<table>
<thead>
<tr>
<th>Title</th>
<th>Order batching and picking in a synchronized zone order picking system</th>
</tr>
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<tr>
<td>Author(s)</td>
<td>Pan, L; Huang, JZ; Chu, SCK</td>
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<tr>
<td>Issued Date</td>
<td>2011</td>
</tr>
<tr>
<td>URL</td>
<td><a href="http://hdl.handle.net/10722/158883">http://hdl.handle.net/10722/158883</a></td>
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<td>IEEE International Conference on Industrial Engineering and Engineering Management Proceedings. Copyright © IEEE.; ©2011 IEEE. Personal use of this material is permitted. However, permission to reprint/republish this material for advertising or promotional purposes or for creating new collective works for resale or redistribution to servers or lists, or to reuse any copyrighted component of this work in other works must be obtained from the IEEE.; This work is licensed under a Creative Commons Attribution-NonCommercial-NoDerivatives 4.0 International License.</td>
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Order picking has been considered as one of the most critical operations in warehouse. In this study, we propose an analytical approximation model based on probability and queueing network theory to analyze order batching and picking area zoning on the mean order throughput time in a synchronized zone picker-to-part order picking system. The resulting model can be easily applied in the design and selection process of order picking systems.

Keywords - Order batching, order picking, applied probability, queueing

I. INTRODUCTION

Order picking – the process of retrieval stock keeping units from storage (or buffer area) in response to a specific customer request – is the most critical supply chain component for many companies [1].

The total picking time can be roughly divided into three components: set-up time at the depot, time for picking products and traveling time between different locations by walking or driving. According to [1], the traveling time takes up approximately 50% of the total order picking time. Hence, the reduction in travel time will lead to an improvement in order picking throughput time. Besides, for a warehouse with a given layout, a predetermined storage strategy and routing policy, the two major factors that impact on the order picking system performance are zoning and batching. Zoning is to divide the whole storage area into several smaller zones and send pickers to each zone to pick requested items. Batching is a preliminary operation which is to group several orders together before picking. Zoning and batching are closely related issues. However, in most literatures, they are always studied separately. Yu and de Koster [2] study the impact of order batching and zoning in a pick-and-pass order picking system. In this paper, a probability model is constructed to study the average order throughput time in a parallel picker-to-parts order picking system.

II. PROBLEM DESCRIPTION

The order picking system under consideration is a picker-to-parts narrow-aisle and ABC-class strategy synchronized zone warehouse, as shown in Fig.1, having the following properties:

1) Each order requires a variety of items, which arrives at the warehouse according to a Poisson process. When the batch size reaches a prefixed number, those orders in this batch will be released for picking.

2) All picking is done in an active picking area constituted by several smaller picking zones utilizing bin shelving and manual order picking carts. The picking aisles are wide enough to allow a picker to change directions and also allow picking on both sides of the picking aisles.

3) Each order picker picks a single pick list at one time and there is only one picker per zone. Each picking tour begins and ends at a centralized depot where the order picker receives the pick list and deposits the picked items. For simplicity, it is assumed that the capacity of the pick bin is sufficiently large to contain all requested items to pick in this zone and stock outs never happen. Time taken to retrieve an item from a storage location is assumed to be a known constant.

4) A zone is a set of adjacent identical racks, which means one rack cannot belong to more than one zone.

5) ABC-class with across-aisle storage assignment strategy is adopted here with Return routing policy.

Fig.2 gives a schematic illustration of the resulting queueing network including a J-node E/1/G/1 fork-join sub-network with J zones, where node 1 to node J represent the parallel J picking zones and node J+1 represents the sorting station.

