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A New Noncontact Method for the Prediction of Both Internal Thermal Resistance and Junction Temperature of White Light-Emitting Diodes

Xuehui Tao, Huanting Chen, Si Nan Li, and S. Y. Ron Hui, Fellow, IEEE

Abstract—Although critical to the lifetime of LED, the junction temperature of LED cannot be measured easily. Based on the general photoelectrothermal theory for LED systems, the coefficient for the reduction of luminous efficacy with junction temperature is first related to the characteristic temperature of the LED. Then, a noncontact method for estimating the internal junction temperature \( T_j \) and junction-case thermal resistance \( R_{jc} \) of LED from the external power and luminous flux measurements is presented and verified practically. Since these external measurements can be obtained easily, the proposal provides a simple tool for checking \( T_j \) in new LED system designs without using expensive or sophisticated thermal monitoring equipment for the LED junctions. The proposed method has been checked with measurements on LED devices from three different brands with both constant and nonconstant \( R_{jc} \). The theoretical predictions are found to be highly consistent with practical measurements.

Index Terms—Light-emitting diodes (LED) system theory, lighting.

I. INTRODUCTION

Despite the good acceptance of light-emitting diode (LED) technology in decorative, display, signaling, and signage applications, its applications in general or public lighting are still somewhat restricted. In the panel discussion of the International Forum in Shanghai 2009 [1], some of these problems were concluded as eye discomfort caused by the high color temperature (\( >6000 \) K) of LEDs involved, vulnerability to lightning and short lifetime of electronic LED drivers, gradual degradation of luminous output as the LED fixtures warm up, gradual loss of cooling effects of heat sink due to dust deposition and birds’ excretion. LEDs with high color temperature tend to have higher luminous efficacy. The imminent requirement of using warm-colored LEDs (with lower color temperature) signifies the importance of optimal design of the entire LED system, including the right choice of LED devices, the use of highly reliable LED drivers (preferably with lifetime exceeding ten years and high robustness against extreme weather conditions such as lightning and wide temperature range), appropriate thermal management [2]–[6], and proper mechanical design of the lighting fixture. While the mechanical fixture design against dust deposition and bird’s excretion is beyond the scope of this study, this project focuses on the extension of the general photoelectrothermal (PET) theory that links up the relationships of light, power, and heat [7]. This steady-state PET theory provides a useful guidance for choosing the LEDs with lowest junction-to-case thermal resistance \( R_{jc} \) and determining the required thermal resistance for the optimal thermal management. The offline passive LED driver [8] that does not require controlled power switches, electrolytic capacitors, auxiliary power supply, and control integrated circuits offers a reliable solution to meet the long-lifetime requirements. However, one aspect of LED system that is often neglected by design engineers is the prediction of the operating junction temperature \( T_j \) and the junction-case thermal resistance \( R_{jc} \). It has been shown that about 85–90% of LED power is dissipated as heat [9]. In addition, it is not an easy task to measure \( R_{jc} \) [10]. While \( T_j \) cannot be measured easily unless using sophisticated method such as laser or expensive equipment like TeraLED Transient Thermal Tester (T3ster) system [11], [12], it is critical to the lifetime of the LED device and, therefore, the entire LED product. LED manufacturers usually provide typical \( R_{jc} \) values in their data sheets based on the rated power, rather than the actual portion of LED power dissipated as heat. Without any theoretical tool for predicting \( T_j \) and \( R_{jc} \), LED system designers cannot optimize their products easily.

Several methods [13]–[15] have been proposed to estimate the internal junction temperature for white LEDs. Among them, the authors in [15] propose a “noncontact” method based on 1) the measurements of the ratio of the total radiant energy and radiant energy within the blue emission and on 2) the assumption that the thermal resistance of the LED package is constant (using \( R_{jc} \) in the data sheets). This breakthrough provides a convenient way to monitor the internal junction temperature \( T_j \) without using direct monitoring equipment. Based on the general PET theory for LED systems [7], new equations for predicting both \( T_j \) and \( R_{jc} \) based on external luminous flux measurements and LED power are presented in this paper. While the basic concept was briefly described in a conference paper [16], this paper further develops the theory behind this new
noncontact method, with new equations using easily accessible variables and parameters and without the assumption that \( R_{jc} \) is always constant. [Note that \( R_{jc} \) may or may not be a constant depending of the package structures.] Since it is easy to measure luminous flux and LED power externally, this noncontact approach offers a simple method for estimating internal variables and parameters such as \( T_j \) and \( R_{jc} \) so that LED system designers can check if their LED system design will operate within the LED safe operating conditions or not in the design stage before mass production. Practical \( T_j \) and \( R_{jc} \) measurements of several types of LEDs (with both constant and nonconstant \( R_{jc} \)) obtained from the TeraLED T3ster system are used to compare with the theoretical predictions based on external power and luminous flux measurements. Good agreements with measurements and predictions have been obtained.

