The marginal benefit of increasing the investment in evidence collection is:

$$MB(\dot{x}, \dot{y}, \dot{t}, \dot{z}) = \frac{1}{2}U_g + \frac{1}{2}U_b - U_n.$$

In a symmetric equilibrium  $\dot{x} = \dot{y}$ , and by Lemma 3,  $\dot{t} = \dot{u} = \tau(\dot{x}, \theta)$ . We let

$$f(\dot{x}, \theta) \equiv MB(\dot{x}, \dot{x}, \tau(\dot{x}, \theta), \tau(\dot{x}, \theta)).$$

Since sender 1 never hides good evidence,  $U_g = P_{G|g}$ . As in the derivation of  $P_{B|b}$ , we can use the law of iterated expectations to show that

$$U_g = P_{G|g} = \gamma.$$

To calculate  $U_n$ , we use

$$U_n(\dot{x}, \dot{x}, \dot{t}, \dot{t}) = P_{N|n}(\dot{x}, \dot{x}, \dot{t}, \dot{t}) = \frac{1}{2}\dot{x}P_{NB} + \frac{1}{2}\dot{x}(1 - \dot{t})P_{NG} + \left(\frac{1}{2}\dot{x}\dot{t} + 1 - \dot{x}\right)P_{NN},$$

where  $P_{NG}$  and  $P_{NB}$  are given by equations (5) and (7) respectively with  $\dot{y} = \dot{x}$  and  $\dot{u} = \dot{t}$ , and  $P_{NN} = 1/2$  by symmetry. Finally to calculate  $U_b$ , we use  $U_b = P_{B|b} = 1 - \gamma$  if  $\tau(\dot{x}, \theta) < 1$ , or  $U_b = P_{N|b} - \theta$  if  $\tau(\dot{x}, \theta) = 1$ , where  $P_{N|b}$  is given by equation (4) with  $\dot{y} = \dot{x}$  and  $\dot{u} = \dot{t}$ .

Figure 2 shows the plot of  $f(\dot{x}, \theta)$  against  $\dot{x}$  for two different values of  $\theta$ . Consider the lower graph, for example. The upward sloping segment corresponds to the case  $\dot{x} \leq \tilde{x}_1(\theta')$ , when sender 1 hides bad evidence for sure. The downward sloping segment corresponds to the case  $\dot{x} \in (\tilde{x}_0(\theta'), \tilde{x}_1(\theta'))$ , when sender 1 randomizes between concealing and revealing bad evidence. Finally the horizontal segment corresponds to the case  $\dot{x} \geq \tilde{x}_1(\theta')$ , when sender 1 reveals bad evidence for sure. Note that in this case  $U_g = \gamma$ ,  $U_b = 1 - \gamma$  and  $U_n = 1/2$ , so that  $f(\dot{x}, 0) = 0$ . There is no benefit from increasing the investment in investigation if the sender never hides bad evidence.

Let  $\bar{f}(\theta) \equiv f(\tilde{x}_0(\theta), \theta)$  and let  $\underline{f}(\theta) \equiv \lim_{\dot{x} \rightarrow 1} f(\dot{x}, \theta)$ . From Figure 2, we see that  $\bar{f}(\theta)$  is the maximum value of  $f(\dot{x}, \theta)$ . We also note that  $\underline{f}(\theta) > 0$  if and only if  $\tilde{x}_0(\theta) > 1$ . The

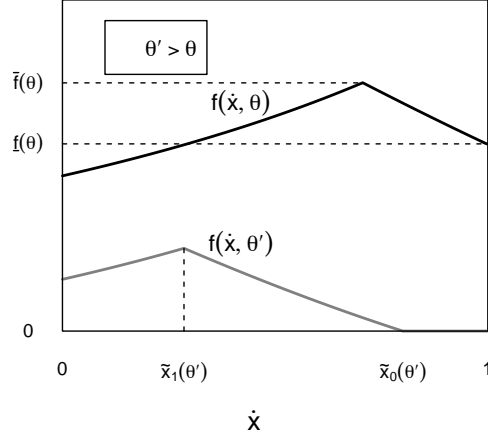


Figure 2. Marginal benefit of investigation in a symmetric equilibrium

following result summarizes the symmetric equilibrium of our model.

**Proposition 3.** For any  $\theta$  that satisfies assumption (1), there exists a symmetric equilibrium with a unique  $x^* = y^* > 0$ .

- (i) If  $C'(1) \leq \underline{f}(\theta)$ , then  $x^* = 1$  and  $t^* = 0$ .
- (ii) If  $C'(1) > \underline{f}(\theta)$  and  $C'(\tilde{x}_0(\theta)) < \bar{f}(\theta)$ , then  $x^* \in (\tilde{x}_0(\theta), 1)$  and satisfies  $f(x^*, \theta) = C'(x^*)$ , and  $t^* = \tau(x^*, \theta) \in (0, 1)$ .
- (iii) If  $C'(\tilde{x}_0(\theta)) \geq \bar{f}(\theta)$ , then  $x^* < \tilde{x}_0(\theta)$  and satisfies  $f(x^*, \theta) = C'(x^*)$ , and  $t^* = 1$ .

Proposition 3 is the counterpart to Proposition 1 in the single-sender case. When the marginal cost of information collection is sufficiently low, the equilibrium is a full-information equilibrium with  $x^* = 1$  and  $t^* = 0$ . For such an equilibrium to be possible, we must have  $\underline{f}(\theta) > 0$ , which requires  $\tilde{x}_0(\theta) > 1$ . Since  $\tilde{x}_0(\theta)$  is defined by the value of  $\dot{x}$  that solves  $h(\dot{x}, \dot{x}, 0, 0) = \theta$ , we can solve this equation to get

$$\tilde{x}_0(\theta) = \frac{2\gamma - 1 - 2\theta}{(2\gamma - 1)^3}.$$

Notice that  $\tilde{x}_0(\theta)$  is decreasing in  $\theta$ , which means that the full-information equilibrium ( $x = 1$  and  $t = 0$ ) is possible when  $\theta$  is sufficiently low and  $C'$  is also sufficiently low. Such an equilibrium is supported by off-equilibrium beliefs  $P_{NG}$ ,  $P_{NN}$  and  $P_{NG}$  that makes  $U_n$

small. Indeed, when the punishment to hiding information is low, sender 1 tends to lie more often. Therefore, a low value of  $\theta$  is consistent with a low value of  $\lim_{\dot{x} \rightarrow 1} P_{N|n}$ , which provides the incentive for the sender to acquire full information.

Parts (ii) and (iii) of Proposition 3 say that in a symmetric “interior equilibrium,” the level of investigation is given by the condition that  $f(x^*, \theta) = C'(x^*)$ . Although  $f(x, \theta)$  is not monotone in  $x$ , Proposition 3 establishes that an interior equilibrium is always unique.

In Figure 2, we see that a higher value of  $\theta$  shifts down the marginal benefit of investigation  $f(x, \theta)$ . This turns out to be generally true. Since the equilibrium level of investigation is given by the comparison between marginal benefit and marginal cost, we have the following result.

**Proposition 4.** *An increase in the punishment  $\theta$  of concealing evidence reduces the level of investigation  $x^*$  in a symmetric equilibrium.*

As in the case of the model with a single sender, increasing the punishment to concealing information has two opposing effects on the equilibrium probability  $t^*$  of concealing information. The direct effect is negative because  $\tau(x^*, \theta)$  is decreasing in  $\theta$ , holding  $x^*$  constant. But since a larger  $\theta$  lowers  $x^*$ , and since a lower  $x^*$  induces more concealment (by making the message  $N$  more favorable to the senders), the indirect effect is positive. We cannot make a general statement to determine which of these two effects dominates.

## 5. Do Two Senders Provide More Information than a Single Sender?

In this section, we compare the the two-sender competitive persuasion model and the single-sender persuasion model. We use superscript  $T$  to denote the case of two senders and superscript  $S$  to denote the case of a single sender.

The benefit from hiding bad evidence in the single-sender model is  $h^S(\dot{x}, \dot{t}) = P_N - P_B$ , while the benefit from hiding bad evidence in the two-sender model is  $h^T(\dot{x}, \dot{x}, \dot{t}, \dot{t}) = P_{N|b} - P_{B|b}$ . We have already shown that the law of iterated expectations implies that

$P_B = P_{B|b}$ . To compare  $P_N$  with  $P_{N|b}$ , we can write:

$$\begin{aligned}
P_{N|b} &= \sum_{M \in \{G, B, N\}} \Pr[M_2 = M \mid s_1 = b] P_{NM} \\
&= \sum_{M \in \{G, B, N\}} \Pr[M_2 = M \mid M_1 = N] P_{NM} \\
&\quad + \sum_{M \in \{G, B, N\}} (\Pr[M_2 = M \mid s_1 = b] - \Pr[M_2 = M \mid M_1 = N]) P_{NM} \\
&= P_N + (\mathbb{E}[P_{NM_2} \mid s_1 = b] - \mathbb{E}[P_{NM_2} \mid M_1 = N]), \tag{8}
\end{aligned}$$

where the expectations are taken over possible realizations of sender 2's message  $M_2$ . The distribution of  $M_2$  conditional on  $s_1 = b$  is stochastically lower than the distribution of  $M_2$  conditional on  $M_1 = N$  in the sense of first-order stochastic dominance. Specifically, the probability that  $M_2 = G$  is lower (and the probability that  $M_2 = B$  is higher) conditional on  $s_1 = b$  than it is conditional on  $M_1 = N$ . As a result, the last bracketed term of (8) is negative, and we have  $P_{N|b} < P_N$ . Therefore,

$$h^T(\dot{x}, \dot{x}, \dot{t}, \dot{t}) < h^S(\dot{x}, \dot{t}). \tag{9}$$

Intuitively, when sender 1 receives evidence  $b$  and sends message  $N$ , he is pooling with the type that receives no evidence. Such pooling introduces a divergence in the information sets of sender 1 and of DM. Because sender 1 knows that the evidence is bad ( $s_1 = b$ ), while DM believes that either the evidence is bad or there is no evidence ( $M_1 = N$ ), sender 1 is systematically more *pessimistic* about the state than DM does. Furthermore, because sender 2 tends to send lower messages ( $M_2 = B$  or  $M_2 = N$ ) whenever the state is low ( $\omega = 0$ ), pooling by sender 1 implies that sender 1 is systematically more *pessimistic* about the content of sender 2's message (i.e., he attaches a higher probability to the events  $\{M_2 = B\}$  and  $\{M_2 = B \text{ or } N\}$ ) than DM does. In other words, sender 1 expects that his message  $N$  would be accompanied by a relatively unfavorable message from the other sender in the two-sender model, while no such effect exists in the single-sender model. This explains why  $P_{N|b} < P_N$ . In contrast, when sender 1 receives evidence  $b$  and sends message  $B$ , there is no divergence in the information sets of sender 1 and DM. In this case, sender 1 has the same beliefs about sender 2's message as DM's. This explains why  $P_{B|b} = P_B$ .