III. APPROXIMATION MODEL

In this section, we first derive the first and second moments of service time of an order batch in the picking zones, and the sorting station. Then, we apply the queueing network approximation model to calculate the average throughput time of a random order in the system. The order’s travel route is as sketched in Fig.1, where the picker enters the aisle containing picks from the front cross aisle only, picks the requested item, and then leaves the aisle from the front cross aisle, and the ABC-class strategy is illustrated in Fig.1 with racks nearest to the depot representing the first class type of goods, the racks behind them representing the second class type, and so on. For such type of picking system layout, we define the following notations:

- \( b \) Number of orders in a batch;
First and Second Moments of Service Time in a Zone

First, we derive the first and second moments of service time in a pick zone for the picking system. The travel time, $T_{tw}$, in zone $j$, given zone $j$ is visited and $q_j$ items to be picked, consists of two components: (1) travel time within the aisles, $T_{tw}$, and (2) travel time across the aisles, $T_{ta}$.

Under the assumption that items of the same class stored in the same rack are randomly distributed, the cumulative distribution function that the travel distance in zone $j$, given that $q'$ picks in this aisle with the farthest pick in class $k$, is

\[ P[d_{r} \leq s | q', q_j, F = r] = \sum_{r=1}^{k} \left( \sum_{i=1}^{n} P_{ia} \right)^{i} \left( \sum_{i=1}^{n} P_{ia} \right) - \sum_{i=1}^{n} P_{ia} \left( \sum_{i=1}^{n} P_{ia} \right) \left( \frac{s}{r} \right)^{r} \]

Then the mean travel time within aisle $k$, given $q'$ picks in this aisle with the farthest pick in class $r$, is

\[ E[T_{tw} | q', q_j, F = r] = \frac{1}{V} \sum_{r=1}^{k} \left( \sum_{i=1}^{n} P_{ia} \right)^{i} \left( \sum_{i=1}^{n} P_{ia} \right) \frac{1}{r} dP[\text{travel distance in } r \leq s | q', F = r] \]

Therefore, the expected travel time in aisle $k$ given $q'$ picks in $k$ can simply be represented as

\[ E[T_{tw} | q', q_j] = \sum_{r=1}^{k} E[T_{tw} | q', q_j, F = r] \cdot P[F = i | q', q_j] \]

where

\[ P[F = r | q', q_j] = \left( \sum_{i=1}^{n} P_{ia} \right)^{i} \frac{1}{r} \sum_{i=1}^{n} P_{ia} \left( \sum_{i=1}^{n} P_{ia} \right)^{r} \]

is the probability that, given $q'$ picks in aisle $k$, the farthest pick is in class $r$.

The expected travel time within one aisle given that $q_j$ picks to be picked in this zone can be calculated as

\[ E[L | q_j] = \sum_{q=1}^{n} E[T_{tw} | q', q_j] \cdot P[q' \text{ picks in aisle } k | q_j] \]

where

\[ P[q' \text{ picks in aisle } k | q_j] = \left( \frac{q}{q_j} \right) \left( \frac{1}{M} \right) \left( \frac{M - 1}{M} \right)^{q_j - q} \]

Hence, the expected within aisle travel time in zone $j$ given there are $q_j$ picks in $j$ can be approximated as twice the expected within aisle travel times the...
where \( P(\mathbf{Q}, \mathbf{b}) \) is the probability that a batch has \( Q \) items to be picked given that it contains \( b \) orders. An order should at least contain one order line. Here, the distribution is taken as negative binomial distribution:

\[
P(\mathbf{Q}, \mathbf{b}) = P(\mathbf{Q} = \mathbf{b} + n | Q) = \binom{x + \alpha - 1}{\alpha - 1} \left( \frac{\alpha}{\alpha + \nu} \right)^{\mathbf{b} + n} \left( 1 - \frac{\alpha}{\alpha + \nu} \right)^{x}
\]

where \( 0 < \nu < 1 \), and \( \alpha > 0 \) are constant parameters, and \( x = 0, 1, \ldots \) as used in [5]. The advantage of adopting this distribution is that a large number of distributions can be approximated by adjusting \( \alpha \) and \( \nu \).

