

AN EFFICIENT METHOD FOR TIME-DEPENDENT ANALYSIS OF COMPOSITE BEAMS

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ABSTRACT

Composite beams have found wide applications in buildings and bridges. As the time-dependent effects, including the ageing, creep and shrinkage of concrete, affect the interaction between concrete and steel, it is essential to account for them properly to ensure satisfactory service and safety. This paper presents an efficient method to investigate the time-dependent performance of composite beams. Firstly, the age-adjusted elasticity modulus to account for concrete creep and the shrinkage-adjusted elasticity modulus to account for concrete shrinkage and its interaction with creep are introduced. Then an effective single-step numerical method for time-dependent analysis is developed in conjunction with the finite element method. Results obtained for the time-dependent behaviour and long-term redistribution of stresses agree well with published results. Numerical results show that the interaction among concrete ageing, creep and shrinkage increases the long-term deformation and results in stress redistribution, particularly at the interface.

Keywords: Concrete ageing, concrete creep, concrete shrinkage, single-step method, steel-concrete composite beams, time-dependent analysis.

1. INTRODUCTION

Steel-concrete composite beams have found wide applications in buildings and bridges. A reinforced concrete slab is mechanically connected to the top flange of a steel beam through a connection system, which facilitates rapid construction and substantially increases the stiffness. Under service loads, only elastic deformations occur to the steel beam, while significant inelastic deformations take place in the concrete slab with the passage of time because of time-dependent effects, including the ageing, creep and shrinkage of concrete. These inelastic deformations in the concrete deck will redistribute stresses in the composite beam and induce considerable deformations. Therefore it is essential to account for these effects properly to ensure satisfactory service and safety.

Analyses of cross sections of composite steel-concrete members under sustained loads can be performed independently by assuming stiff connectors to provide full interaction between components (Gilbert 1989; Partov and Kantchev 2009). When the deformability of the connection

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system is considered, e.g. flexible shear connections, consideration of the global behaviour of the structural system is required (Tarantino and Dezi 1992). The finite element method has been widely adopted in conjunction with a step-by-step numerical procedure to study the long-term behaviour of composite structures at serviceability limit state with or without cracking, and even collapse analyses (Fragiacomo et al. 2004; Macorini et al. 2006). However, although it is accurate, it becomes an enormous computational problem for complex structures, as the creep strains within a time interval depend on the entire previous stress history. Thus simplified methods such as the single-step method using age-adjusted effective modulus or age-adjusted elasticity modulus (AAEM) have been developed (Bažant 1972; Ghali et al. 2002). Shrinkage analysis is often performed using AAEM assuming that the development of shrinkage strain is the same as that of creep. Although it gives satisfactory approximations in cases in which shrinkage effect is minor, it cannot give accurate solutions to problems of restrained shrinkage (Au et al. 2007; Au et al. 2009). To overcome this, the shrinkage-adjusted elasticity modulus (SAEM) was introduced (Au et al. 2007). This paper presents a general single-step method by using the AAEM to account for the effect of concrete creep and initial loading and the SAEM to account for the effect of concrete shrinkage and its interaction with concrete creep. Then numerical studies are conducted to validate the proposed method.

2. METHODOLOGY

2.1. Numerical model

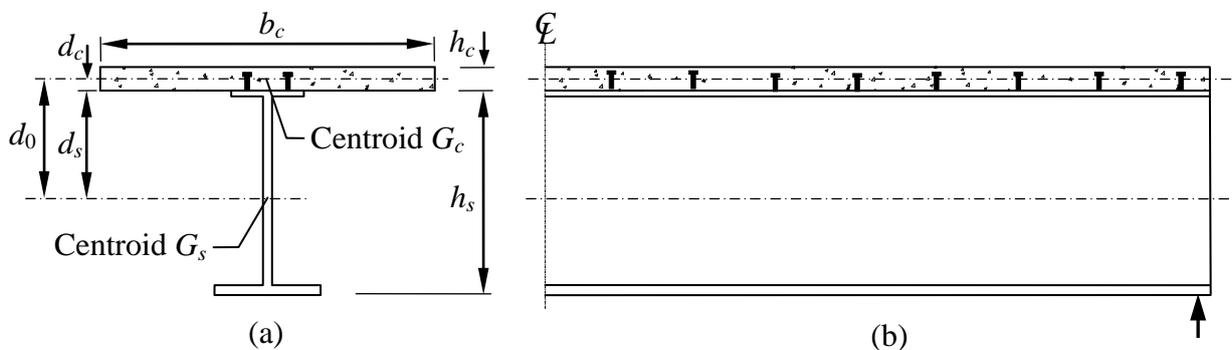


Figure 1: Concrete-steel composite beam: (a) Cross section; (b) Elevation.