II. REVISIT OF THE PET THEORY

The general PET theory is first revisited briefly before it is extended to derive the new equations for predicting \( T_j \) and \( R_{jc} \). Assuming a general LED system with \( N \) LED devices mounted on a heat sink, Fig. 1 shows a simplified thermal equivalent circuit. In practice, heat sink compound or equivalence may be used between the LEDs and the heat sink to ensure good thermal contact. The thermal resistance of such thermal compound is smaller than 0.0045 \(^\circ\)C·in\(^2\)/W [18], [19] and is relatively small when compared with \( R_{jc} \) of LEDs (typically in the order of several \(^\circ\)C/W) and is neglected in the following analysis.

The total luminous flux \( \phi_o \) of an LED system consisting of \( N \) identical LED devices can be expressed as

\[
\phi_o = N \times E \times P_d \tag{1}
\]

where \( E \) is the luminous efficacy (lm/W) and \( P_d \) is the real power of one LED (W). The emission intensity \( (I) \) of LEDs decreases with increasing temperature. Near room temperature, the emission intensity follows an exponential decay function [17]:

\[
I = I_{25} \exp \left( -\frac{(T_j - 25^\circ C)}{T_1} \right) \tag{2}
\]

where \( T_1 \) is the characteristic temperature.

The exponential curve of (2) within this practical range of operating temperature is fairly linear and will be approximated as

\[
E = E_o \left[ 1 - \frac{1}{T_1} (T_j - T_o) \right] \tag{3a}
\]

\[
E = E_o \left[ 1 + k_e (T_j - T_o) \right] \quad \text{for } T_j \geq T_o \text{ and } E \geq 0 \tag{3b}
\]

where \( E_o \) is the rated efficacy at the rated temperature \( T_o \) (typically 25 \(^\circ\)C in some LED data sheets). The derivation of (3a) from (2) is given in the Appendix. It should be noted that \( k_e \) is the relative rate of reduction of luminous efficacy with increasing temperature. Comparison of (3a) and (3b) shows that the coefficient \( k_e \) used in the PET theory is

\[
k_e = -\frac{1}{T_1}. \tag{4}
\]

Based on the model in Fig. 1, the steady-state heat sink temperature can be expressed as

\[
T_{hs} = T_o + R_{hs} (N P_{heat}) = T_o + R_{hs} (N k_h P_d) \tag{5}
\]

where \( T_o \) is the ambient temperature. From Fig. 1 and (5), the junction temperature of each LED is therefore

\[
T_j = T_{hs} + R_{jc} k_h P_d = T_o + (R_{jc} + N R_{hs}) k_h P_d. \tag{6}
\]

Combining (6) and (3b) gives

\[
E = E_o \left[ 1 + k_e (T_o - T_j) + k_e k_h (R_{jc} + N R_{hs}) P_d \right]. \tag{7}
\]

The total luminous flux \( \phi_o \) is

\[
\phi_o = N E P_d
\]

\[
\phi_o = N E_o \left[ \frac{1 + k_e (T_o - T_j)}{P_d} \right]
\]

\[
+ k_e k_h (R_{jc} + N R_{hs}) P_d^2. \tag{8}
\]

In summary, (6) in the original PET theory provides the equation for the internal junction temperature of the LED package. It must, however, be stressed that while \( k_e \) is given in some LED data sheets or can be obtained from (4), \( k_h \) is usually not available and \( R_{jc} \) is not a constant even though it is assumed as constant in data sheets. Therefore, the practical procedures for determining \( k_h \) and \( R_{jc} \) are essential before \( T_j \) can be estimated accurately. In the next section, we focus on such procedures with the emphasis that \( T_j \) can be obtained based on externally measurable variables and parameters.