The second moment of service time in zone \( j \) given there are \( q_j \) items to be picked in that zone is:

\[
E[S^2_j | q_j] = E[Tw_j + Ta_j + q_j \cdot t + \tau \cdot k \cdot \nu | q_j]
\]

\[
= E[Tw_j | q_j] + E[Ta_j | q_j] + 2E[Tw_j \cdot Ta_j | q_j] + (q_j \cdot t + \tau \cdot k) + 2q_j 
\]

(1)

Similarly as before, the calculation of the first term in (1), the second moment of within aisle travel time in zone \( j \) assuming \( q_j \) items can be approximated by:

\[
E[Tw_j, q_j] = E(2\sum_{i=1}^{q_j} Tw_i^2 | q_j) = 4E[M^2_j | q_j] \cdot E[L^2_j | q_j].
\]

(2)

To calculate \( E[M^2_j | q_j] \), first notice the variance of number of aisles visited given \( q_j \) picks is just the occupancy probability. Thus it can be computed as:

\[
Var(M_j | q_j) = M_j - 1/2 \left( M_j - 1/2 \right) \left( M_j - 1/2 \right) - \left( M_j - 1/2 \right)^2.
\]

(3)

Hence, we can have:

\[
E[M^2_j | q_j] = Var(M_j | q_j) + E[M_j | q_j]^2.
\]

Similarly as calculating its first moment, the second moment of the travel time within aisle \( k \), given \( q'_j \) picks in the aisle with the farthest pick class being \( i \), is:

\[
E[Tw_k | q'_j, q_j, F = r] = \int_{0}^{\infty} \left( \sum_{j=1}^{q'_j} f_j \right) dP(q_j,F = r)
\]

\[
= \int_{s}^{\infty} \left( \sum_{j=1}^{q'_j} f_j \right) dP(\text{travel distance in } r < s | q'_j, F = r)
\]

Then, we can show that the second moment of the travel time in aisle \( k \) given that \( q'_j \) picks in \( k \) is:

\[
E[Tw_k | q'_j, q_j, F = r] = \sum_{i=1}^{q_j} E[Tw_k | q'_j, q_j, F = r] \cdot P(q_j | q'_j)
\]

And the second moment of the travel time within one aisle given that \( q'_j \) picks in aisle \( j \) is:

\[
E[L^2_j | q_j] = \sum_{i=1}^{q_j} E[L^2_j | q'_j] \cdot P(q'_j | q_j)
\]

Therefore, \( E[Tw_k | q_j] \) can be approximated as in (2).

The second moment of the cross-aisle travel time given \( q_j \) picks in zone \( j \), \( E[T_{acj}^2 | q_j] \), can be calculated as two conditions according to whether the number of aisles is odd or even. Similarly as the analysis in calculating \( E[Ta_j | q_j] \), we can have

\[
E[T_{acj}^2 | q_j, M_j \text{ is even}]
\]

\[
\left( \frac{4\nu^2}{\nu M_j} \right) \sum_{k \geq 0} \left( \frac{\nu M_j}{2} \right)^{k+1} M_j^{-1} \left( 1 - \frac{M_j}{2} \right)_{k} + 2 \sum_{k \geq 0} \left( k^k + M_j \right) M_j^{-1}
\]

As for the product term \( E[Tw_j \cdot Ta_j | q_j] \) in (1), by assuming \( Tw_j \) and \( Ta_j \) are independent, we have:

\[
E[Tw_j \cdot Ta_j | q_j] = E[Tw_j | q_j] \cdot E[Ta_j | q_j]
\]

Therefore, \( E[S^2_j | q_j] \) can be computed as stated in (1). Then, the second moment of service time in zone \( j \) given that a batch consists of \( b \) orders can be calculated as:

\[
E[S^2_j | b] = \sum_{q_j=0}^{b} E[S^2_j | q_j] \cdot P(q_j | Q) \cdot P(Q, Q) = \sum_{q_j=0}^{b} \sum_{q'_j=0}^{q_j} E[S^2_j | q_j] \cdot P(q_j | Q) \cdot P(Q, Q)
\]

B. Mean and SCV of Maximum Service Time in Pick Zones

It is difficult to model job service time in fork-join synchronization models analytically. Indeed, to date, exact analytical results exist only for the mean response time of a two server system consisting of homogeneous \( M/M/1 \) queues [6]. In the previous section, we have derived the first and second moments of the service time in each zone node. Now, we consider \( J \) picking zones as one service node in a simple sequential queue, by using the derived approximation formula below to approximate the first and second moments of maximum service time among these zones.

