Harmonic Analysis and Comparison of Permanent Magnet Vernier and Magnetic-Geared Machines

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In this paper, harmonic analysis and comparison of permanent magnet vernier (PMV) and magnetic-geared (MGd) machines are presented. The key is to analytically solve the governing Laplacian/quasi-Poissonian field equations in the air-gap regions. By using the time-stepping finite-element method, the analytical method is verified. Hence, the performances of the PMV machine are quantitatively compared with that of the MGd machine. The results show that the PMV machine can offer higher no-load EMF and higher torque-handling capability than its counterpart. Finally, the experimental result is given to further verify the validity of the analysis.

Index Terms—Analytical method, harmonic analysis, low-speed machine, permanent magnet machine, vernier machine.

I. INTRODUCTION

THE permanent magnet vernier (PMV) machine is a promising candidate for low-speed high-torque applications such as direct-drive wind power generation [1]. The corresponding analysis is generally based on magnetic circuit methods in which only the fundamental component is considered, whereas the harmonic components are ignored [2]. On the other hand, some viable analytical methods have been proposed for magnetic gears [3] and slotless PM machines [4]. However, the presence of stator slots has a large influence on the air-gap magnetic field and hence the motor performances. In recent years, more accurate analytical methods have been developed for the PM machines, which take into account the slotting effect [5]–[8].

In this paper, an analytical method for harmonic analysis of the PMV machine is presented. Hence, the corresponding harmonic spectra, no-load EMF, and electromagnetic torque are analytically calculated, and then compared with another low-speed PM machine which is called the magnetic-geared (MGd) machine [9]. The time-stepping finite-element method (TS-FEM) is used to provide numerical results to verify the accuracy of the analytical method.

II. MACHINE TOPOLOGY

The structure of a newly developed PMV machine [1] is shown in Fig. 1, which adopts an outer-rotor arrangement to facilitate direct-drive applications. The PMs are made of Nd–Fe–B, which are mounted on the inner surface of the rotor and are magnetized radially. The key is the introduction of the flux-modulation poles (FMPs) in the outer part of the inner stator, which functions to modulate the high-frequency rotating field of the armature windings and the low-frequency rotating PM field of the outer rotor. Thus, it can offer low-speed motion for direct-drive applications while can enable high-frequency rotating field design to maximize the power density. The corresponding relationship is governed by \( p_r = N_s - p_s \), where \( N_s \) is the number of FMPs in the stator, \( p_r \) is the number of PM pole-pairs in the rotor, and \( p_s \) is the number of armature winding pole pairs in the stator.

III. MAGNETIC FIELD ANALYSIS

In order to perform the analytical formulation for harmonic analysis, there are some basic assumptions: the permeability of both stator and rotor cores are infinite; the relative permeability of PMs is unity, the same as the air; and the end-effects are neglected. First, the machine model without considering the FMPs is analyzed. Second, the model with FMPs is analyzed. Both models are shown in Fig. 2. Meanwhile, the magnetic scalar potential is adopted to analyze the magnetic field of the machine.

A. Magnetic Field Without FMPs

Fig. 2(a) depicts the machine model without the FMPs. The corresponding scalar potential distribution in the PM subdomain I is governed by Poisson’s equation, while that in the air-gap subdomain II is by Laplace’s equation, which are expressed as follows:

\[
\frac{\partial^2 \varphi_1}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_1}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi_1}{\partial \theta^2} = \frac{\mu_0}{r} \frac{\partial M_r}{\partial \theta} \tag{1}
\]

\[
\frac{\partial^2 \varphi_{II}}{\partial r^2} + \frac{1}{r} \frac{\partial \varphi_{II}}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \varphi_{II}}{\partial \theta^2} = 0 \tag{2}
\]
where $\mu_0$ is the permeability of the vacuum and $M_r$ is the radial magnetization of the PM.

The general solutions of their magnetic scalar potentials can be expressed as follows:

$$
\varphi_I = \sum_{n=1}^{\infty} \left[ A_I(n) r^n + B_I(n) r^{-n} - \frac{M_n}{\mu_r (n^2 - 1)} \right] \cos(n\theta) \\
+ \sum_{n=1}^{\infty} \left[ C_I(n) r^n + D_I(n) r^{-n} - \frac{N_n}{\mu_r (n^2 - 1)} \right] \sin(n\theta)
$$

(3)

$$
\varphi_{II} = \sum_{n=1}^{\infty} \left[ A_{II}(n) r^n + B_{II}(n) r^{-n} \right] \cos(n\theta) \\
+ \sum_{n=1}^{\infty} \left[ C_{II}(n) r^n + D_{II}(n) r^{-n} \right] \sin(n\theta)
$$