III. REARRANGING EQUATIONS FOR \( R_{jc}, k_h, \text{ and } T_j \)

A. Determination of \( k_h \)

The LED power can be defined as \( P_d = V_d \times I_d \), where \( I_d \) is the diode current and \( V_d \) is the diode voltage. But only part of the power will be dissipated as heat. Thus, the heat generated in one LED is defined as

\[
P_{heat} = k_h P_d \tag{9}
\]

where \( k_h \) is less than 1.

The heat dissipation coefficient \( k_h \) can be determined by using the thermal measurement method detailed in [9]. Such method provides accurate results but takes a fairly long time to obtain all the results because, for each measurement, the silicon oil takes a few hours to reach its steady-state temperature. A second method is described here as an alternative.

The wall-plug efficiency \( \eta_w \) indicates the useful portion of the electrical power and it is defined as the ratio of optical...
power $P_{\text{opt}}$ to electrical power $P_d$. For LED, both the junction temperature $T_j$ and electrical power $P_d$ affect the optical power $P_{\text{opt}}$. So, $\eta_w$ is a 2-D parameter [14]

$$\eta_w = \frac{P_{\text{opt}}}{P_d}. \quad (10)$$

The heat dissipation coefficient can be related to the wall-plug efficiency as

$$k_h = \frac{P_{\text{heat}}}{P_d} = \frac{P_d - P_{\text{opt}}}{P_d} = 1 - \eta_w. \quad (11)$$

The polynomial method [20] can be applied to solve the $\eta_w$ equation, which is a function of both $T_j$ and $P_d$. First, the dependence of $\eta_w$ on $T_j$ is established. By keeping $P_d$ at a constant value of $P_{d0}$, experiments can be conducted to plot the wall-plug efficiency $\eta_w$ expressed as a function of the $T_j$

$$\eta_w(T_j, P_{d0}) = \alpha T_j + \beta$$

where the coefficients $\alpha$ and $\beta$ are constant values that can be obtained from the plots of measurements (as will be demonstrated in Section IV).

It is, however, more convenient to replace $T_j$ with $T_{\text{hs}}$ in (12) because it is easier to measure the heatsink temperature than the junction temperature. Therefore, it is further proposed in this paper a new equation for $\eta_w$. Using (6), (11), and (12), the wall-plug efficiency $\eta_w$ at constant electrical power can be expressed as

$$\eta_w(T_j, P_{d0}) = \alpha [T_{\text{hs}} + R_{jc} P_{d0}(1 - \eta_w)] + \beta.$$ \hspace{1cm} (13)

Rearranging (13) leads to

$$\eta_w(T_j, P_{d0}) = \frac{\alpha(T_{\text{hs}} + R_{jc} P_{d0}) + \beta}{1 + \alpha P_{d0} R_{jc}}.$$ \hspace{1cm} (14)

At a constant electrical power $P_{d0}$, the specific form of $\eta_w(T_j, P_{d0})$ in (14) can be rearranged in terms of $T_{\text{hs}}$ as follows:

$$\eta_w(T_{\text{hs}}, P_{d0}) = \sigma T_{\text{hs}} + \tau$$ \hspace{1cm} (15)

where $\sigma = [\alpha/(1 + \alpha P_{d0} R_{jc})]$ and $\tau = (\alpha R_{jc} P_{d0} + \beta)/(1 + \alpha P_{d0} R_{jc})$. The parameters $\sigma$ and $\tau$ can be practically determined as demonstrated in Section IV.

Second, the dependence of $\eta_w$ on $P_d$ is developed. Here, the heatsink temperature is fixed at $T_{\text{hs,0}}$ and measurements of the wall-plug efficiency $\eta_w$ are obtained for a range of $P_d$ and plotted as the following function:

$$\eta_w(T_{\text{hs,0}}, P_d) = \chi P_d^2 + \delta P_d + \gamma$$ \hspace{1cm} (16)

where the coefficients $\chi$, $\delta$, and $\gamma$ are constant values that can be obtained as shown in Section IV.