Since each sender compares the benefit from hiding evidence to the punishment  $\theta$ ,

equation (9) immediately implies that

$$t^T(\dot{x}, \theta) \leq t^S(\dot{x}, \theta). \quad (10)$$

In other words, *holding the level of investigation the same*, an sender is less likely to hide unfavorable evidence in the two-sender model than in the single-sender model. Loosely speaking, this effect arises because the marginal impact of each sender on DM's belief by sending message  $N$  instead of message  $B$  is smaller in the two-sender case than in the single-sender case. The smaller marginal influence of each sender, however, should not be viewed as simply a statistical property of inference with multiple signals.<sup>9</sup> Rather, the smaller marginal influence is the result of disparate information sets induced by pooling, as explained in the derivation of equation (8).

The result  $t^T(\dot{x}, \theta) \leq t^S(\dot{x}, \theta)$  would suggest that DM benefits from having multiple senders if the collection of evidence is exogenous: not only that having two signals is better than having one, but each signal has a smaller chance of being strategically concealed. However this conclusion has to be modified when endogenous information collection is taken into account. With multiple senders, each sender is less likely to conceal unfavorable evidence because he has a smaller marginal influence on DM's belief. But precisely because his marginal influence is smaller, the incentive to collect evidence in the first place is also diminished. We have the following result.

**Proposition 5.** *The level of investigation per sender is lower in the symmetric equilibrium of the two-sender model than in the equilibrium of the single-sender model.*

The key to understanding this result is that the marginal benefit from evidence gathering is smaller in the two-sender case than in the single-sender case:

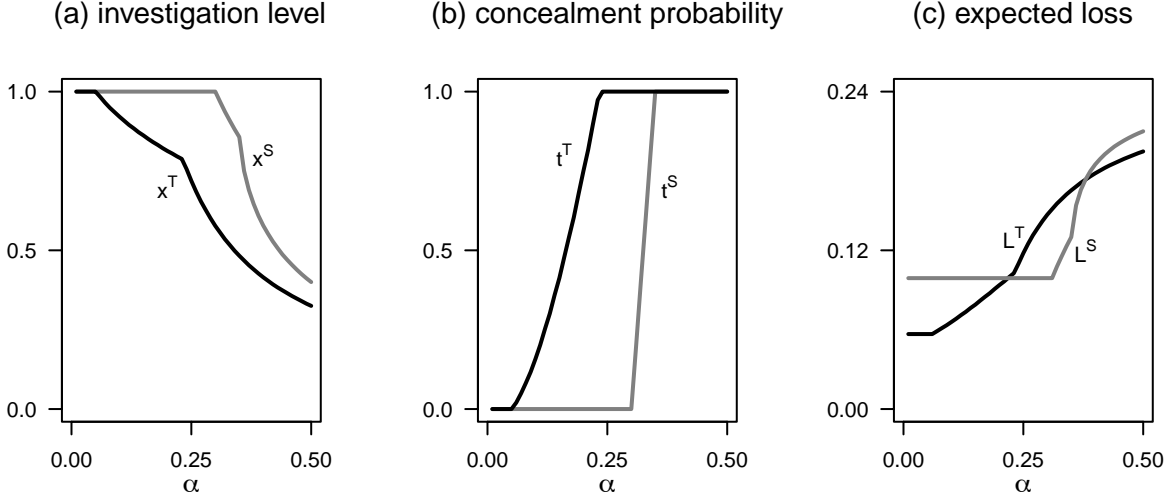
$$f^T(\dot{x}, \theta) < f^S(\dot{x}, \theta).$$

For example, suppose  $\dot{x}$  is such that sender 1 would randomize between concealing bad evidence and revealing it. Then the marginal benefit from gathering evidence is  $f^T(\dot{x}, \theta) = 1/2 - P_{N|n}$  in the two-sender case, and is  $f^S(\dot{x}, \theta) = 1/2 - P_N$  in the single-sender case. But the same intuition that makes  $P_{N|b} < P_N$  also explains why  $P_{N|n} > P_N$ . Specifically, we can write:

$$P_{N|n} = P_N + (\mathbb{E}[P_{NM_2} | s_1 = n] - \mathbb{E}[P_{NM_2} | M_1 = N]).$$

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<sup>9</sup>It is not always the case that the marginal influence of one message decreases in the presence of another message. For example, suppose  $M_2 = G$ . It is easy to verify that  $P_{NG} - P_{BG} > P_N - P_B$ .



**Figure 3.** Comparison between the two-sender model (T) and the single-sender model (S) for different marginal cost parameters

The term in parentheses is positive because the belief about the state (and hence the belief about sender 2's message) is more *optimistic* given  $s_1 = n$  than it is given  $M_1 = N$ . Since sender 1 expects that the signal  $s_1 = n$  would be accompanied by a relatively favorable message  $M_2$ , his expected payoff  $U_n$  is higher in the two-sender case than in the single-sender case, thus diminishing his incentive to obtain evidence.

Because  $x^T > x^S$  by Proposition 5, the equilibrium probability of hiding evidence  $t^T$  in the two-sender model may be greater than or smaller than  $t^S$  in the single-sender model, depending on whether the direct effect (equation (10)) fixing the same investigation level is dominated by the indirect effect that works through the difference in the equilibrium investigation levels. In Figure 3, we assume that the cost function is quadratic,  $C(x) = \alpha x^2/2$ , and plot the equilibrium investigation levels and concealment probabilities against the marginal cost parameter  $\alpha$  (we set  $\theta = 0.1$  and  $\gamma = 0.9$ ). Consistent with Proposition 5, panel (a) shows that  $x^T \leq x^S$  for all values of  $\alpha$ . In panel (b), we see that  $t^T(x^T, \theta) \geq t^S(x^S, \theta)$ . In other words, the indirect effect due to lower investment in evidence collection causes each sender to hide unfavorable evidence with a greater probability in the competitive persuasion case than in the single-sender case. We note, however, that such this conclusion is not general. If the marginal cost function is highly convex (i.e., it is near vertical when it intersects both  $f^T(\dot{x}, \theta)$  and  $f^S(\dot{x}, \theta)$ ), then the difference between  $x^T$  and  $x^S$  would be very small, in which case we can have  $t^T(x^T, \theta) < t^S(x^S, \theta)$ .

Panel (c) of Figure 3 shows that even when  $x^S > x^T$  and  $t^S < t^T$ , DM does not neces-

sarily always prefer to have one sender than to have two senders. When the marginal cost curve is very flat ( $\alpha < \underline{f}(\theta)$ ), there is full incentive to collect and to reveal information in either model. Expected loss for DM is smaller in the two-sender case for the simple reason that two signals is better than one signal. When the marginal cost is very steep ( $\alpha$  is high), the marginal cost curve  $C'(x)$  cuts the marginal benefit curves  $f^T(x, \theta)$  and  $f^S(x, \theta)$  at nearly the same level of  $x$ . In Figure 3, we see that  $x^S$  is only slightly higher than  $x^T$  for large  $\alpha$ , and  $t^S = t^T = 1$ . The slightly larger equilibrium level of investigation in the single-sender case is not sufficient to offset the advantage brought by having two signals in the two-sender case. Again DM prefers to have access to two senders. However, for intermediate values of  $\alpha$ , DM's expected loss is higher when there are two competing senders, because each sender invests less in evidence gathering and conceals more often when unfavorable evidence is obtained.

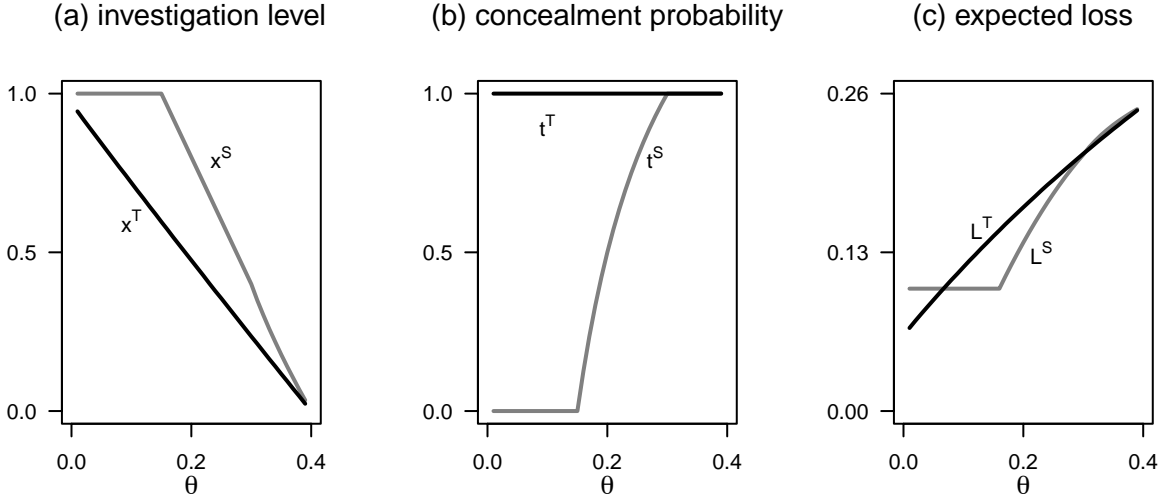
In Figure 4 we compare the two-sender model with the single-sender model for different punishment cost of hiding evidence. In this example, we vary the level of  $\theta$  while fixing  $\alpha = 0.25$  and  $\gamma = 0.9$ . Panels (a) and (b) shows that there is more information collection and less information concealment in the single-sender case. Panel (c) shows that DM benefits from having one sender instead of two if either  $\theta$  is very small or  $\theta$  is very large. In the former case,  $t^T = 1$  while  $t^S = 0$ , so having two signals is not better than having just one because the two senders always hides an unfavorable evidence. When  $\theta$  is large, the difference between  $x^T$  and  $x^S$  is minor, and again having two signals is better than having just one. For intermediate values of  $\theta$ , however, DM is better off with just a single sender.

Although the comparison of the expected loss for DM under the two models is generally ambiguous, we can establish a sufficient condition for the expected loss to be lower under the single-sender case.

**Proposition 6.** *If  $C'(1) < 1/2 - \theta$ , then for  $\gamma$  sufficiently large, DM's equilibrium payoff with two senders is lower than her equilibrium payoff with a single sender.*

Given the restriction on  $C'(1)$ , when  $\gamma$  is sufficiently large, we have  $C'(1) < \gamma - 1/2 - \theta$ . By Proposition 1, we have  $x^S = 1$  and  $t^S = 0$  in the single-sender model. In contrast, in the two-sender model, for  $\gamma$  large enough,  $\underline{f}(\theta) = 0$ , so we must have  $x^T < 1$  and  $t^T > 0$ . When the signal precision is very high, a single-sender who always obtains evidence and reveals it fully almost induces the first-best: the value of having another signal from a second sender is very small to DM. Therefore, the benefit from a higher  $x^*$  and a lower  $t^*$  under the single-sender case dominates the benefit from having a second signal.