(4)

where $A_I(n), B_I(n), C_I(n), D_I(n), A_{II}(n), B_{II}(n), C_{II}(n), D_{II}(n)$ are the coefficients determined by the boundary conditions. By applying the boundary conditions and the radial magnetization function, the air-gap flux density can be deduced as [6], [7] follows:

$$
B_{II} = \sum_{n=1}^{\infty} \frac{\mu_0 n (M_n \cos(n\theta) + N_n \sin(n\theta))}{\rho (n^2 - 1)} \\
\times \left( (n-1) \left( \frac{R_m}{R_t} \right)^{2n} + 2 \left( \frac{R_m}{R_t} \right)^{n-1} - (n+1) \right) \\
\times \left( \left( \frac{r}{R_m} \right)^{n-1} + \left( \frac{R_s}{R_m} \right)^{n-1} \left( \frac{R_s}{r} \right)^{n+1} \right) \\
+ \sum_{n=1}^{\infty} \frac{\mu_0 n}{R_s} \left( A_{stator}(n, \omega) \cos(n\theta) \right. \\
\left. + B_{stator}(n, \omega) \sin(n\theta) \right) \\
\times \frac{1}{\rho} \left( (\mu_r + 1) + (\mu_r - 1) \left( \frac{R_m}{R_t} \right)^{2n} \left( \frac{R_s}{r} \right)^{n+1} \right) \\
\times \left( (\mu_r - 1) + (\mu_r + 1) \left( \frac{R_m}{R_t} \right)^{2n} \left( \frac{R_s}{r} \right)^{n+1} \right) \right)
$$

(8)

$$
B_{II} = \sum_{n=1}^{\infty} \frac{\mu_0 n (N_n \cos(n\theta) - M_n \sin(n\theta))}{\rho (n^2 - 1)} \\
\times \left( (n-1) \left( \frac{R_m}{R_t} \right)^{2n} + 2 \left( \frac{R_m}{R_t} \right)^{n-1} - (n+1) \right) \\
\times \left( \left( \frac{r}{R_m} \right)^{n-1} + \left( \frac{R_s}{R_m} \right)^{n-1} \left( \frac{R_s}{r} \right)^{n+1} \right) \\
+ \sum_{n=1}^{\infty} \frac{\mu_0 n}{R_s} \left( B_{stator}(n, \omega) \cos(n\theta) \\
- A_{stator}(n, \omega) \sin(n\theta) \right) \\
\times \frac{1}{\rho} \left( (\mu_r + 1) + (\mu_r - 1) \left( \frac{R_m}{R_t} \right)^{2n} \left( \frac{R_s}{r} \right)^{n+1} \right) \\
\times \left( (\mu_r - 1) + (\mu_r + 1) \left( \frac{R_m}{R_t} \right)^{2n} \left( \frac{R_s}{r} \right)^{n+1} \right) \right)
$$

(9)

where $A_{stator}$ and $B_{stator}$ are the expansions of Fourier series; $M_n$ and $N_n$ are the Fourier coefficients of the radial magnetization scalar; and $\rho$ is given by

$$
\rho = (\mu_r - 1) \left( \left( \frac{R_m}{R_t} \right)^{2n} - \left( \frac{R_s}{R_m} \right)^{2n} \right) \\
+ (\mu_r + 1) \left( 1 - \left( \frac{R_s}{r} \right)^{2n} \right).
$$

(10)

B. Magnetic Field With FMPs

Fig. 2(b) depicts the machine model with the FMPs. The modulation effect of the FMPs can be described by using the concept of complex relative air-gap permeance. The permeance parameters are calculated by using the Schwarz–Christoffel transformation in such a way that the slotted structure can be transformed from the original plane to a new plane in which the structure becomes smooth. So, the analytical expression of magnetic flux density in the air gap is given by

$$
B_M = B_0 \lambda^* 
$$

(11)

where $B_0$ and $B_0$ are the air-gap flux densities with and without the FMPs, respectively, and $\lambda^*$ is the complex relative air-gap permeance. In polar coordinates, they can be written as

$$
B_M = B_{II\text{FMP}} \rho^* + B_{\theta\text{FMP}} \rho \theta^* \\
B_0 = B_{II\text{FMP}} + B_{\theta\text{FMP}} \theta 
$$

(12)

$$
\lambda^* = \lambda_{\rho} \rho^* - \lambda_{\theta} \theta^* 
$$

(13)

$$
\lambda_0 = \lambda_0 + \sum_{k=1}^{\infty} \lambda_{ak} \cos(kN \theta) \\
\lambda_{ak} = \sum_{k=1}^{\infty} \lambda_{ak} \sin(kN \theta) 
$$

(14)

where $\lambda_0$, $\lambda_{ak}$, and $\lambda_{ak}$ are the Fourier coefficients.

IV. PERFORMANCE CALCULATION

A. Electromagnetic Torque Calculation

The electromagnetic torque $T_e$ is generated by the interaction of the magnetic field generated by the PMs and the armature currents. It can readily be deduced by using the Maxwell stress tensor [8]:

$$
T_e = \frac{L R_e^2}{\rho_0} \int_0^{2\pi} B_{II\text{FMP}}(R_e, \theta) \cdot B_{\theta\text{FMP}}(R_e, \theta) \cdot d\theta 
$$

(15)

where $R_e$ is the effective radius of the integration path in the air gap, and $L$ is the axial length of the motor. Substituting (8)–(14) into (15), the analytical expression of the electromagnetic torque becomes

$$
T_e = \frac{\pi L R_e^2}{\rho_0} \sum_{n=1}^{\infty} \left( W_n X_n + Y_n Z_n \right) 
$$

(16)

where $W_n, X_n, Y_n,$ and $Z_n$ are the Fourier expansions that can readily be determined using the boundary conditions.