The 2-D function of $\eta_w$ can then be established by combining (15) and (16) as

$$\eta_w(T_{\text{hs}}, P_d) = \frac{(\sigma T_{\text{hs}} + \tau)(\chi P_d^2 + \delta P_d + \gamma)}{\mu} = (\alpha' T_{\text{hs}} + \beta') (\chi P_d^2 + \delta P_d + \gamma)$$ \hspace{1cm} (17)

where $\mu$ is a constant, which is the value of $\eta_w$ at point $(T_{\text{hs,0}}, P_{d0})$, and $\alpha' = \sigma/\mu$ and $\beta' = \tau/\mu$ [21]. Substituting (17) into (11), the new $k_h$ expression is finally established as

$$k_h = 1 - (\alpha' T_{\text{hs}} + \beta') (\chi P_d^2 + \delta P_d + \gamma).$$ \hspace{1cm} (18)

The parameters $\alpha'$, $\beta'$, $\chi$, $\delta$, and $\gamma$ can be determined practically as shown in Section IV.

B. Determination of $R_{jc}$

If the thermal resistance $R_{jc}$ is a constant as quoted in the manufacturers’ data sheets, (6) provides a means to calculate the internal junction temperature of an LED. However, in some cases, $R_{jc}$ is not a constant because heat flow is, in practice, a 3-D process instead of a 1-D one. In order to estimate $T_j$ accurately, it is necessary to determine $R_{jc}$ first.

In order to predict $R_{jc}$ and $T_j$, (8) can be rearranged so that $R_{jc}$ becomes the subject of the equation as follows:

$$R_{jc} = \frac{\phi_v}{N E_o k_c k_h} P_d^2 - 1 + \frac{k_v(T_a - T_0)}{k_c k_h} P_d^{-1}$$ \hspace{1cm} (19)

$$- N R_{hs}$$ \hspace{1cm} for $P_d > 0$

where $k_h$ obeys (18).

Because the heat sink temperature can be easily measured by a thermal sensor in practice, it is more convenient to express (19) in terms of $T_{\text{hs}}$ instead of $R_{hs}$ and $T_a$ with the use of (6). The new junction thermal resistance equation can be rearranged as

$$R_{jc} = \frac{\phi_v}{N E_o k_c k_h} P_d^2 - 1 + \frac{k_v(T_{\text{hs}} - T_0)}{k_c k_h} P_d^{-1}$$ \hspace{1cm} (20)

It can now be seen that (20) gives a new method to estimate the $R_{jc}$ in terms of externally measurable parameters and variables. Here, $k_c$ can be obtained from data sheets and $k_h$ can be obtained either from experiment [9] or (18). LED power $P_d$, luminous flux $\phi_v$, and heat sink temperature $T_{\text{hs}}$ can all be measured.

C. Determination of $T_j$

With $k_h$ and $R_{jc}$ determined, (6) can now be used to obtain $T_j$:

$$T_j = T_{\text{hs}} + R_{jc} k_h P_d = T_a + (R_{jc} + N R_{hs}) k_h P_d.$$ \hspace{1cm} (21)

If $k_h$ from (18) is used instead of using the method in [9], (6) can be expressed as

$$T_j = (R_{jc} + N R_{hs}) [P_d - (\alpha' T_{\text{hs}} + \beta') (\chi P_d^2 + \delta P_d + \gamma P_d)] + T_a.$$ \hspace{1cm} (22)

Equations (20) and (21) will be used in the next section for calculating the theoretical values of $R_{jc}$ and $T_j$, respectively.

IV. PRACTICAL EVALUATION AND VERIFICATION

In order to confirm that $R_{jc}$ and $T_j$ can be estimated with a reasonable degree of accuracy using externally measured luminous flux, a TeraLED T3ster system (see Fig. 2) is used to provide the practical measurements for comparison with the theoretical predictions. The T3ster system has an actively temperature-controlled mounting plate for the LED device. This sophisticated
LED equipment allows internal variables, such as the junction temperature and thermal resistance, and also the external variables, such as the luminous flux and the electric power of an LED, to be measured. With the help of this TeraLED T3ster system to provide both the internal and external measurements, experiments have been set up for several types of LED devices in order to check if the modified equations offer accurate predictions of $R_{jc}$ and $T_j$ or not.