Our approach is inspired by [7] for approximating the mean of the maximum of multiple random variables in a split-merge queue. In this queue, a job splits into a number of subtasks which are serviced in parallel. Only when all the subtasks finish servicing and rejoin can the next job split and start servicing. Hence, the fork-join time of the task is the maximum of the subtasks’ processing times.

For generally distributed random variables, similarly as Harrison and Zertal [7], we derive the first moment of the maximum of \( n \) independent, non-negative random variables with first moment \( e = (e_1, \ldots, e_n) \) and \( \mathbf{a} = (\alpha_1, \ldots, \alpha_n) \) is the Laplace form of the probability density function of the maximum of \( m \) exponential random variables with parameters \( \alpha = (\alpha_1, \ldots, \alpha_n) \) and \( \mathbf{a} = (\alpha_1, \ldots, \alpha_n, \alpha_m, \ldots, \alpha_n) \).

We now derive an approximation for the second moment of the maximum of a set of independent, non-negative random variables. First, consider \( T \)
= \max(T_1, T_2) \) for two non-negative exponential random variables \( T_1, T_2 \) with parameters \( \lambda_1, \lambda_2 \) respectively. Then,

\[
E[T^2] = E[T_1^2] + P(T_1 > T_2)E[T_1^2] + E[T_2^2] - T_1^2 = E[T_1^2] + 2E(\alpha_x)/\lambda^2
\]

Hence, the second moment of the maximum of \( n \) independent non-negative random variables can be approximated by \( \Phi^2(k, \alpha, \mathbf{E}) \) defined by the recurrence, for \( k = 2, \ldots, n \), with \( \Phi^2(1, \alpha_x, \mathbf{E}_x) = E \) as

\[
\Phi^2(k, \alpha, \mathbf{E}) = \sum_{m=1}^{k} \Phi^2(k-1, \alpha, \mathbf{E}_x) + (2E - \epsilon_k)J_{\epsilon_k}(\alpha_x, \alpha_x)
\]

Therefore, the first and second moments of maximum service time in zone \( j \) given \( b \) orders in a batch, \( E[SE]_b\mid b \) and \( E[SE^2]_b\mid b \), can be expressed by

\[
E[SE]_b\mid b = \Phi(M_2, \alpha, \mathbf{E}), \quad E[SE^2]_b\mid b = \Phi(M_2, \alpha, \mathbf{E})
\]

where \( \alpha = (E[SE]_b\mid b), \ldots, E[SE^2]_b\mid b) \), \( \mathbf{E} = (E[SE]_b\mid b), \ldots, E[SE^2]_b\mid b) \).

Hence, the squared coefficient of variation (SCV) of the maximum service time among all zones, given that there are \( b \) orders in a batch can be calculated by

\[
c_b^2 = \frac{E[SE^2]_b\mid b - E[SE]_b\mid b^2}{E[SE]_b\mid b^2}
\]

C. Mean and SCV of Service Time at the Sorting Station

Since orders are batched before picking, they need to be sorted again into the original orders and then sent for packing and transportation. Here, at the sorting station, the service time of a batch is modeled as a constant setup plus a sorting time and the sorting time per item is also a constant. Then, the service time for sorting a batch containing \( Q \) items can be represented as

\[
SE = sc + s, \quad Q
\]

where \( sc \) is the setup time and \( s, \) is the sorting time per item.

Hence, the first moment of the service time of sorting a batch containing \( b \) orders is

\[
E[SE]_b\mid b = sc + s \sum_{q=0}^{Q} Q \cdot P_2(Q, b) = sc + s \cdot E[Q]
\]

And the second moment of the service time of a batch containing \( b \) orders is

\[
E[SE^2]_b\mid b = sc^2 + 2scs + s^2 \cdot E[Q] + s^2 \cdot E[Q^2]
\]

where

\[
E[Q^2] = \sum_{q=0}^{\infty} Q^2 \cdot P_2(Q, b)
\]

