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<td><strong>Author(s)</strong></td>
<td>Ng, SKK; Zhong, J</td>
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Smart Dispatch of Controllable Loads with High Penetration of Renewables

Simon K. K. Ng, Student Member, and J. Zhong, Senior Member, IEEE

Abstract—This paper presents a bi-level aggregator-utility optimization model to schedule an energy consumption pattern of controllable loads in a power system with a high penetration of renewables. The upper level is an aggregator’s problem which aims to minimize the electricity payment by managing the energy consumption of three types of controllable loads. On the other hand, the lower level is a utility’s problem which is assumed to be a follower. The utility’s problem is a market-clearing model which provides a spot price to the aggregator’s problem. We derive the Karush-Kuhn-Tucker (KKT) optimality conditions of the lower-level utility’s problem as the equilibrium constraint in the upper-level aggregator’s problem. Therefore, the bi-level formulation is converted into a form of mathematical program with equilibrium constraints (MPECs) which can be solved analytically. A numerical example is conducted to demonstrate the performance of the proposed model.

Index Terms—Controllable load, load dispatch, bilevel programming, smart grid, mathematical program with equilibrium constraints (MPECs).

I. NOMENCLATURE

A. Indices

* \( t \)  * time index
* \( i \)  * type I load index
* \( j \)  * type II load index
* \( l \)  * type III controllable load index

B. Set

* \( T \)  * set of indices of time
* \( I \)  * set of indices of generators
* \( J \)  * set of indices of type I loads
* \( K \)  * set of indices of type II loads
* \( L \)  * set of indices of type III loads
* \( \Omega^I \)  * set of indices of allowed operating hours of type I loads
* \( \Omega^J \)  * set of indices of allowed operating hours of type II loads
* \( \Omega^L \)  * set of indices of allowed operating hours of type III loads

C. Constant

* \( p^I_t \)  * Rated power output of type I load \( j \)
* \( H^I_t \)  * Total operating period of type I load \( j \)

D. Continuous variables

* \( p^I_t \)  * Power output of type I load \( j \) at period \( t \)
* \( p^II_t \)  * Power output of type II load \( k \) at period \( t \)
* \( p^III_t \)  * Power output of type III load \( l \) at period \( t \)
* \( \mu_t \)  * Lagrange multiplier associated with the power balance equation at period \( t \)
* \( \gamma^I_t \)  * Lagrange multiplier associated with the upper bound for the power output of generator \( i \) at period \( t \)
* \( \gamma^J_t \)  * Lagrange multiplier associated with the lower bound for the power output of generator \( i \) at period \( t \)

E. Binary variables

* \( x^I_{j,t} \)  * 0/1 variable that is equal to 1 if type I load \( j \) is on and otherwise is equal to 0 at period \( t \)
* \( x^{II}_{k,t} \)  * 0/1 variable that is equal to 1 if type II load \( k \) is on and otherwise is equal to 0 at period \( t \)
* \( x^{III}_{l,t} \)  * 0/1 variable that is equal to 1 if type III load \( l \) is on and otherwise is equal to 0 at period \( t \)

F. Random variables

* \( p^w_t \)  * Forecasted wind power generation at period \( t \)

II. INTRODUCTION

With increasing emphasis on improving efficiency and utilizing more renewable energy to mitigate climate change effects, power industry is confronted with many new challenges. Traditionally, power balance is achieved by regulating the controllable generation to meet the uncontrollable loads within the prescribed level of reliability [1]. To reduce emissions, renewables which are primarily in wind energy are gradually being integrated into the grid in rising proportion. However, renewables are intermittent and...
uncertain by nature, making them difficult to predict accurately [2].

A sudden change of wind conditions may cause a large surplus or lack of power output and subsequently affecting security or even adequacy in some cases. To maintain stability and power balance of a system, one has to provide sufficient reserve capacities. However, reserve capacity is usually provided by conventional generators which are flexible and easily controlled but also likely to have higher cost and carbon emissions (e.g. gas turbines or diesel generators). Consequently, carbon reduction in having more renewable penetration may require more conventional generators to support in order to meet stability and power balancing needs.