**Figure 4.** Comparison between the two-sender model (T) and the single-sender model (S) for different penalties of hiding evidence

## 6. Same Bias versus Opposite Biases

So far, we have assumed that the two senders in the competitive persuasion model have diametrically opposing preferences: sender 1 prefers to maximize DM’s posterior, and sender 2 prefers to minimize it. While the idea that the “competition of ideas” is conducive to information revelation has a long intellectual history (Smith 1981; Coase 1974), and the role of adversarial interests is emphasized in the recent literature (Dewatripont and Tirole 1999; Krishna and Morgan 2001), our model of competitive persuasion does not rely on the assumption that information providers has opposing interests in influencing the decision maker’s beliefs. In this section, we provide a brief outline of the model when sender 2 has identical preferences as sender 1, but their signals (if any) are private and conditionally independent and they are not acting in concert.

Specifically, let sender 2’s utility from DM’s action be  $a$ , so that he also wants to maximize DM’s belief. Suppose sender 2 chooses investigation level  $y$ . If he obtains good evidence, he reveals it; if he obtains bad evidence, he hides it with probability  $u$ . Given the beliefs  $\hat{y}$  and  $\hat{u}$  about sender 2’s strategy, the benefit from hiding bad evidence to sender 1 is  $h^{TS}(\hat{x}, \hat{y}, \hat{t}, \hat{u}) = P_{N|b} - P_B$  (the superscript  $TS$  stands for “two senders, same

bias"), where

$$\begin{aligned}
P_{N|b} &= [(1 - \gamma)\dot{y}\gamma + \gamma\dot{y}(1 - \gamma)]P_{NG} \\
&\quad + [(1 - \gamma)(\dot{y}(1 - \gamma)\dot{u} + 1 - \dot{y}) + \gamma(\dot{y}\gamma\dot{u} + 1 - \dot{y})]P_{NN} \\
&\quad + [(1 - \gamma)\dot{y}(1 - \gamma)(1 - \dot{u}) + \gamma\dot{y}\gamma(1 - \dot{u})]P_{NB},
\end{aligned}$$

where  $P_{NG}$  and  $P_{NB}$  are given by equations (5) and (7) as before but  $P_{NN}$  is not given by (6). Instead,

$$P_{NN} = \frac{P_N(\dot{y}(1 - \gamma)\dot{u} + 1 - \dot{y})}{P_N(\dot{y}(1 - \gamma)\dot{u} + 1 - \dot{y}) + (1 - P_N)(\dot{y}\gamma\dot{u} + 1 - \dot{y})}.$$

In a symmetric equilibrium with  $\dot{x} = \dot{y}$  and  $\dot{t} = \dot{u} > 0$ , we have  $P_{NN} < 1/2$ . Nevertheless we can use the same reasoning as in equation (8) to show that  $P_{N|b} < P_N$ , and the law of iterated expectation to show that  $P_B = 1 - \gamma$ . As a result,  $h^{TS}(\dot{x}, \dot{x}, \dot{t}, \dot{t}) < h^S(\dot{x}, \dot{t})$ . This implies that, holding  $\dot{x}$  and  $\dot{t}$  constant, sender 1 has less incentive to conceal information when he faces another sender with the same bias than when he is acting alone. Indeed we establish a stronger result:  $h^{TS}(\dot{x}, \dot{x}, \dot{t}, \dot{t}) < h^{TO}(\dot{x}, \dot{x}, \dot{t}, \dot{t})$ , where the superscript  $TO$  stands for "two senders, opposite biases."

**Proposition 7.** *Given the same beliefs about the investigation level  $\dot{x}$  and the probability of hiding unfavorable evidence  $\dot{t}$ , the benefit from hiding unfavorable evidence is lower when two senders have the same bias than when they have opposite biases.*

To understand Proposition 7, it is easier to think of DM as receiving the two senders' messages in sequence—first  $M_2$  and then  $M_1$  (even though the senders choose their messages simultaneously.) An  $N$  message from sender 1, compared to a  $B$  message, pulls DM's posterior upward. The gap is the value of hiding bad evidence for sender 1. If sender 2 is of an opposite bias, then DM's posterior after receiving  $M_2$  would be relatively unfavorable to sender 1, so sender 1 has a lot of room to change the DM's posterior through concealing bad evidence. But if sender 2 is of the same bias, then DM's posterior after receiving  $M_2$  would be already quite favorable to sender 1, so sender 1 has little room to further increase DM's posterior. Therefore, holding  $\dot{x}$  and  $\dot{t}$  constant, sender 1 has less incentive to hide unfavorable evidence when he faces another sender with the same bias than when the other sender is of an opposite bias.

However, the same reason that same bias gives a low marginal influence from hiding evidence also gives a low marginal benefit from investing in evidence. In particular, we can show that  $f^{TS}(\dot{x}, \theta) < f^{TO}(\dot{x}, \theta)$ . Since the level of investigation is determined by the

condition that the marginal benefit from investigation equals its marginal cost, the next result follows.

**Proposition 8.** *In a symmetric equilibrium, the equilibrium investigation level is strictly lower when two senders have the same bias than when they have opposite biases.*

Proposition 8 shows that the main disadvantage of having two senders with the same bias is that each sender has less incentive to collect evidence. But because given the same investigation level, senders with same bias have more incentive to reveal evidence once it is obtained, no general conclusion can be given regarding the comparison of the equilibrium  $t^*$  under the case of same bias versus the case of opposite biases. What we have shown in this section, however, is that the comparison between the single-sender model and the two-sender model is robust to assumptions about whether the competing senders have the same bias or opposite biases.

## 7. Conclusion

This paper studies the interaction between senders' incentive to investigate for evidence and their incentive to hide evidence in an attempt to influence the decision maker. Paradoxically, the possibility of hiding unfavorable evidence provides an incentive for the sender to invest in obtaining evidence in the first place. When there is a lower exogenous cost of concealment, a sender invests more in investigation. A lower cost of concealment may lead to less concealment by the sender because of a feedback effect: a higher investigation level means that when the sender does not report anything it is more likely that the sender is hiding something, so the interpretation of silence is more unfavorable to the sender, reducing his incentive to conceal evidence.

We also find that a sender's payoff from hiding unfavorable evidence is lower when there is a competing sender trying to influence the same decision maker, regardless of whether the other sender has the same or the opposite bias. This is because, when a sender obtains bad evidence and hides it, he is more pessimistic about the messages sent by the other sender than is the decision maker, as pooling introduces a divergence in information sets between himself and the decision maker. Since the payoff from telling the truth after obtaining unfavorable evidence is not affected by the presence of another sender, the incentive to hide unfavorable evidence is lower when there is another sender.

On the other hand, when a sender is truly ignorant, he is more optimistic about the messages sent by the other sender than is the decision maker. As a result, the sender's payoff from being ignorant is better when there is another sender. To sum up, when there

is a competing sender, each sender's payoff from receiving a favorable evidence does not change, the payoff from receiving a unfavorable evidence drops, while the payoff from being ignorant is higher. Together, these imply that the incentive to investigate to obtain evidence is lower. Furthermore, the feedback effect suggests that a lower investigation level tends to raise the probability of concealment. As a result, the quality of information, i.e., the chance that the decision maker receives informative evidence supplied by each sender, can be lower when two competing senders are trying to influence the same decision maker. Indeed the deterioration in the quality of information can be so great that it outweighs the benefit from having two signals instead of one.

## References

- Ambrus, A. and S. Takahashi (2008) "Multi-sender Cheap Talk with Restricted State Spaces." *Theoretical Economics* 3: 1–27.
- Argenziano, R., F. Squintani and S. Severinov (2011) "Acquisition and Transmission of Information." Working paper, University of British Columbia.
- Austen-Smith, D. (1990) "Information Transmission in Debate." *American Journal of Political Science* 34(1): 124–152.
- Battaglini, M. (2002) "Multiple Referrals and Multidimensional Cheap Talk." *Econometrica* 70(4): 1379–1401
- Bhattacharyay, S. and A. Mukherjee (2011) "Strategic Information Revelation when Experts Compete to Influence." Working paper, Michigan State University.
- Bourjade S. and B. Jullien (2011) "The Roles of Reputation and Transparency on the Behavior of Biased Experts." *RAND Journal of Economics* 42(3): 575–594.
- Chan, J. and W. Suen (2009) "Media as Watchdogs: The Role of News Media in Electoral Competition." *European Economic Review* 53: 799–814.
- Che, Y. and N. Kartik (2009) "Opinions as Incentives." *Journal of Political Economy* 117(5): 815–860.
- Coase, R. (1974) "The Economics of the First Amendment: The Market for Goods and the Market for Ideas." *American Economic Review* 64(2): 384–391.
- Dahm, M., P. Gonzales and N. Porteiro (2009) "Trials, Tricks and Transparency: How Disclosure Rules Affect Clinical Knowledge." *Journal of Health Economics* 28: 1141–1153
- Damiano, E., H. Li and W. Suen (2008) "Credible Ratings." *Theoretical Economics* 3: 325–365.
- Dewatripont, M. and J. Tirole. (1999) "Advocates." *Journal of Political Economy* 107(1): 1–39.
- Gentzkow, M. and E. Kamenica. (2012) "Competition in Persuasion." Working paper, University of Chicago.

- Gentzkow, M. and J. Shapiro (2008) "Competition and Truth in the Market for News." *Journal of Economic Perspectives* 22(2): 133–154.
- Gerardi, D., R. McLean and A. Postlewaite (2009) "Aggregation of Expert Opinions." *Games and Economic Behavior* 65: 339–371.
- Gilligan, T., Krehbiel, K. (1989) "Asymmetric Information and Legislative Rules with a Heterogenous Committee." *American Journal of Political Science* 33(2): 459–490.
- Grossman, S. (1981) "The Informational Role of Warranties and Private Disclosure about Product Quality." *Journal of Law and Economics* 24(3): 461–483.
- Kartik, N. (2009) "Strategic Communication with Lying Costs." *Review of Economic Studies* 76(4):1359–1395.
- Kamenica, E. and M. Gentzkow (2011) "Bayesian Persuasion." *American Economic Review* 101(6): 2590–2615.
- Krishna, V. and J. Morgan (2001) "A Model of Expertise." *Quarterly Journal of Economics* 116(2): 747–775.
- Milgrom, P. (1981) "Good News and Bad News: Representation Theorems and Applications." *Bell Journal of Economics* 12: 380–391.
- Milgrom, P. (2008) "What the Seller Won't Tell You: Persuasion and Disclosure in Markets." *Journal of Economic Perspectives* 22(2): 115–131.
- Milgrom, P. and J. Roberts (1986) "Relying on the Information of Interested Parties." *RAND Journal of Economics* 17: 18–32.
- Morris, S. (2001) "Political Correctness." *Journal of Political Economy* 109(2): 231–265.
- Shavell, S. (1994) "Acquisition and Disclosure of Information Prior to Sale." *RAND Journal of Economics* 25: 20–36.
- Shin, H. (1994) "News Management and the Value of Firms." *RAND Journal of Economics* 25: 58–71.
- Shin, H. (1998) "Adversarial and Inquisitorial Procedures in Arbitration." *RAND Journal of Economics* 29(2): 378–405.
- Smith, J. (1981) "Freedom of Expression and the Marketplace of Ideas Concept from Milton to Jefferson." *Journal of Communication Inquiry* 7(1): 47–63.