Therefore, the SCV of the service time at the sorting station, given \( b \) orders in a batch can be expressed as

\[
c_b^2 = \frac{E[SE^2]_b\mid b - E[SE]_b\mid b^2}{E[SE]_b\mid b^2}
\]

D. Mean Throughput Time of \( b \) Orders in the System

The calculation of mean throughput of \( b \) orders in the parallel zoning order picking system is based on the open queueing network approximation model described in [8]. Fig. 3 is the simplified 2-stage tandem queue, after considering all \( J \) zones as one service node (node \( p \)). Each node operates like a \( E_1/G_1/1 \) queueing system. The approximation analysis uses two parameters, the mean service time and the SCV, to characterize the arrival process and the service time at each node.
\[ E[T | b] = E[T_p | b] + E[T_s | b] \]

E. Mean Throughput Time of a Random Order

The mean throughput time of an arbitrary order in the system consists of two components: the expected waiting time for the order to form a batch and the mean throughput time of picking and sorting in the system. Since the arrival of orders is a Poisson process, if \( t_n \) is the time of the \( n \)-th arrival, then \( t_n \) follows an Erlang distribution. Hence, for the \( n \)-th arrival of a Poisson process with parameter \( \lambda \), the expected arrival time is \( n/\lambda \). Accordingly, the expected waiting time to form a batch of \( b \) orders can be estimated as

\[ E[W_b | b] = \sum_{r=1}^{b} (r-1)/\lambda b \]

Therefore, the mean throughput time of a random order in the system is given by

\[ E[T_r | b] = E[W_b | b] + E[T | b] \]

F. Numerical Example

A simulation model of the paralleled zoning order picking system with 3 classes’ items has been implemented in C with the parameters of the warehouse in Table 1. The simulation model was run 100,000 times for each set of system parameters examined and so that the 98% confidence intervals are within 1% of the mean order picking throughput time in the system. It can be seen from Fig. 4 that the approximate model provides sufficient accuracy in estimating the expected throughput times.

IV. CONCLUSION

In this paper, we developed an analytical probability model based on an open queueing network approximation model to evaluate the performance of a synchronized picker-to-parts order picking system with online order arrivals. Firstly, the first and second moments of service times of each pick zone are derived and then an approximation formula is used to obtain the first and second moments of the maximum service time among all zones. After that, the order picking system is modeled as a 2-stage tandem queueing network with \( E_r/G/1 \) queues on each node. This study extends previous work by considering the synchronized zoning issue and the ABC-class storage strategy, which are widely used in practice. This paper also takes the subsequent sorting operation

<table>
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<th>Parameters</th>
<th>Value</th>
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<td>Number of zones</td>
<td>2</td>
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<tr>
<td>Order arrival rate</td>
<td>Poisson (0.0045)</td>
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<tr>
<td>Number of items per order</td>
<td>Negbinomial (4, 0.55) + 2</td>
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<tr>
<td>Number of aisles per zone</td>
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<tr>
<td>Length of an aisle</td>
<td>20 meters</td>
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<tr>
<td>Center-to-center distance between 2 aisles</td>
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<td>Conveyor transportation time</td>
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<td>Picker’s travel speed</td>
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<tr>
<td>Picking setup time in each zone</td>
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<tr>
<td>Picking time per item</td>
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<td>Sorting setup time</td>
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<td>Sorting time per item</td>
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<tr>
<td>Storage assignment (A:B:C)</td>
<td>(20:70) : (30:20) : (50:10)</td>
</tr>
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</table>

TABLE 1

Parameters used in the experiment

Fig. 4. The impact of batch size on mean order picking throughput time for ABC-class storage.

after picking into consideration.

Although this study is particularly aimed at synchronized zone order picking, it considers mainly common characteristics of many order picking systems. It is possible to extend our model to different order arrivals, zoning layout, storage strategies and routing policies, where the open queueing network approximation model can still be applied.

ACKNOWLEDGMENT

This study is partially supported by the Hong Kong RGC General Research Fund (GRF) Awards: HKU 7126/05E and HKU 7017/08P, and Shenzhen New Industry Development Fund under Grant No. CXB201005250021A.

REFERENCES