In addition to managing the generation side of the power balance equation, Demand Side Management (DSM) is one of the approaches for utilities to improve energy efficiency and facilitate power balance at demand side. One of the means of DSM is Demand Response (DR). DR refers to changing the consumption patterns by customers in response to prices, monetary incentives or system needs.

In general, demand response is divided into two main groups: (1) Direct Load Control (DLC) [3]-[8] and (2) Indirect Load Control (e.g. time-of-use pricing (TOU) [9]-[11]). For DLC, control systems are mainly designed to curtail thermostatically controlled loads such as air conditioners [7] and water heaters [8] in peak hours based on some pre-set agreements. On the other hand, ILC relies on variable tariff and/or economic incentives to encourage customers to shift their consumption patterns to improve efficiency and reduce peak demand. Although ILC is voluntary and non-intrusive, its implementation involves many complex issues such as regulatory and economic considerations. In addition, its response is difficult to predict and the re-shaping of the load profile may have other implications (e.g. stressing other parts of the grid in a different time).

In this paper, we therefore focus on using controllable loads with a smart DLC algorithm to provide fast balancing services to the system with renewable sources. In this paper, the controllable loads are defined as a load which is not essential to customers and it can be controlled by utility if needed. Some typical examples include hot water tanks, dehumidifiers etc. On the contrary, the non-controllable load may include desktop computers, cookers and emergency lighting.

With increasing deployment of smart meters and Advanced Metering Infrastructure (AMI), an advanced two-way communication system is established between the utility and consumers. With such new infrastructures, a more sophisticated DLC algorithm is essential in a smart grid environment [1]. Du and Lu [12] proposed an appliance commitment to find an optimal schedule of thermostatically controlled appliances based on price and consumption forecasts. Mohsenian-Rad and Leon-Garcia [13] introduced an optimization framework to schedule the operation and energy consumption of residential appliances with the objective of minimizing the household’s electricity payment. Pedrasa et al [14] described a distributed energy resources scheduling algorithm to maximize the end user’s revenue in a smart home case study.

The recent DR approaches with the smart grid technologies mainly focus on the scheduling of appliances in an automated manner. However, those approaches generally formulated the price signals with a predefined value which ignored the system conditions.

Therefore, we propose a bi-level aggregator-utility optimization model to use the controllable loads in response to the systems with high accommodation of renewables such as wind energy. The upper level is an aggregator’s problem which is assumed to be a leader in the bi-level model. The objective of the aggregator’s problem is to minimize the electricity payment by scheduling the consumption pattern of controllable loads given the spot price signals are present. Since the number of controllable loads in a single household/premise is small, we propose to use the aggregator to aggregate the controllable loads from different premises for DLC. In the aggregator’s problem, we classify the controllable loads into three types based on their natures and operational characteristics.

On the other hand, the lower level is a utility’s problem which is assumed to be a follower. The utility’s problem is a simple market-clearing model which neglects the network constraints. The spot prices can be derived from a set of Lagrange multipliers in the power balance equations of the utility’s problem [15].

Since the utility’s problem is continuous and convex, we can derive the Karush-Kuhn-Tucker (KKT) optimality conditions of the lower-level utility’s problem as the equilibrium constraint in the upper-level aggregator’s problem. Therefore, the bi-level formulation is converted into a form of mathematical program with equilibrium constraints (MPECs) [16] which can be solved in GAMS [17].

This paper is organized as follow. Section III gives a control framework of the aggregator. Section IV formulates the proposed bi-level model. Section V provides a solution approach of the problem. Section VI gives a numerical example of the model and finally, some conclusions are drawn in Section VII.

III. LOAD CONTROL AGGREGATOR MODEL

In this section, we propose a framework which uses the “aggregator” to aggregate controllable loads as shown in Fig. 1. Instead of shedding load when supply is short, the aggregator can manipulate groups of loads from different premises and turn off those loads which are less important to customers at the time. The aggregator keeps track of the appliance usage and turns on those controlled loads again when there is sufficient supply. As such, the aggregation of controllable loads can allow more versatile and optimized control for the system and lessen the impact among different premises. Using the aggregator can also reduce the control complexity of a centralized control which requires the system operator to manipulate millions or even more of load entities.
IV. BI-LEVEL AGGREGATOR-UTILITY SMART DISPATCH MODEL

Next, we provide a mathematical formulation of the bi-level aggregator-utility model. The upper level is the aggregator’s problem which aims to use controllable loads to minimize the payment. The lower level utility’s problem aims to clear the market.