Wolinsky, A. (2002) "Eliciting Information from Multiple Experts." *Games and Economic Behavior* 41: 141–160.

## Appendix

**Proof of Lemma 1.** Suppose  $x \in (0, \tilde{x}_1(\theta)]$  and  $\dot{t} < 1$ . Since  $h$  is strictly decreasing in  $\dot{x}$  and strictly decreasing in  $\dot{t}$ ,

$$h(\dot{x}, \dot{t}) > h(\dot{x}, 1) \geq h(\tilde{x}_1(\theta), 1) = \theta.$$

This forms a contradiction. Therefore,  $\dot{t} = t = 1$  if  $x \in (0, \tilde{x}_1(\theta)]$ .

Suppose  $x \in (\tilde{x}_1(\theta), 1)$ . For any  $\dot{x}$ ,  $h$  is strictly decreasing in  $\dot{x}$ . Therefore,

$$h(\dot{x}, 1) < h(\tilde{x}_1(\theta), 1) = \theta,$$

which implies  $t < 1$ . Also,  $h(\dot{x}, 0) = \gamma - 1/2 > \theta$ , so  $t > 0$ . Since  $t \in (0, 1)$ , the sender must be indifferent between hiding and not hiding, i.e.,  $h(\dot{x}, \dot{t}) = \theta$ , which implies  $\dot{t} = \tau(\dot{x}, \theta)$ . Because in equilibrium DM's belief about the sender's strategy is consistent with his actual strategy,  $t = \tau(x, \theta)$ . ■

**Proof of Proposition 1.** We first establish the following claim.

*Claim 1.* There exists a unique solution of  $x \in (0, 1)$  to the equation  $f(x, \theta) = C'(x)$  if and only if  $C'(1) > \gamma - 1/2 - \theta$ .

Assume for the moment that the marginal cost function is linear, i.e.,  $C'(x) = \alpha x$  for some constant  $\alpha$ . Let

$$g_1(x, \theta) \equiv \frac{1}{2} \frac{2\gamma - 1}{2 - x} - \frac{1}{2} \theta,$$

$$g_2(\theta) \equiv \gamma - \frac{1}{2} - \theta.$$

Note that  $g_2(\theta) < g_1(x, \theta)$  for  $x > \tilde{x}_1(\theta)$ , and  $f(x, \theta) = \min\{g_1(x, \theta), g_2(\theta)\}$ . Define  $\hat{\alpha}$  and  $\hat{x}$  such that  $C'(x)$  is tangent to  $g_1(x, \theta)$  if  $\alpha = \hat{\alpha}$  and  $x = \hat{x}$ . Direct calculation shows that  $\hat{x} > \tilde{x}_1(\theta)$ . Note that  $g_1(x, \theta) - C'(x)$  is convex in  $x$ , with  $g_1(0, \theta) - C'(0) > 0$ . Therefore the equation  $g_1(x, \theta) - C'(x) = 0$  has no solution if  $\alpha < \hat{\alpha}$ , and has two solutions if  $\alpha > \hat{\alpha}$ . When the equation has two solutions, the two roots  $x'$  and  $x''$  must satisfy  $x' < \hat{x} < x''$ , and  $g_1(x, \theta) - C'(x) < 0$  for all  $x \in (x', x'')$ . Let  $\alpha'$  be such that when  $\alpha = \alpha'$ , the smaller root  $x'$  is equal to  $\tilde{x}_1(\theta)$ . Note that  $\alpha' > \hat{\alpha}$ . There are three cases to consider.

- (a) If  $\alpha < \hat{\alpha}$ , then  $g_1(x, \theta) - C'(x) = 0$  has no solution. We have  $g_1(x, \theta) > C'(x)$  for all  $x < \tilde{x}_1(\theta)$ .



- (b) If  $\alpha \in [\hat{\alpha}, \alpha')$ , then the smaller root to the equation  $g_1(x, \theta) - C'(x) = 0$  is greater than  $\tilde{x}_1(\theta)$ . Again, we have  $g_1(x, \theta) > C'(x)$  for all  $x < \tilde{x}_1(\theta)$ .
- (c) If  $\alpha \geq \alpha'$ , then there is an  $x'$  such that  $g_1(x, \theta) > C'(x)$  for  $x < x'$  and  $g_1(x, \theta) < C'(x)$  for  $x \in (x', \tilde{x}_1(\theta))$ .

In cases (a) and (b),  $f(x, \theta) > C'(x)$  for  $x \in [0, \tilde{x}_1(\theta)]$ . Since  $f(x, \theta)$  is a constant for  $x \in [\tilde{x}_1(\theta), 1]$ , while  $C'(x)$  is strictly increasing, by the intermediate value theorem there exists a unique solution  $x^* \in (\tilde{x}_1(\theta), 1)$  to  $f(x, \theta) = C'(x)$  if and only if  $C'(1) > f(1, \theta) = \gamma - 1/2 - \theta$ . In case (c),  $x^* = x'$  is the unique solution to  $f(x, \theta) = C'(x)$  for  $x \in [0, \tilde{x}_1(\theta)]$  as  $x'' > \hat{x} > \tilde{x}_1(\theta)$ . Furthermore, since  $f(x, \theta) - C'(x)$  is strictly decreasing for  $x \in [\tilde{x}_1(\theta), 1]$  and is non-positive at  $x = \tilde{x}_1(\theta)$ , there cannot be another solution to  $f(x, \theta) = C'(x)$ . Note finally that case (c) applies if and only if  $C'(\tilde{x}_1(\theta)) \geq \gamma - 1/2 - \theta$ , which implies that  $C'(1) > \gamma - 1/2 - \theta$ . This proves the claim when the marginal cost function is linear.

Next suppose that the marginal cost function is convex (i.e.,  $C''' \geq 0$ ). There are two cases. First, suppose  $C'(\tilde{x}_1(\theta)) < g_1(\tilde{x}_1(\theta), \theta)$ . If we let  $\alpha = C'(\tilde{x}_1(\theta))/\tilde{x}_1(\theta)$ , then  $\alpha < \alpha'$  defined above. Moreover, the analysis of cases (a) and (b) and the convexity of  $C'$  implies that  $g_1(x, \theta) > \alpha x \geq C'(x)$  for  $x < \tilde{x}_1(\theta)$ . The rest of the analysis is the same. There can only be one solution  $x^* \in (\tilde{x}_1(\theta), 1)$  to be equation  $f(x, \theta) = C'(x)$ . Next, suppose  $C'(\tilde{x}_1(\theta)) \geq g_1(\tilde{x}_1(\theta), \theta)$ . Then there must exist an  $x' \leq \tilde{x}_1(\theta)$  such that  $C'(x') = g_1(x', \theta)$ . Let  $\alpha = C'(x')/x'$ , then  $\alpha \geq \alpha'$ . The analysis of case (c) and the convexity of  $C'$  implies that  $g_1(x, \theta) > \alpha x \geq C'(x)$  for  $x < x'$ , and  $g_1(x, \theta) < \alpha x \leq C'(x)$  for  $x \in (x', \tilde{x}_1(\theta))$ . The rest of the analysis is the same. There is a unique solution  $x^* = x' \in (0, \tilde{x}_1(\theta)]$  to the equation  $f(x, \theta) = C'(x)$ . This establishes Claim 1.

Case (i). Suppose  $C'(1) \leq \gamma - 1/2 - \theta$ . Then  $x^* = 1$  and  $t^* = \tau(1, \theta) = 0$  is an equilibrium. Since message  $N$  does not occur in equilibrium, we can assign the off-equilibrium belief  $P_N = 1 - \gamma + \theta$ . Note that this value is equal to  $\lim_{\hat{x} \rightarrow 1} P_N(\hat{x}, \tau(\hat{x}, \theta))$ . We then have  $MB(1, 0) = \gamma - 1/2 - \theta \geq C'(1)$ , so the sender has no incentive to lower  $x^*$ . Suppose there exists another interior equilibrium  $x' < 1$ . Such  $x'$  must satisfy  $f(x', \theta) = C'(x')$ , which contradicts Claim 1.

Cases (ii) and (iii). Suppose  $C'(1) > \gamma - 1/2 - \theta$ . Consider the candidate equilibrium with  $x = \hat{x} = x^*$  and  $t = \hat{t} = \tau(x^*, \theta)$ , where  $x^*$  satisfies  $f(x^*, \theta) = C'(x^*)$ . Recall that  $MB(\hat{x}, \hat{t})$  depends only on DM's belief and is constant with respect to the actual choice of  $x$  made by the sender. Therefore, the convexity of  $C$  implies that the sender's payoff is concave in  $x$ . Thus  $x$  maximizes his payoff if and only if  $x = x^*$ . Claim 1 establishes that  $x^*$  is unique. Moreover, by Lemma 1, the corresponding equilibrium probability of concealment is  $\tau(x^*, \theta)$ . ■

**Proof of Proposition 2.** By Proposition 1, equilibrium  $x^*$  is determined by the condition that  $f(x^*, \theta) = C'(x^*)$  if  $x^* < 1$  and  $f(x^*, \theta) \geq C'(x^*)$  if  $x^* = 1$ . Note that  $C'$  is increasing, and the solution is unique. Therefore, to establish that  $x^*$  decreases in  $\theta$ , it suffices to show that  $f(\dot{x}, \theta)$  is strictly decreasing in  $\theta$  for any fixed  $\dot{x}$ .

Pick any  $\theta' > \theta \in (0, \gamma - 1/2)$ , we have  $\tilde{x}_1(\theta') < \tilde{x}_1(\theta)$ . Using the functions  $g_1$  and  $g_2$  defined in the proof of Proposition 1, since both functions are decreasing in  $\theta$ ,  $f(\dot{x}, \theta)$  decreases in  $\theta$  as well. This proves the first part of the proposition.