B. No-Load EMF Calculation

In order to compute the no-load EMF of a three-phase machine, the first step is to calculate the magnetic scalar potential of each slot $j$ at a given rotor position. Considering that the
current density is uniformly distributed over the slot area, the magnetic scalar potential can be averaged over the slot area to represent the coil as

$$\varphi_j = \frac{L}{S_{\text{slot}}} \int \int A_j(r, \theta) r \cdot dr \cdot d\theta.$$  \hspace{1cm} (17)

Hence, the three-phase flux vector can be expressed as

$$\begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix}' = n_{\text{turn}} \cdot [C] \cdot (\varphi_1 \varphi_2 \cdots \varphi_{2Q-1} \varphi_{2Q})' \hspace{1cm} (18)$$

where \(n_{\text{turn}}\) is the number of turns in series per phase and \([C]\) is a connecting matrix that represents the stator winding distribution in the slots. Consequently, at a given position \(\Delta\), the three-phase no-load EMF vector is represented by

$$\begin{pmatrix} E_a \\ E_b \\ E_c \end{pmatrix}' = \Omega \cdot \frac{d}{d\Delta} \cdot \begin{pmatrix} \psi_a \\ \psi_b \\ \psi_c \end{pmatrix}', \hspace{1cm} (19)$$

V. RESULTS AND VERIFICATION

The working principle of the FMPs in the PMV machine is similar to that of stationary ferromagnetic segments (SFSs) in the MGd machine. Thus, the aforementioned analytical derivation for the PMV machine can be easily extended to the MGd machine. Since the PMV machine is evolved from the MGd machine, a quantitative comparison between them can help justify the merits of the PMV machine. Consequently, both analytical calculation and TS-FEM are applied to both machines.

First, the TS-FEM is applied to analyze the magnetic field distributions of both the PMV machine and the MGd machine under no load at two extreme rotor positions, namely at 0° and 90° electrical degree, as shown in Fig. 3. It can be observed that the magnetic field rotates 90° electrical degree, while the rotor rotates only 3.75° mechanical degree. This indicates that this machine can provide low-speed high-torque operation. It also can be observed that the flux lines per stator tooth of the PMV machine can pass through the FMPs separately, and that of the MGd machine can pass through the SFSs separately, hence verifying the desired flux modulation in both machines. Actually, the main difference between them lies on the use of FMPs and SFSs. Notice that the PMV machine involves only one air gap, whereas the MGd machine involves two air gaps.

Second, the air-gap flux density waveforms and their harmonic spectra of both machines are simulated by using both the proposed analytical calculation and the TS-FEM. Figs. 4 and 5 depict the radial and tangential air-gap flux density components of the PMV machine, respectively. It is clearly seen that the 3rd, 24th, 48th, 72nd, and 120th harmonics have prominent values due to the slotting effect. Quantitatively, the averaged values of the radial and tangential components are 0.67 and 0.18 T, respectively. A larger radial component is preferable since it accounts for the effective torque, whereas a smaller tangential component is beneficial to limit the torque ripple. The corresponding analytical results agree well with the TS-FEM results.

On the other hand, the radial and tangential inner air-gap flux density components of the MGd are shown in Figs. 6 and 7, respectively. Their averaged values are 0.13 and 0.014 T, respectively. As expected, the results obtained by the analytical calcu-
Fig. 8. No-load EMF waveforms. (a) PMV machine. (b) MGd machine.

Fig. 9. Electromagnetic torque characteristics. (a) Waveforms. (b) Harmonic spectra.

Fig. 10. Final system.

Fig. 11. Measured waveforms of PMV machine. (a) No-load EMF (50 V/div, 2 ms/div). (b) Torque and current (2 N m/div, 2 A/div, 5 ms/div).

analysis. Fig. 11 shows the measured no-load EMF, torque, and current waveforms of the PMV machine. It can be found that the analytical and TS-FEM results agree well with the experimental results. Thus, the proposed analytical method and TS-FEM simulation for the PMV machine are verified, hence the validity for the MGd machine.

VI. CONCLUSION

In this paper, an analytical method for harmonic analysis of the PMV machine has been proposed and implemented. The technique is to analytically solve the governing Laplacian/quasi-Poissonian field equations in the air-gap regions. By using the TS-FEM and experimental results, the analytical method is verified. Also, the performances of the PMV machine are quantitatively compared with that of the MGd machine, confirming that the PMV machine can offer a higher no-load EMF by 15.6% and a higher torque-handling capability by 14.6%.

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