A. Upper-level Aggregator’s Problem

The aggregator obtains the hourly price signal from the utility and manages the energy consumption of the controllable loads in order to minimize the payment. The controllable loads are divided into three types according to their natures and operational characteristics. The mathematical formulation of the aggregator problem is shown as follows:

\[
\min \sum_{t \in T} \left( \sum_{j \in J} p_j^l + \sum_{k \in K} p_{k,t}^l + \sum_{l \in L} p_{l,t}^u \right)
\]

subject to

1) Constraints of type I load:
   \[
   p_{j,t}^l = x_{j,t}^l p_j^l, \forall j \in J
   \]
   \[
   \sum_{t \in T} x_{j,t}^l = H_j^l, \forall j \in J
   \]

2) Constraints of type II load:
   \[
   p_{k,t}^l = x_{k,t}^l p_{k,t}^l, \forall k \in K
   \]
   \[
   x_{k,t}^l + x_{k,t+1}^l + \ldots + x_{k,t+q_k}^l \geq d_k^l, \forall k \in K, \forall t
   \]
   \[
   t = 1, 2, \ldots, t_{\text{max}} - b_k^l
   \]

3) Constraints of type III load:
   \[
   x_{l,t}^{\text{III,min}} \leq \sum_{t \in T} \left( p_{l,t}^{\text{III}, \Delta t} \right) \leq x_{l,t}^{\text{III,max}}
   \]

4) Operating period constraints:
   \[
   \left\{ \begin{array}{l}
   x_{j,t}^l \in [0,1] | t \in \Omega_j, \forall j \\
   x_{j,t}^l = 0 | t \in \Omega_j, \forall j \\
   x_{l,t}^{\text{III}, \Delta t} \in [0,1] | t \in \Omega_k, \forall k \\
   x_{k,t}^l = 0 | t \in \Omega_k, \forall k
   \end{array} \right.
   \]

The objective function of the aggregator is to minimize the total cost as shown in (1). \(i_t\) is the hourly spot price and it can be derived from the Lagrange multiplier of the power balance constraint in the lower level utility’s problem.

A type I load is defined as a load which consumes fixed amount of total energy consumption. Some typical examples of type I loads are water pump, dish washer, washing machine and electric vehicle charging. Equation (2) imposes the type I load as a single-mode appliance. It can be on and off arbitrary within the day but the fixed energy has to be given before the last allowed operating hour as shown in (3).

The operation of a type II load is similar to the type I load but with time constraint. For example, a refrigerator can be temporally switched off when the electricity price is high but in order to keep the food fresh, it cannot be turned off consecutively more than certain period (e.g. 2 hours). Some other typical examples are freezer and chiller. The type II load is formulated as a single-mode appliance as shown in (4). Equation (5) ensures at least certain energy has to be given to the type II load constrained with the time limit \(b_k^l\).

A type III load is defined as multi-mode appliance such as air conditioners and water heaters as shown in (6). We assume that the aggregator can control their operating modes in this paper. In order to ensure maintaining certain comfortability to customers, a minimum energy is required to be given within the schedule period as shown in (7).

Equations (8)-(10) impose operating period constraints of different types of controllable loads. In order to increase the control flexibility from the demand sector, the consumer is required to inform the set of allowed operating hours of controllable loads to the utility. For the hours that excludes in the set, the corresponding decision variables are forced to 0.

B. Lower-level Utility Problem

The objective of the utility’s problem is to minimize the cost of the dispatched energy in a day-ahead market as shown in (11):

\[
\min \left\{ \sum_{t \in T} \sum_{i \in I} C_{i,t} P_{i,t}^g \right\}
\]

The objective function is subject to following constraints:

1) Power balance constraints:
   \[
   \sum_{i \in I} p_{i,t}^g + p_{i,t}^w = \sum_{j \in J} p_{j,t}^l + \sum_{k \in K} p_{k,t}^l + \sum_{l \in L} p_{l,t}^u + P_{t}^{BL}, \forall t \in T
   \]

2) Power production constraints:
   \[
   p_{i,t}^{g,min} \leq p_{i,t}^g \leq p_{i,t}^{g,max}, \forall i \in I, \forall t \in T
   \]

The utility’s objective function to be minimized is shown in (11). For simplicity, the network constraints are ignored and the utility’s problem in this paper only focuses on the power balance constraints as shown in (12). The approximate wind
power is first forecasted in the day-ahead market. Equation (13) shows the power production limits. The Lagrange multipliers associated with equations (12) and (13) are given in corresponding parentheses. It should be noted that all the Lagrange multipliers are non-negative numbers.