Suppose now the quadratic cost function is  $C(x) = \alpha x^2/2$ . Let  $I$  denote the set of  $\theta$  for which  $t^* < 1$  is the equilibrium. Let  $F$  denote the set of  $\theta$  for which  $t^* = 1$  is the equilibrium. For a quadratic cost function, if  $\alpha < \frac{1}{2}(\gamma - \frac{1}{2})$  then  $I = [0, \gamma - \frac{1}{2})$  and  $F = \emptyset$ ; if  $\alpha \in [\frac{1}{2}(\gamma - \frac{1}{2}), \gamma - \frac{1}{2}]$ , then  $I = [0, 2(\gamma - \frac{1}{2} - \alpha))$  and  $F = [2(\gamma - \frac{1}{2} - \alpha), \gamma - \frac{1}{2}]$ ; and if  $\alpha > \gamma - \frac{1}{2}$ , then  $I = \emptyset$  and  $F = [0, \gamma - \frac{1}{2})$ .

Take any  $\theta \in I$ . Solving the equilibrium condition  $g_2(\theta) = C'(x^*)$  gives  $x^* = \frac{1}{\alpha}(\gamma - \frac{1}{2} - \theta)$ . Plugging this value of  $\dot{x} = x^*$  into  $\tau(\dot{x}, \theta)$  gives  $t^* = \frac{2}{\theta}(\alpha - \gamma + \frac{1}{2} + \theta)$ . Therefore,

$$\frac{dt^*}{d\theta} = \frac{2}{\theta^2} \left( \gamma - \frac{1}{2} - \alpha \right),$$

which is positive because  $I \neq \emptyset$  implies that  $\alpha < \gamma - \frac{1}{2}$ . Thus, if  $\theta' > \theta$  and both belong to  $I$ , then  $t^*(\theta') > t^*(\theta)$ . If  $\theta' \in F$  and  $\theta \in I$ , then  $t^*(\theta') = 1 > t^*(\theta)$ . If  $\theta, \theta' \in F$ , then the proposition is trivially true because both  $t^*(\theta')$  and  $t^*(\theta)$  are equal to 1.

Finally, given any posterior  $P$  upon receiving a message from the sender, the expected loss of DM is  $P(1 - P)^2 + (1 - P)P^2 = P(1 - P)$ . In equilibrium, DM gets message  $G$  with probability  $x^*/2$ , message  $B$  with probability  $x^*(1 - t^*)/2$ , and message  $N$  with the remaining probability. Therefore, her equilibrium loss is

$$\begin{aligned} L^* &= \frac{1}{2}x^*P_G(1 - P_G) + \frac{1}{2}x^*(1 - t^*)P_B(1 - P_B) + \left(1 - x^* + \frac{1}{2}x^*t^*\right)P_N(1 - P_N) \\ &= \left(x^* - \frac{1}{2}x^*t^*\right)\gamma(1 - \gamma) + \frac{1}{2} \frac{(1 - x^* + x^*(1 - \gamma)t^*)(1 - x^* + x^*\gamma t^*)}{2(1 - x^*) + x^*t^*} \end{aligned}$$

Taking derivative with respect to  $x^*$  and  $t^*$ , for  $x^* \in (0, 1)$  and  $t^* \in (0, 1)$ , we have

$$\begin{aligned} \frac{\partial L^*}{\partial x^*} &= -\frac{(2\gamma - 1)^2 (2(1 - x^*)^2(1 - t^*) + t^* + (t^*x^*)^2)}{2(2(1 - x^*) + x^*t^*)^2} < 0, \\ \frac{\partial L^*}{\partial t^*} &= \frac{(2\gamma - 1)^2 x^*(1 - x^*)}{2(2(1 - x^*) + x^*t^*)^2} > 0. \end{aligned}$$

Since an increase in  $\theta$  lowers  $x^*$  and raises  $t^*$  (given a quadratic cost function), it raises the equilibrium loss to DM. ■

**Proof for Lemma 2.** If  $h(\dot{x}, \dot{y}, 1, \dot{u}) - \theta > 0$ , then sender 1 strictly prefers hiding bad evidence to revealing it. So  $t = 1$  is a best response. Further, since  $h$  is strictly decreasing in  $\dot{t}$ ,  $t = 1$  is the unique best response. Similarly, if  $h(\dot{x}, \dot{y}, 0, \dot{u}) - \theta < 0$ , then  $t = 0$  is the unique best response. When neither of these two conditions hold, there is a unique  $\dot{t} \in [0, 1]$  that solves  $h(\dot{x}, \dot{y}, \dot{t}, \dot{u}) - \theta = 0$ . At such a  $\dot{t}$ , sender 1 is indifferent between concealing and revealing evidence. So choosing  $t = \dot{t}$  is a unique best response. That is,  $t^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta)$  is the best response function for sender 1. Similar reasoning shows that  $u^{BR}(\dot{t}, \dot{x}, \dot{y}, \theta)$  is the best response function for sender 2. In a perfect Bayesian equilibrium with investigation levels  $x$  and  $y$ ,  $t = \dot{t}$  and  $u = \dot{u}$  must satisfy the fixed point property that  $t = t^{BR}(u, x, y, \theta)$  and  $u = u^{BR}(t, x, y, \theta)$ . ■

**Proof of Lemma 3.** In a symmetric equilibrium,  $x = y = \dot{x} = \dot{y}$  and  $t = u = \dot{t} = \dot{u}$ . Therefore,

$$h(x, x, t, t) = x(\gamma^2 + (1 - \gamma)^2)(P_{NB} - P_{BB}) + 2\gamma(1 - \gamma)x(1 - t)(P_{NG} - P_{BG}) \\ + (1 - x + 2\gamma(1 - \gamma)tx)(P_{NN} - P_{BN}).$$

By symmetry,  $P_{NN} = P_{BG} = 1/2$  and  $P_{BN} = 1 - P_{NG}$ . Thus  $h(x, x, t, t)$  simplifies to

$$x(\gamma^2 + (1 - \gamma)^2) \left( P_{NB} - \frac{(1 - \gamma)^2}{\gamma^2 + (1 - \gamma)^2} \right) + (1 - x + 2\gamma(1 - \gamma)x) \left( P_{NG} - \frac{1}{2} \right),$$

where

$$P_{NB} = \frac{(1 - \gamma)(1 - x + (1 - \gamma)tx)}{1 - x + (\gamma^2 + (1 - \gamma)^2)tx}, \\ P_{NG} = \frac{\gamma(1 - x + (1 - \gamma)tx)}{1 - x + 2\gamma(1 - \gamma)tx}.$$

It can be verified that both  $P_{NB}$  and  $P_{NG}$  are decreasing in  $t$ . Hence  $h(x, x, t, t)$  is decreasing in  $t$ . Moreover, the derivative of  $h(x, x, t, t)$  with respect to  $x$  is

$$(\gamma^2 + (1 - \gamma)^2) \left( P_{NB} - \frac{(1 - \gamma)^2}{\gamma^2 + (1 - \gamma)^2} - P_{NG} + \frac{1}{2} \right) \\ + x(\gamma^2 + (1 - \gamma)^2) \frac{\partial P_{NB}}{\partial x} + (1 - x + 2\gamma(1 - \gamma)x) \frac{\partial P_{NG}}{\partial x}.$$

It can be verified that each of these three terms are negative. Hence  $h(x, x, t, t)$  is decreasing in  $x$ . It follows immediately that  $\tilde{x}_0(\theta) > \tilde{x}_1(\theta)$ .

Suppose  $x \in (0, \tilde{x}_1(\theta)]$ , which implies that  $h(x, x, 1, 1) \geq \theta$ . If  $t < 1$ , the benefit of hiding bad evidence is  $h(x, x, t, t) > h(x, x, 1, 1) \geq \theta$ , so each sender would want to choose  $t = 1$ , which is a contradiction. Therefore,  $t = 1$ .

Suppose  $x \in (\tilde{x}_1(\theta), \tilde{x}_0(\theta))$ , which implies that  $h(x, x, 1, 1) < \theta$  and  $h(x, x, 0, 0) > \theta$ . If  $t = 1$ , then the benefit of hiding bad evidence is  $h(x, x, t, t) = h(x, x, 1, 1) < \theta$ , which is a contradiction. If  $t = 0$ , then  $h(x, x, t, t) = h(x, x, 0, 0) > \theta$ , again a contradiction. Therefore we must have  $t \in (0, 1)$ . Since sender 1 must be indifferent between sending message  $N$  and sending message  $B$  when  $t \in (0, 1)$ , the only value of  $t$  that is consistent with indifference must satisfy  $h(x, x, t, t) = \theta$ , which implies  $t = \tau(x, \theta)$ .

Suppose  $x \geq \tilde{x}_0(\theta)$ , which implies that  $h(x, x, 0, 0) \leq \theta$ . If  $t > 0$ , then  $h(x, x, t, t) < h(x, x, 0, 0) \leq \theta$ , so each sender would want to choose  $t = 0$ , which gives a contradiction. Therefore,  $t = 0$ .

Finally,  $\tau(x, \theta)$  is continuous and decreasing in  $x$  because  $h(x, x, t, t)$  is continuous and decreasing in both  $x$  and  $t$ . For the limit value of  $\tau(x, \theta)$ , there are two cases to consider. Case (i). If  $\tilde{x}_0(\theta) < 1$ , then for  $x \in [\tilde{x}_0(\theta), 1)$ ,  $\tau(x, \theta) = 0$ , so  $\lim_{x \rightarrow 1} \tau(x, \theta) = 0$ . Case (ii). If  $\tilde{x}_0(\theta) \geq 1$ , then for  $x \in [\tilde{x}_1(\theta), 1)$ ,  $\tau(x, \theta) = \tau(x, \theta)$ . Take any  $\epsilon > 0$ . One can find  $x_\epsilon$  such that  $h(x_\epsilon, x_\epsilon, \epsilon, \epsilon) = \theta$ . This is because  $h(1, 1, \epsilon, \epsilon) = 0 < \theta$  and  $h(0, 0, \epsilon, \epsilon) = \gamma - \frac{1}{2} > \theta$ . Since  $h(1, 1, \epsilon, \epsilon) = 0$  and  $h(x_\epsilon, x_\epsilon, \epsilon, \epsilon) = \theta$ , for any  $x \in (x_\epsilon, 1)$ , it must be that  $0 < h(x, x, \epsilon, \epsilon) < \theta$ . By definition,  $h(x, x, \tau(x, \theta), \tau(x, \theta)) = \theta$ . This implies  $\tau(x, \theta) < \epsilon$ . That is, for any  $\epsilon > 0$ , there exists  $x_\epsilon$  such that for all  $x \in (x_\epsilon, 1)$ ,  $\tau(x, \theta) < \epsilon$ . Therefore,  $\lim_{x \rightarrow 1} \tau(x, \theta) = 0$ . ■

**Proof of Proposition 3.** We first establish some properties of the  $f$  function.

*Claim 2.*  $f(x, \theta)$  is strictly increasing in  $x$  for  $x \in (0, \tilde{x}_1(\theta))$ , strictly decreasing in  $x$  for  $x \in (\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\})$ , and equal to 0 for  $x \geq \tilde{x}_0(\theta)$ .