LED devices from Cree, Philips, and Sharp are used as examples in this evaluation. Individually, each type of LED is mounted on the cold plate of the T3ster system, and the temperature of the plate is kept constant at 30°C. This means that the heat sink temperature $T_{hs}$ for the LED is 30°C in the analysis.

The T3ster system is then used to measure $T_j$, $R_{jc}$, $\phi_v$, and $E_v$.

The efficacy $E_v$ used in the calculation is measured by operating the LED at 25°C for a short time.

### A. Cree XR-LED

The parameters used in the analysis for the Cree LED are listed in Table I. The model number is XREWHT-L1-0000-007F5.

Under the constant power ($P_{d0} = 1.06$ W), the measured $\eta_w$ points and the traced polynomial line versus different LED case temperature are plotted in Fig. 3; here, the case temperature is equivalent to the heat sink temperature $T_{hs}$ because the temperature of the mounting plate is actively controlled.

Thus, for this case, the wall-plug efficiency in (15) becomes

$$\eta_w(T_{hs}, 1.06) = -0.000276 \times T_{hs} + 0.207198.$$  \hfill (22)

Under the constant case temperature ($T_{hs} = 30$ °C), the measured $\eta_w$ points and the traced polynomial line versus different LED power are plotted in Fig. 4.

Thus, $\eta_w$ in (16) becomes

$$\eta_w(30, P_d) = 0.015299P_d^2 - 0.078292P_d + 0.26422.$$  \hfill (23)

From (17), the entire 2-D wall-plug efficiency $\eta_w$ equation is

$$\eta_w(T_{hs}, P_d) = \left( -0.000276T_{hs} + 0.207 \right) \frac{(0.015P_d^2 - 0.078P_d + 0.264)}{0.2014}.$$  \hfill (24)

Comparing the coefficients of (17) and (24), the coefficients are found to be

$$\alpha' = -0.000276 \quad \beta' = 0.207198 \quad \chi = 0.015299 \quad \delta = -0.078292 \quad \gamma = 0.26422.$$  \hfill (25)
By submitting these coefficients of (25) into the \( k_h \) parameter equation (18), the calculated heat dissipation coefficient \( k_h \) can be obtained. The measured and calculated \( k_h \) parameters are shown in Fig. 5.

By substituting the calculated \( k_h \) in Fig. 5 together with the parameters in Table I into (20), the junction thermal resistance \( R_{jc} \) can be calculated. Fig. 6 shows the measured and calculated \( R_{jc} \). The calculated values derived from the externally measured \( \phi_v \) and \( P_d \) are found to be fairly accurate.

With the calculated \( R_{jc} \), according to (21), \( T_j \) can be calculated and is plotted in Fig. 7 with the measured junction temperature. It can be seen that the calculated and measured junction temperature values agree well with each other.

### Parameters Used for Philips LED \( R_{jc} \) and \( T_j \) Calculation

<table>
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<tr>
<th>( I(A) )</th>
<th>( K_c )</th>
<th>( N )</th>
<th>( T_{hs}(^\circ C) )</th>
<th>( T_j( ^\circ C) )</th>
<th>( E_p(\text{Lum/Watt}) )</th>
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<td>1</td>
<td>30</td>
<td>25</td>
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**B. Philips LXHL Cool-White LED**

LED from Philips with model number LXHL-PW01 is used to repeat the experiments. Table II tabulates the parameter values used in the theoretical calculation.

Under the constant power \( (P_d = 1.10 \text{ W}) \), the measured \( \eta_w \) points and the traced polynomial line versus different LED case temperature are plotted in Fig. 8.

Thus, for this case, the wall-plug efficiency in (15) becomes

\[
\eta_w (T_{hs}, 1.10) = -0.000188 \times T_{hs} + 0.13666.
\]  

Under the constant case temperature \( (T_{hs} = 30 \ ^\circ C) \), the measured \( \eta_w \) points and the traced polynomial line versus different LED power are plotted in Fig. 9.