V. SINGLE-LEVEL EQUIVALENT

In order to solve the proposed bi-level model analytically, we have to derive the first order KKT optimality conditions of the lower-level utility problem as the equilibrium constraints in the upper-level aggregator problem. The bi-level model is then converted to the MPECs’ problem. The derivation of the KKT conditions is applicable since the decision variables of lower-level utility’s problem are continuous and therefore, the utility problem is convex. Also, the binary variables which appear in the power balance constraints as shown in (12) can be regarded as parameters to the utility’s problem.

Therefore, the original bi-level problem (1)-(13) is converted into the MPECs’ problem:

\[
\min \sum_{t \in T} \mu_t \left( \sum_{j \in J} P_{j,t}^l + \sum_{k \in K} P_{k,t}^l + \sum_{t \in T} P_{l,t}^{ll} \right) 
\]

subject to

\[
P_{j,t}^l = x_{j,t}^l p_j^l, \forall j \in J 
\]

\[
\sum_{t \in T} x_{j,t}^l = H_j^l, \forall j \in J 
\]

\[
P_{k,t}^{ll} = x_{k,t}^{ll} p_k^{ll}, \forall k \in K 
\]

\[
x_{k,t}^{ll} + x_{k,t+1}^{ll} + \ldots + x_{k,t+q_k^l}^{ll} \geq d_k^l, \forall k \in K, \forall t 
\]

\[= 1,2, \ldots, t_{max} - b_k^l 
\]

\[
x_{k,t}^{ll} \geq 0, \forall k \in K 
\]

\[
x_{k,t}^{ll} \leq x_{k,t+q_k^l}^{ll}, \forall l \in L 
\]

\[= p_{ll}^{l, min} \leq p_{ll}^{l, max} = E_{l,t}^{l, max} \leq \sum_{t \in T} (p_{ll}^{l, max} \Delta t) \leq E_{ll}^{l, max} \]

\[
\left\{ \begin{array}{l} x_{j,t}^l \in (0,1) | t \in \Omega_l \\ x_{j,t}^l = 0 | t \in \Omega_l, \forall j \in J \end{array} \right. 
\]

\[
\left\{ \begin{array}{l} x_{j,t}^l \in (0,1) | t \in \Omega_l \\ x_{j,t}^l = 0 | t \in \Omega_l, \forall k \in K \end{array} \right. 
\]

\[
\left\{ \begin{array}{l} x_{k,t}^{ll} \in (0,1) | t \in \Omega_l \\ x_{k,t}^{ll} = 0 | t \in \Omega_l, \forall l \in L \\ x_{k,t}^{ll} = 0 | t \in \Omega_l, \forall l \in L 
\end{array} \right. 
\]

\[
\sum_{t \in T} P_{l,t}^q + P_{l,t}^w = \sum_{j \in J} P_{j,t}^l + \sum_{k \in K} P_{k,t}^{ll} + \sum_{t \in T} P_{l,t}^{ll} 
\]

\[= P_{l,t}^{q, min} \leq P_{l,t}^{q, max}, \forall i \in I, \forall t \in T 
\]

\[= C_{l,t} - (\mu_t + y_{l,t}^{min} + y_{l,t}^{max}) = 0, \forall i \in I, \forall t \in T 
\]

\[= y_{l,t}^{max} \geq 0, \forall i \in I, \forall t \in T 
\]

\[= y_{l,t}^{min} \geq 0, \forall i \in I, \forall t \in T 
\]

\[= P_{l,t}^{q, max} - P_{l,t}^{q, min} = 0, \forall i \in I, \forall t \in T 
\]

Equations (14)-(23) are consistent with the original upper-level consumer’s problem as shown in (1)-(10). Equations (24)-(30) represent the KKT optimality conditions of the lower-level utility’s problem (11)-(13). Equations (24)-(25) represent the primal constraints. The dual constraints and complementarity slackness conditions are given in (26)-(28), and (29) and (30), respectively.