For  $x \leq \tilde{x}_1(\theta)$ ,  $\tau(x, \theta) = 1$ . Therefore,

$$U_b(x, x, \tau(\theta, x), \tau(x, \theta)) = P_{N|b} - \theta = (\gamma^2 + (1 - \gamma)^2)xP_{NB} + (1 - (\gamma^2 + (1 - \gamma)^2)x)\frac{1}{2} - \theta,$$

$$U_n(x, x, \tau(\theta, x), \tau(x, \theta)) = P_{N|n} = \frac{1}{2}xP_{NB} + (1 - \frac{1}{2}x)\frac{1}{2};$$

where

$$P_{NB} = \frac{(1-\gamma)(1-\gamma x)}{1-2\gamma(1-\gamma)x}.$$

Since  $f(x, \theta) = (U_g + U_b)/2 - U_n$ , we obtain:

$$f(x, \theta) = \frac{2\gamma - 1}{4(1 - 2\gamma(1 - \gamma)x)} - \frac{1}{2}\theta,$$

which is increasing in  $x$ .

For  $x \in (\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\})$ ,  $t(x, \theta) = \tau(x, \theta) \in (0, 1)$ . Using the fact that  $P_{N|n} = (P_{N|g} + P_{N|b})/2$ , we obtain:

$$f(x, \theta) = \frac{1}{2}(\gamma - P_{N|g}) + \frac{1}{2}(P_{B|b} - P_{N|b}) = \frac{1}{2}(\gamma - P_{N|g}) - \frac{1}{2}\theta.$$

Therefore, we just need to prove that  $P_{N|g}$  is increasing in  $x$  given that  $t = \tau$  (for which we drop the arguments to save on notation). Note that

$$\begin{aligned} P_{N|g} &= [xP_{NB} + (1-x)P_{NN}] + (1-2\gamma(1-\gamma))x[(1-\tau)P_{NG} + \tau P_{NN} - P_{NB}], \\ P_{N|b} &= [xP_{NB} + (1-x)P_{NN}] + 2\gamma(1-\gamma)x[(1-\tau)P_{NG} + \tau P_{NN} - P_{NB}]. \end{aligned}$$

Therefore,

$$P_{N|g} = \frac{1-2\gamma(1-\gamma)}{2\gamma(1-\gamma)}P_{N|b} - \frac{1-4\gamma(1-\gamma)}{2\gamma(1-\gamma)}[xP_{NB} + (1-x)P_{NN}].$$

Since  $P_{N|b}$  is fixed at  $1 - \gamma + \theta$  when  $t = \tau$ , and since  $1 - 4\gamma(1 - \gamma) > 0$ ,  $P_{N|g}$  is increasing in  $x$  if and only if  $xP_{NB} + (1-x)P_{NN}$  is decreasing in  $x$ .

Suppose to the contrary that  $xP_{NB} + (1-x)P_{NN}$  is increasing in  $x$ . Then, since  $P_{NN} = 1/2$ , we have

$$x \frac{dP_{NB}}{dx} + (P_{NB} - P_{NN}) \geq 0,$$

which implies that  $dP_{NB}/dx > 0$  because  $P_{NB} - P_{NN} < 0$ . From equations (5) and (7), we see that both  $P_{NG}$  and  $P_{NB}$  depend on  $x$  through  $P_N$ , and that both are increasing in  $P_N$ . So, if  $dP_{NB}/dx > 0$ , we must have  $dP_{NG}/dx > 0$ . Since  $P_{N|b}$  is fixed at  $1 - \gamma + \theta$  when

$t = \tau$ , its derivative with respect to  $x$  is equal to zero. This condition gives:

$$x \frac{dP_{NB}}{dx} + (P_{NB} - P_{NN}) = 2\gamma(1 - \gamma) \left[ x \frac{dP_{NB}}{dx} - x(1 - \tau) \frac{dP_{NG}}{dx} - x(P_{NN} - P_{NG}) \frac{\partial \tau}{\partial x} - [(1 - \tau)P_{NG} + \tau P_{NN} - P_{NB}] \right] \geq 0.$$

Since  $2\gamma(1 - \gamma) < 1$ , this equation implies

$$x(1 - \tau) \frac{dP_{NG}}{dx} + x(P_{NN} - P_{NG}) \frac{\partial \tau}{\partial x} + \leq -(1 - \tau)[P_{NG} - P_{NN}].$$

This is a contradiction because each of the two terms on the left-hand-side is positive, while the right-hand-side is negative. We conclude that  $xP_{NB} + (1 - x)P_{NN}$  must be strictly decreasing in  $x$ , which implies that  $f(x, \theta)$  is strictly decreasing in  $x$  in this interval.

Finally, for  $x \geq \tilde{x}_0(\theta)$ ,  $t(x, \theta) = 0$ . Therefore,  $P_{N|n} = 1/2$  and  $f(x, \theta) = 0$ . This proves Claim 2.

*Claim 3.* There exists a unique solution of  $x \in (0, 1)$  to the equation  $f(x, \theta) = C'(x)$  if and only if  $C'(1) > \underline{f}(\theta)$ .

Assume for the moment that the marginal cost function is linear, i.e.,  $C'(x) = \alpha x$  for some constant  $\alpha$ . Let

$$g_1(x, \theta) \equiv \frac{2\gamma - 1}{4(1 - 2\gamma(1 - \gamma)x)} - \frac{1}{2}\theta.$$

Note that  $g_1(x, \theta) = f(x, \theta)$  for  $x \leq \tilde{x}_1(\theta)$ , and  $g_1(x, \theta) > f(x, \theta)$  for  $x > \tilde{x}_1(\theta)$ . Define  $\hat{\alpha}$  and  $\hat{x}$  such that  $C'(x)$  is tangent to  $g_1(x, \theta)$  if  $\alpha = \hat{\alpha}$  and  $x = \hat{x}$ . Direct calculation shows that

$$\hat{x} = \frac{1}{4\gamma(1 - \gamma)} > 1 > \tilde{x}_1(\theta).$$

Note that  $g_1(x, \theta) - C'(x)$  is convex in  $x$ , with  $g_1(0, \theta) - C'(0) > 0$ . Therefore the equation  $g_1(x, \theta) - C'(x) = 0$  has no solution if  $\alpha < \hat{\alpha}$ , or has two solutions if  $\alpha > \hat{\alpha}$ . When the equation has two solutions, the two roots  $x'$  and  $x''$  must satisfy  $x' < \hat{x} < x''$ , and  $g_1(x, \theta) - C'(x) < 0$  for all  $x \in (x', x'')$ . Let  $\alpha'$  be such that when  $\alpha = \alpha'$ , the smaller root  $x'$  is equal to  $\tilde{x}_1(\theta)$ . Note that  $\alpha' > \hat{\alpha}$ . There are two cases to consider.

- (a) If  $\hat{\alpha} \leq \alpha < \alpha'$ , then the smaller root to the equation  $g_1(x, \theta) - C'(x) = 0$  is greater than  $\tilde{x}_1(\theta)$ . We have  $g_1(x, \theta) > C'(x)$  for all  $x < \tilde{x}_1(\theta)$ .
- (b) If  $\alpha \geq \alpha'$ , then there is an  $x'$  such that  $g_1(x, \theta) > C'(x)$  for  $x < x'$  and  $g_1(x, \theta) < C'(x)$  for  $x \in (x', \tilde{x}_1(\theta))$ .

In cases (a),  $f(x, \theta) > C'(x)$  for  $x \in [0, \tilde{x}_1(\theta)]$ . Claim 2 establishes that  $f(x, \theta)$  is decreasing for  $x \in [\tilde{x}_1(\theta), 1]$ , while  $C'(x)$  is strictly increasing. By the intermediate value theorem there exists a unique solution  $x^* \in (\tilde{x}_1(\theta), 1)$  to  $f(x, \theta) = C'(x)$  if and only if  $C'(1) > \underline{f}(\theta)$ . In case (b),  $x^* = x'$  is the unique solution to  $f(x, \theta) = C'(x)$  for  $x \in [0, \tilde{x}_1(\theta)]$ . Furthermore, since  $f(x, \theta) - C'(x)$  is strictly decreasing for  $x \in [\tilde{x}_1(\theta), 1]$  and is non-positive at  $x = \tilde{x}_1(\theta)$ , there cannot be another solution to  $f(x, \theta) = C'(x)$ . Note finally that case (b) applies if and only if  $C'(\tilde{x}_1(\theta)) \geq \bar{f}(\theta)$ , which implies that  $C'(1) > \underline{f}(\theta)$ . This proves the claim when the marginal cost function is linear.

Next suppose that the marginal cost function is convex (i.e.,  $C''' \geq 0$ ). There are two cases. First, suppose  $C'(\tilde{x}_1(\theta)) < \bar{f}(\theta)$ . If we let  $\alpha = C'(\tilde{x}_1(\theta))/\tilde{x}_1(\theta)$ , then  $\alpha < \alpha'$ . Moreover, the analysis of case (a) and the convexity of  $C'$  implies that  $g_1(x, \theta) > \alpha x \geq C'(x)$  for  $x < \tilde{x}_1(\theta)$ . The rest of the analysis is the same. Next, suppose  $C'(\tilde{x}_1(\theta)) \geq g_1(\tilde{x}_1(\theta), \theta)$ . Then there must exist an  $x' \leq \tilde{x}_1(\theta)$  such that  $C'(x') = g_1(x', \theta)$ . Let  $\alpha = C'(x')/x'$ , then  $\alpha \geq \alpha'$ . The analysis of case (b) and the convexity of  $C'$  implies that  $g_1(x, \theta) > \alpha x \geq C'(x)$  for  $x < x'$ , and  $g_1(x, \theta) < \alpha x \leq C'(x)$  for  $x \in (x', \tilde{x}_1(\theta))$ . The rest of the analysis is the same. This establishes Claim 3.

We are now in the position to prove the proposition.