Thus, \( \eta_w \) in (16) becomes

\[
\eta_w (30, P_d) = 0.01475 P_d^2 - 0.066037 P_d + 0.186311.
\]
From (17), the entire wall-plug efficiency \( \eta_w \) equation is

\[
\eta_w(T_{hs}, P_d) = \frac{(-0.00188T_{hs} + 0.137) (0.0148P_d^2 - 0.066P_d + 0.186)}{\eta_w(30, 1.10)} = \frac{(0.0148P_d^2 - 0.066P_d + 0.186)}{0.13145}.
\]

Comparing (17) and (28), the coefficients are

\[
\alpha' = -0.00188 = -0.0014 \quad \beta' = 0.13666 = 1.0396 \quad \chi = 0.01475 \quad \delta = -0.066037 \quad \gamma = 0.186311.
\]

Once these coefficients are found, the \( k_h \) equation can be defined easily by submitting the coefficients of (29) into (18).

Putting the calculated \( k_h \) in Fig. 10 together with the parameters in Table II into (20), the junction thermal resistance \( R_{jc} \) can be calculated. Fig. 11 shows the measured and calculated \( R_{jc} \). The calculated values derived from the measured \( \phi_v \) and \( P_d \) are found to be fairly accurate.

With the calculated \( R_{jc} \) and using (21), \( T_j \) is calculated and plotted in Fig. 12 with the measured junction temperature. Unlike the Cree LED, the \( R_{jc} \) of this Philips LED is not a constant. This illustrates the important fact that the assumption of a constant \( R_{jc} \) [15] is not necessarily valid. It can be seen that the calculated and measured junction temperature values are consistent with each other.

### C. Sharp LED

The same sets of tests are then carried out using the Sharp GW5BWC15L02 LED. Table III shows the parameters used in the theoretical calculation. The measured and calculated \( R_{jc} \) curves based on (20) are plotted in Fig. 16. The measured and calculated \( T_j \) curves based on (21) are shown in Fig. 17. Again, very good agreements between the measurements and predictions have been obtained.
Under the constant power \( (P_d) = 7.76 \text{ W} \), the measured \( \eta_w \) points and the traced polynomial line versus different LED case temperature are plotted in Fig. 13.

Under the constant case temperature \( (T_{hs}) = 60 \, ^\circ \text{C} \), the measured \( \eta_w \) points and the traced polynomial line versus different LED power are plotted in Fig. 14.

From (17), the entire 2-D wall-plug efficiency \( \eta_w \) equation is

\[
\eta_w(T_{hs}, P_d) = \left( -0.00014T_{hs} + 0.15 \right) \left( 0.00025P_d^2 - 0.013P_d + 0.229 \right) \times 0.142.
\]

According to (30), the coefficients are

\[
\alpha' = \frac{-0.000141}{0.142} = -0.00099
\]
\[
\beta' = \frac{0.150766}{0.142} = 1.062
\]
\[
\chi = 0.000253, \quad \delta = -0.01315, \quad \gamma = 0.228579.
\]

Putting these parameters in (18), the calculated \( k_h \) is plotted in Fig. 15. The theoretical and practical results of the junction thermal resistance \( R_{jc} \) and junction temperature \( T_j \) are in Figs. 16 and 17, respectively. The good agreements confirm the validity of the proposed equations that are based on external measurements.

The practical and theoretical results obtained in these three types of LED show the validity of the proposed noncontact approach for estimating the \( R_{jc} \) and \( T_j \). One important observation is that \( R_{jc} \) is not always a constant. LED manufacturers usually provide a constant value of \( R_{jc} \) measured at its rated power in their data sheet and do not give the information of \( R_{jc} \) variation with applied power. This theoretical method provides a valid tool to estimate the \( R_{jc} \) and its slight variation with power, which is very important to the research on thermal management of LED.

V. Conclusion

The general PET theory for LED systems has been used to derive new equations for predicting the thermal resistance and junction temperature of LED devices based on externally measurable parameters and variables. Unlike previous noncontact method that relies on the assumption of constant thermal resistance of LED, this new method enables the estimation of both thermal resistance and junction temperature. For predicting the internal junction temperature and thermal resistance of the LED packages, which are critical to the lifetime of the devices, the proposed approach avoids the need for using sophisticated LED monitoring equipment. Since the proposed method requires only the measurements of luminous flux, LED power, and heat sink temperature that can be obtained easily, it can be adopted as a simple tool by design engineers to check if the internal temperature of the LED may exceed its safety limit before finalizing their designs. This method has been practically verified with LEDs of different brands with constant and nonconstant thermal resistance. Measurements and theoretical predictions have very
good agreements, confirming the validity of the new equations and the accuracy of this method.