VI. CASE STUDY

In order to evaluate the proposed DLC algorithm, we conduct a simple example with a 24-hour period (i.e. t = 1,2, ...,24) to schedule the controllable loads with data presented in Table I-VI. Table I-III show the data of type I, II and III loads, respectively. Table IV lists the data of the generators. Table V shows the hourly base-load and forecasted wind power profiles.

Some simulation assumptions are made as follows:

1. The energy offer of each generator is consistent in each time period.
2. The wind speed forecasting error is not considered.
3. No operating period constraints are imposed to controllable loads.
4. The generation cost of wind power output is zero.

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<tr>
<th>Load / (kW)</th>
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<th>2</th>
<th>3</th>
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<td>P_i^l</td>
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<td>3</td>
<td>6</td>
<td>8</td>
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<tr>
<td>H_i</td>
<td>6</td>
<td>3</td>
<td>4</td>
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<tr>
<td>P_i^l</td>
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<td>5</td>
<td>6</td>
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<tr>
<td>a_i^l</td>
<td>2</td>
<td>1</td>
<td>1</td>
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<td>b_i^l</td>
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<td>P_i^ll</td>
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<td>0</td>
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<td>0</td>
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<td>e_i^ll</td>
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<tr>
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</tr>
<tr>
<td>e_i^g</td>
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<th>FORECASTED WIND AND BASELOAD POWER PROFILE</th>
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<tr>
<td>P_i^w (kW)</td>
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In this case study, we create a base case to benchmark the results from the proposed DLC algorithm. The base case uses the same set of data but without optimally scheduling the load pattern. Fig. 2 compares the hourly load pattern of the base case with the optimal result from the proposed model. From Fig. 2, it is observed that the peak load occurs from Hr19 to Hr22 in the base case. The result from the proposed DLC algorithm shaves the peak demand at that time period and its load pattern is more correlated with the wind power output.

Table VI compares the system performance between base case and with the proposed DLC algorithm. From Table VI, it is observed that the total payment with DLC (i.e. $4,155) is much lower than the base case (i.e. $8,255). It is also interesting to note that the total consumed energy with DLC is actually higher than the base case. This is due to the fact that the proposed DLC model allows some controllable loads (e.g. type II and type III loads) to absorb excessive wind energy. It is also observed that the average spot price with DLC (i.e. 7.5) is much lower than the base case (i.e.). By dispatching the controllable loads intelligently, the expensive generator (i.e. Generator 3) is dispatched for one hour (i.e. on t = 9) only as shown in Table VII. However, for the base case, there are 5 hours operating Generator 3 and this explains why its average spot price is much higher than that with DLC.

VII. CONCLUSION

The increasing penetration of renewable sources in a power system imposes the challenge of the system operator to maintain the system balance. At the same time, the latest developed information and communication technologies in a smart grid brings alternative approach to control the demand with intermittent renewables. This paper proposes a bi-level aggregator-utility optimization model with spot electricity price to schedule the energy consumption patterns of controllable loads in the system with a high penetration of wind energy. This paper classifies controllable loads into three types based on their natures and operating characteristics. The bi-level formulation is converted into a form of MPECs which can be solved analytically. We then used a numerical example to demonstrate that the proposed model can improve energy efficiency and facilitate power balance with intermittent wind power output.

ACKNOWLEDGMENT

We would like to acknowledge Dr. J. W. M. Cheng from CLP Research Institute Ltd. for his advices and supports.

VIII. REFERENCES


IX. BIOGRAPHIES

Simon K.K. Ng received the B.Eng. (Hons.) and M.Phil. degrees in electrical engineering from The University of Hong Kong, Hong Kong, China, in 2005 and 2007, respectively. He is currently pursuing the Ph.D. degree of power systems in the same institution. He was a Modeling Specialist with the CLP Research Institute Ltd. from 2008 to 2009. His main research activities include load signature analysis and load modeling.

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