Case (i). Suppose  $C'(1) \leq \bar{f}(\theta)$ . Then  $x^* = 1$  and  $t^* = \tau(1, \theta) = 0$  is an equilibrium. Since message  $N$  does not occur in equilibrium, we can assign the off-equilibrium beliefs:  $P_{M_1 M_2} = \lim_{\dot{x} \rightarrow 1} P_{M_1 M_2}(\dot{x}, \tau(\dot{x}, \theta))$  for any  $M_1 = N$  or  $M_2 = N$ . Note that this implies  $MB(1, 1, 0, 0) = \underline{f}(\theta) \geq C'(1)$ , so the sender has no incentive to lower  $x^*$ . Suppose there exists another interior equilibrium  $x' < 1$ . Such  $x'$  must satisfy  $f(x', \theta) = C'(x')$ , which contradicts Claim 3.

Cases (ii) and (iii). Suppose  $C'(1) > \bar{f}(\theta)$ . Consider the candidate equilibrium with  $x = \dot{x} = x^*$  and  $t = \dot{t} = \tau(x^*, \theta)$ , where  $x^*$  satisfies  $f(x^*, \theta) = C'(x^*)$ . Recall that  $MB(\dot{x}, \dot{t})$  depends only on DM's belief and is constant with respect to the actual choice of  $x$  made by the sender. Therefore, the convexity of  $C$  implies that the sender's payoff is concave in  $x$ . Thus  $x$  maximizes his payoff if and only if  $x = x^*$ . Claim 3 establishes that  $x^*$  is unique. Moreover, by Lemma 3, the corresponding equilibrium hiding probability is  $\tau(x^*, \theta)$ . ■

**Proof for Proposition 4.** When  $x < \tilde{x}_1(\theta)$ ,

$$f(\theta, x) = \frac{2\gamma - 1}{4(1 - 2\gamma(1 - \gamma)x)} - \frac{1}{2}\theta,$$

which is strictly decreasing in  $\theta$ . When  $x \in [\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\}]$ ,

$$f(\theta, x) = \frac{1}{2}(\gamma - P_{N|g}) - \frac{1}{2}\theta.$$

We observe that  $P_{N|g}$  is decreasing in  $\tau(x, \theta)$ . By Lemma 2,  $\tau(\theta, x)$  is decreasing in  $\theta$ . So  $P_{N|g}$  is increasing in  $\theta$ , which implies that  $f(x, \theta)$  is strictly decreasing in  $\theta$ . Since an increase in  $\theta$  shifts down the marginal benefit curve  $f(\cdot, \theta)$ , and since  $x^*$  satisfies  $f(x^*, \theta) = C'(x^*)$  if  $x^* < 1$  and  $\lim_{x \rightarrow 1} f(x, \theta) > C'(1)$  if  $x^* = 1$ , we conclude that an increase in  $\theta$  lowers the equilibrium  $x^*$ . ■

**Proof of Proposition 5.** We claim that  $\tilde{x}_1^T(\theta) < \tilde{x}_1^S(\theta)$ . Suppose not, i.e.,  $\tilde{x}_1^T(\theta) \geq \tilde{x}_1^S(\theta)$ . Then we have

$$\theta = h^T(\tilde{x}_1^T(\theta), \tilde{x}_1^T(\theta), 1, 1) \leq h^T(\tilde{x}_1^S(\theta), \tilde{x}_1^S(\theta), 1, 1) < h^S(\tilde{x}_1^S(\theta), 1) = \theta,$$

a contradiction. Therefore,  $\tilde{x}_1^T(\theta) < \tilde{x}_1^S(\theta)$ .

Case (i).  $\dot{x} \leq \tilde{x}_1^T(\theta)$ . The senders are believed to hide unfavorable evidence with probability one in both models. Since  $P_{N|b} < P_N$  and  $P_{N|n} > P_N$ , we have

$$f^T(\dot{x}, \theta) = \frac{1}{2}\gamma + \frac{1}{2}(P_{N|b} - \theta) - P_{N|n} < \frac{1}{2}\gamma + \frac{1}{2}(P_N - \theta) - P_N = f^S(\dot{x}, \theta).$$

Case (ii).  $\dot{x} \in (\tilde{x}_1^T(\theta), \tilde{x}_1^S(\theta)]$ . The senders are believed to randomize under the two-sender model, but a single sender is believed to hide unfavorable evidence with probability one. Since  $f^T(\cdot, \theta)$  is decreasing while  $f^S(\cdot, \theta)$  is increasing in this interval, we have

$$f^T(\dot{x}, \theta) < f^T(\tilde{x}_1^T(\theta), \theta) < f^S(\tilde{x}_1^T(\theta), \theta) < f^S(\dot{x}, \theta).$$

Case (iii).  $\dot{x} \in (\tilde{x}_1^S(\theta), \min\{\tilde{x}_0^T(\theta), 1\})$ . The senders are believed to randomize in both models. Since  $P_{N|n} > P_N$ , we have

$$f^T(\dot{x}, \theta) = \frac{1}{2}\gamma + \frac{1}{2}(1 - \gamma) - P_{N|n} < \frac{1}{2}\gamma + \frac{1}{2}(1 - \gamma) - P_N = f^S(\dot{x}, \theta).$$

Case (iv).  $\dot{x} \geq \tilde{x}_0^T(\theta)$  (with  $\tilde{x}_0^T(\theta) < 1$ ). The senders are believed to always reveal unfavorable evidence in the two-sender model, but a single sender is believed to randomize between revealing and concealing evidence. In this case,  $f^T(\dot{x}, \theta) = 0 < f^S(\dot{x}, \theta)$ .



Finally,  $\dot{x} = 1$  in the two-sender model if

$$C'(1) \leq \underline{f}(\theta) = \lim_{\dot{x} \rightarrow 1} f^T(\dot{x}, \theta) < \lim_{\dot{x} \rightarrow 1} f^S(\dot{x}, \theta) = \gamma - \frac{1}{2} - \theta.$$

Therefore, whenever  $x^* = \dot{x} = 1$  in the two-sender case, we have  $x^* = \dot{x} = 1$  in the single-sender case as well. ■

**Proof for Proposition 6.** Denote the loss of the DM under the single-sender model by  $L^S(x^S, t^S, \gamma)$  and that under the two-sender model by  $L^T(x^T, t^T, \gamma)$ . Let  $L^{S*}$  and  $L^{T*}$  denote the equilibrium loss of DM in these two models. Then,  $L^S(1, 0, 1) - L^T(1, 0, 1) = 0$ . By the loss function's continuity in  $\gamma$ , for any  $\epsilon > 0$ , there exists a cutoff  $\hat{\gamma}(\epsilon)$  such that for any  $\gamma \in (\hat{\gamma}(\epsilon), 1)$ ,  $\epsilon > L^S(1, 0, \gamma) - L^T(1, 0, \gamma) > 0$ . Also,  $L^i(x^i, t^i, \gamma)$  is strictly decreasing in  $x^i$  for any  $t^i \in [0, 1]$  and any  $\gamma \in [0, 1]$  for  $i = T, S$ .

Given that  $C'(1) < 1/2 - \theta$ , we have  $C'(1) < \gamma - 1/2 - \theta$  for  $\gamma$  sufficiently close to 1. For such values of  $\gamma$ ,  $x^S = 1$  and  $t^S = 0$  in the single-sender model. Therefore the equilibrium loss for DM is  $L^{S*} = L^S(1, 0, \gamma)$ .

In the two-sender model, the condition that  $\tilde{x}_0(\theta)$  satisfies  $h(\tilde{x}_0(\theta), \tilde{x}_0(\theta), 0, 0) = \theta$  gives

$$\tilde{x}_0(\theta) = \frac{2\gamma - 1 - 2\theta}{(2\gamma - 1)^3}.$$

Since  $\tilde{x}_0(\theta) = 1 - 2\theta$  when  $\gamma = 1$ , we have  $\tilde{x}_0(\theta) < 1$  for  $\gamma$  sufficiently close to 1, which implies that  $C'(1) > \underline{f}(\theta) = 0$  for such  $\gamma$ . Since  $C'(x)$  is strictly positive, we have  $x^T < 1$  and  $t^T > 0$  in the symmetric equilibrium of the two-sender model. Let  $\epsilon = L^{T*} - L^T(1, 0, \gamma)$ , which is strictly positive by the strict monotonicity of the loss function. Then for any  $\gamma > \max\{\hat{\gamma}(\epsilon), C'(1) + \theta + 1/2\}$ , we have  $L^{T*} - L^{S*} = L^T(1, 0, \gamma) + \epsilon - L^S(1, 0, \gamma) > -\epsilon + \epsilon = 0$ . ■

**Proof of Proposition 7.** Direct calculation shows that  $h^{TS}(x, x, t, t) - h^{TO}(x, x, t, t)$  is equal to

$$\begin{aligned} & \frac{-(2\gamma - 1)^5 t^2 (1 - x)^2 x^2}{2(1 - x + 2\gamma(1 - \gamma)tx)(1 - x + (\gamma^2 + (1 - \gamma)^2)tx)} \\ & \times \frac{(1 - x + tx)}{(2 - 4x + 2tx + 2x^2 - 2tx^2 + t^2x^2 - 2\gamma t^2x^2 + 2\gamma^2 t^2x^2)} \end{aligned}$$

which is negative. ■

**Proof of Proposition 8.** We first establish the following claims.

*Claim 4.* Let  $\tilde{x}_1^{TS}(\theta)$  be the value of  $\dot{x}$  that solves  $h^{TS}(\dot{x}, \dot{x}, 1, 1) = \theta$ . Let  $\tilde{x}_0^{TS}(\theta)$  be the value of  $\dot{x}$  that satisfies  $h(\dot{x}, \dot{x}, 0, 0) = \theta$ . Let  $\tilde{x}_1^{TO}(\theta)$  and  $\tilde{x}_0^{TO}(\theta)$  represent the counterparts in the two-sender model with opposite biases. Then  $\tilde{x}_1^{TS}(\theta) < \tilde{x}_1^{TO}(\theta)$  and  $\tilde{x}_0^{TS}(\theta) = \tilde{x}_0^{TO}(\theta)$ .

Suppose to the contrary that  $\tilde{x}_1^{TS}(\theta) \geq \tilde{x}_1^{TO}(\theta)$ . Then, by Proposition 7 and by the fact that  $h^{TO}(\dot{x}, \dot{x}, 1, 1)$  is decreasing in  $\dot{x}$ , we have

$$\theta = h^{TS}(\tilde{x}_1^{TS}(\theta), \tilde{x}_1^{TS}(\theta), 1, 1) < h^{TO}(\tilde{x}_1^{TS}(\theta), \tilde{x}_1^{TS}(\theta), 1, 1) < h^{TO}(\tilde{x}_1^{TO}(\theta), \tilde{x}_1^{TO}(\theta), 1, 1) = \theta.$$

which is a contradiction. Therefore,  $\tilde{x}_1^{TS}(\theta) < \tilde{x}_1^{TO}(\theta)$ .