**APPENDIX**

The luminous intensity represents the light intensity of a source as perceived by the human eye. The luminous intensity is measured in units of candela (cd). The definition of luminous intensity is as follows: a monochromatic light source emitting an optical power of (1/683) W at 555 nm has a luminous intensity of 1 cd. [17]

The luminous intensity \( I \) of LEDs decreases with increasing temperature. Near room temperature, the luminous intensity follows an exponential decay function [17]:

\[
I = I_{25\,\text{C}} \exp \left( \frac{-(T_j - 25\,\text{C})}{T_1} \right) \tag{A1}
\]

where \( T_1 \) is called the characteristic temperature.

The luminous flux represents the light power of a source as perceived by the human eye. It is defined as follows: a monochromatic light source emitting an optical power of (1/683) W at 555 nm has a luminous flux of 1 lm. [17]

By comparing the definitions for the luminous intensity and the luminous flux, we can draw this conclusion that if an LED with viewing angle of \( \theta \) (sr) and luminous intensity of \( I \) (cd), then it will have a luminous flux as

\[
\phi_v = \int I \cdot d\theta. \tag{A2}
\]

Equation (A2) realized the conversion between the luminous flux and the luminous intensity. Again, the luminous efficacy here refers to the ratio of luminous flux to the electrical power. That is,

\[
E = \frac{\phi_v}{P_d}. \tag{A3}
\]

By substituting (A2) into (A3), we can get efficacy expression

\[
E = \frac{\int I \cdot d\theta}{P_d}. \tag{A4}
\]

Where \( I \) is luminous intensity and in units of lm/sr (cd), \( \theta \) is beam angle in unit of steradian (sr), and \( P_d \) is the electrical power in unit of W.

Substituting (A1) into (A4), the relationship between luminous efficacy and junction temperature can be obtained

\[
E = \frac{\int I \cdot d\theta}{P_d} = \frac{\int I_{25\,\text{C}} \exp \left( \frac{-(T_j - 25\,\text{C})}{T_1} \right) \cdot d\theta}{P_d} = \frac{\int I_{25\,\text{C}} \cdot d\theta}{P_d} \exp \left( \frac{-(T_j - 25\,\text{C})}{T_1} \right) = E_{25\,\text{C}} \exp \left( \frac{-(T_j - 25\,\text{C})}{T_1} \right). \tag{A5}
\]

That is,

\[
E = E_{25\,\text{C}} \exp \left( \frac{-(T_j - 25\,\text{C})}{T_1} \right). \tag{A6}
\]

Then, by the Taylor series expansions of exponential functions [22], the item of \( \exp\left(-(T - 25\,\text{C})/T_1\right) \) can be expressed as

\[
\exp\left(\frac{-(T_j - 25\,\text{C})}{T_1}\right) = 1 + \left(\frac{-(T_j - 25\,\text{C})}{T_1}\right) + \frac{\left(-(T_j - 25\,\text{C})/T_1\right)^2}{2!} + \frac{\left(-(T_j - 25\,\text{C})/T_1\right)^3}{3!} + \cdots \tag{A7}
\]

The characteristic temperature \( T_1 \) is thousand orders of magnitude for white LED, so the item of \( T - 25\,\text{C}/T_1 \) is far less than 1. Equation (A7) can be simplified as

\[
\exp\left(\frac{-(T_j - 25\,\text{C})}{T_1}\right) \approx 1 + \left(\frac{-(T_j - 25\,\text{C})}{T_1}\right). \tag{A8}
\]

So, (A6) becomes

\[
E = E_{25\,\text{C}} \left[ 1 - \frac{1}{T_1} (T_j - 25\,\text{C}) \right]. \tag{A9}
\]

Equation (A9) gives the relationship of luminous efficacy \( E \) and junction temperature \( T_j \). The procedure from (A1) to (A9) leads to the conversion from luminous intensity to luminous efficacy.

**REFERENCES**

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