Furthermore, from the proof of Proposition 7, we see that  $h^{TS}(\dot{x}, \dot{x}, 0, 0) - h^{TO}(\dot{x}, \dot{x}, 0, 0) = 0$ . Thus,  $\tilde{x}_0^{TS}(\theta) = \tilde{x}_0^{TO}(\theta)$ . This establishes Claim 4.

*Claim 5.*  $f^{TS}(x, \theta)$  is strictly decreasing in  $x$  for  $x \in (\tilde{x}_1^{TS}(\theta), \min\{\tilde{x}_0^{TS}(\theta), 1\})$ .

For ease of notation, we will drop the superscript  $TS$  in establishing this claim. For  $x \in (\tilde{x}_1(\theta), \min\{\tilde{x}_0(\theta), 1\})$ ,  $t(x, \theta) = \tau(x, \theta) \in (0, 1)$ . As in the proof of Claim 3, we have

$$f(x, \theta) = \frac{1}{2}(\gamma - P_{N|g}) - \frac{1}{2}\theta.$$

Therefore, to show that  $f(x, \theta)$  is decreasing in  $x$ , we just need to prove that  $P_{N|g}$  is increasing in  $x$  given that  $t = \tau$ . Note that

$$\begin{aligned} P_{N|g} &= [xP_{NG} + (1-x)P_{NN}] + 2\gamma(1-\gamma)x[(1-\tau)P_{NB} + \tau P_{NN} - P_{NG}], \\ P_{N|b} &= [xP_{NG} + (1-x)P_{NN}] + (1-2\gamma(1-\gamma))x[(1-\tau)P_{NB} + \tau P_{NN} - P_{NG}]. \end{aligned}$$

Therefore,

$$P_{N|g} = \frac{2\gamma(1-\gamma)}{1-2\gamma(1-\gamma)}P_{N|b} + \frac{1-4\gamma(1-\gamma)}{1-2\gamma(1-\gamma)}[xP_{NG} + (1-x)P_{NN}].$$

Since  $P_{N|b}$  is fixed at  $1 - \gamma + \theta$  when  $t = \tau$ , and since  $1 - 4\gamma(1 - \gamma) > 0$ ,  $P_{N|g}$  is increasing in  $x$  if and only if  $xP_{NG} + (1 - x)P_{NN}$  is increasing in  $x$ .

Suppose to the contrary that  $xP_{NG} + (1 - x)P_{NN}$  is decreasing in  $x$ . Since  $P_{N|b}$  is fixed at  $1 - \gamma + \theta$  when  $t = \tau$ , its derivative with respect to  $x$  is equal to zero. This condition

gives:

$$\begin{aligned} & x \frac{dP_{NG}}{dx} + (1-x) \frac{dP_{NN}}{dx} + (P_{NG} - P_{NN}) \\ = & (1 - 2\gamma(1 - \gamma)) \left[ x \frac{dP_{NG}}{dx} - x\tau \frac{dP_{NN}}{dx} - x(1 - \tau) \frac{dP_{NB}}{dx} \right. \\ & \left. - x(P_{NN} - P_{NB}) \frac{\partial \tau}{\partial x} + [P_{NG} - \tau P_{NN} - (1 - \tau)P_{NB}] \right]. \end{aligned}$$

Note that (a)  $\partial \tau / \partial x < 0$ ; (b) both  $P_{NN}$  and  $P_{NB}$  are decreasing in  $x$ ; (c) both sides are negative; and (d)  $1 - 2\gamma(1 - \gamma) < 1$ . Therefore the above equation implies

$$(1 - x + x\tau) \frac{dP_{NN}}{dx} + x(1 - \tau) \frac{dP_{NB}}{dx} + x(P_{NN} - P_{NB}) \frac{\partial \tau}{\partial x} \geq (1 - \tau)(P_{NN} - P_{NB}).$$

There are two cases to consider. Case (i).  $dP_{NG}/dx \leq 0$ . In this case, the left-hand-side is negative because  $dP_{NB}/dx$  and  $dP_{NG}/dx$  have the same sign, while the right-hand-side is positive—a contradiction. Case (ii).  $dP_{NG}/dx > 0$ . In this case, we can rearrange the inequality to get

$$\begin{aligned} & x(1 - \tau) \frac{dP_{NB}}{dx} + (1 - x) \frac{dP_{NN}}{dx} + (P_{NG} - P_{NN}) \\ & \geq -x\tau \frac{dP_{NN}}{dx} - x(P_{NN} - P_{NB}) \frac{\partial \tau}{\partial x} + [P_{NG} - \tau P_{NN} - (1 - \tau)P_{NB}]. \end{aligned}$$

The right-hand-side of the above is positive. The left-hand-side is less than the derivative of  $xP_{NG} + (1 - x)P_{NN}$  because  $0 < dP_{NB}/dx < dP_{NG}/dx$ . But since by assumption the derivative of  $xP_{NG} + (1 - x)P_{NN}$  is negative, we have a contradiction. We conclude that  $xP_{NG} + (1 - x)P_{NN}$  must be increasing in  $x$ , which implies that  $f(x, \theta)$  is decreasing in  $x$  in this interval. This proves Claim 5.

We are now in the position to prove the proposition.

Case (i). Suppose  $\dot{x} \leq \tilde{x}_1^{TS}(\theta)$ . Then the senders are believed to hide unfavorable evidence with probability one under both the case of same bias and of opposite biases. Direct calculation shows that  $f^{TS}(\dot{x}, \theta) - f^{TO}(\dot{x}, \theta)$  is equal to

$$\frac{-(2\gamma - 1)^5(1 - \dot{x})^2 \dot{x}^2}{4(1 - 2\gamma(1 - \gamma)\dot{x})(1 - \dot{x} + 2\gamma(1 - \gamma)\dot{x})(2(1 - \dot{x}) + \dot{x}^2(\gamma^2 + (1 - \gamma)^2)\dot{x}^2)'}$$

which is negative.

Case (ii). Suppose  $\dot{x} \in (\tilde{x}_1^{TS}(\theta), \tilde{x}_1^{TO}(\theta)]$ . Then the senders are believed to randomize

between revealing and concealing evidence in the case of same bias, but they conceal evidence with probability one in the case of opposite biases. By Claim 2 in the proof of Proposition 3,  $f^{TO}(\dot{x}, \theta)$  is increasing in  $\dot{x}$  in this interval; and by Claim 5,  $f^{TS}(\dot{x}, \theta)$  is decreasing in  $\dot{x}$  in this interval. Therefore,

$$f^{TS}(\dot{x}, \theta) < f^{TS}(\tilde{x}_1^{TS}(\theta), \theta) < f^{TO}(\tilde{x}_1^{TS}(\theta), \theta) < f^{TO}(\dot{x}, \theta).$$

Case (iii).  $\dot{x} \in (\tilde{x}_1^{TO}(\theta), \min\{\tilde{x}_0^{TS}(\theta), 1\})$ . Then the senders are believed to randomize in both models. Let  $t^{TS}$  denote the probability of concealing evidence under the same bias model, and let  $t^{TO}$  denote the corresponding probability under the opposite biases model given the same belief  $\dot{x}$ . By Proposition 7, we have  $t^{TS} < t^{TO}$ . Let  $U_{N|n}^{TS}(\dot{x}, \dot{t})$  denote the expected payoff of sender 1 when he receives no evidence in the symmetric equilibrium of the same bias model with belief  $\dot{x}$  and  $\dot{t}$ . Define  $U_{N|n}^{TO}(t)$  similarly for the opposite biases model:

$$U_{N|n}^{TO}(\dot{x}, \dot{t}) = \frac{1}{2}\dot{x}P_{NB} + \frac{1}{2}\dot{x}(1 - \dot{t})P_{NG} + \left(\frac{1}{2}\dot{x}\dot{t} + 1 - \dot{x}\right)\frac{1}{2}.$$

Note that  $U_{N|n}^{TO}(\dot{x}, \dot{t})$  is decreasing in  $\dot{t}$  because the posteriors  $P_{NB}$  and  $P_{NG}$  are both decreasing in  $\dot{t}$ , and higher  $\dot{t}$  moves more weight from  $1/2$  to  $P_{NG} > 1/2$ . Now,

$$U_{N|n}^{TS}(\dot{x}, \dot{t}) - U_{N|n}^{TO}(\dot{x}, \dot{t}) = \frac{1}{2}\dot{x}\dot{t}(P_{NG} - P_{NB}) - \left(\frac{1}{2}\dot{x}\dot{t} + 1 - \dot{x}\right)\left(\frac{1}{2} - P_{NN}^{TS}\right).$$

A direct calculation shows that this is equal to

$$\frac{(2\gamma - 1)^5 \dot{t}^3 (1 - \dot{x}) \dot{x}^3 (1 - \dot{x} + \dot{x}\dot{t})}{4(1 - \dot{x} + 2\gamma(1 - \gamma)\dot{x}\dot{t})(1 - \dot{x} + (\gamma^2 + (1 - \gamma)^2)\dot{x}\dot{t})(((1 - \gamma)\dot{x}\dot{t} + 1 - \dot{x})^2 + (\gamma\dot{x}\dot{t} + 1 - \dot{x})^2)},$$

which is positive. Thus,

$$U_{N|n}^{TS}(\dot{x}, t^{TS}) > U_{N|n}^{TO}(\dot{x}, t^{TS}) > U_{N|n}^{TO}(\dot{x}, t^{TO}),$$

where the last inequality follows from the fact that  $U_{N|n}^{TO}(\dot{x}, \dot{t})$  is decreasing in  $\dot{t}$ . Since  $f(\dot{x}, \theta) = 1/2 - U_{N|n}$  for both the same bias model and the same bias model and opposite biases model in this interval, we have  $f^{TS}(\dot{x}, \theta) < f^{TO}(\dot{x}, \theta)$ .

Case (iv).  $\dot{x} \geq \tilde{x}_0^{TS}(\theta)$  (when  $\tilde{x}_0^{TS}(\theta) < 1$ ). Then the senders are believed to hide with zero probability in both cases. We have  $f^{TS}(\dot{x}, \theta) = f^{TO}(\dot{x}, \theta) = 0$ .

We have shown that  $f^{TS}(\dot{x}, \theta) < f^{TO}(\dot{x}, \theta)$  for  $\dot{x} < \tilde{x}_0^{TS}(\theta)$ . Since equilibrium investigation  $x^*$  is given by the condition that  $f(x^*, \theta) = C'(x^*)$  if  $x^* < 1$ ,  $\lim_{\dot{x} \rightarrow 1}(\dot{x}, \theta) \geq C'(1)$

if  $x^* = 1$ , and  $x^* < \min\{\tilde{x}_0^{TS}(\theta), 1\} = \min\{\tilde{x}_0^{TO}(\theta), 1\}$ , we conclude that the equilibrium investigation levels in these two models satisfy  $x^{TS} < x^{TO}$ . ■