Figure 1 shows a composite beam modelled by finite elements. The ageing, creep and shrinkage of the concrete slab can be easily introduced for analysis under service loads by the single-step method. The composite beam is discretized into several segments longitudinally. In each segment, the concrete slab and steel girder are modelled separately by using proper elements, such as beam elements or shell elements. Proper spring elements can be provided to simulate the interaction between adjacent components due to shear connectors, relative slips and possible uplifts at the interface. Actually, this numerical model applies not only to steel-concrete composite beams, but also to concrete-concrete composite systems.

2.2. Age-adjusted elasticity modulus

The creep induced by time-varying stresses can be determined by the principle of superposition. The total strain $\varepsilon_c(t)$ can be expressed by the following superposition equation for stress history $\sigma_c(t)$ since the time of loading t_0 (Ghali et al. 2002; Partov and Kantchev 2009) as

$$\varepsilon_c(t) = \varepsilon_{cs}(t, t_0) + \sigma_c(t_0)J(t, t_0) + \int_{t_0}^{\Delta\bar{\sigma}_c} J(t, \tau)d\sigma \quad (1)$$

where $\varepsilon_{cs}(t, t_0)$ is the free shrinkage, $J(t, t_0)$ is the creep compliance at time t and $\Delta\bar{\sigma}_c$ is the stress increment from time t_0 to t . Assuming that the stress history is a linear algebraic function of the relaxation function $r_c(t, t_0)$ that gives the stress variation due to a unit strain introduced at time t_0 and kept constant henceforth, the total strain $\varepsilon_c(t)$ due to the gradually introduced stress increment $\Delta\bar{\sigma}_c$ during the period (t, t_0) can be reduced using equation (1) to

$$\varepsilon_c(t) = \varepsilon_{cs}(t, t_0) + [1 + \varphi(t, t_0)][\sigma_c(t_0)/E_c(t_0)] + [1 + \chi_c(t, t_0)\varphi(t, t_0)][\Delta\bar{\sigma}_c(t)/E_c(t_0)] \quad (2a)$$

$$\bar{E}_c(t, t_0) = E_c(t_0)/[1 + \chi_c(t, t_0)\varphi(t, t_0)] \quad (2b)$$

where $E_c(t_0)$ is the modulus of elasticity at the time of loading t_0 , $\bar{E}_c(t, t_0)$ is the AAEM and $\chi_c(t, t_0)$ is the ageing coefficient to account for time-dependent effects due to concrete creep, which can be expressed in terms of the relaxation function $r_c(t, t_0)$ as

$$\chi_c(t, t_0) = E_c(t_0)/[E_c(t_0) - r_c(t, t_0)] - 1/\varphi(t, t_0) \quad (3)$$

However, the method results in low accuracy when the strain function is different from the creep coefficient, such as in problems of restrained shrinkage (Au et al. 2007; Au et al. 2009).

2.3. Shrinkage-adjusted elasticity modulus (SAEM)

In order to predict the long-term performance of composite structures efficiently and accurately, one may use the AAEM to account for creep and the SAEM to account for the effect of concrete shrinkage and its interaction with creep. The present problem can be approximately analyzed as two separate parts. AAEM is used in the first part to account for the effects of initial stresses and creep giving the strain component $\bar{\varepsilon}_{cc}(t)$ and ‘‘creep’’ stress increment $\Delta\bar{\sigma}_{cc}(t)$. In the second part, SAEM is utilised to account for the interaction between shrinkage and creep giving the strain component $\bar{\varepsilon}_{cs}(t)$ and ‘‘shrinkage’’ stress increment $\Delta\bar{\sigma}_{cs}(t)$ with zero initial strain condition $\bar{\varepsilon}_{cs}(t_0)$. Therefore noting the above conditions of strain $\varepsilon_c(t) = \bar{\varepsilon}_{cc}(t) + \bar{\varepsilon}_{cs}(t)$ and stress increment $\Delta\bar{\sigma}_c = \Delta\bar{\sigma}_{cc} + \Delta\bar{\sigma}_{cs}$, equation (1) can be rewritten as (Si et al. 2009)

$$\varepsilon_c(t) = [1 + \varphi_c(t, t_0)][\sigma_c(t_0)/E_c(t_0)] + \Delta\bar{\sigma}_{cc}(t)/\bar{E}_c(t, t_0) + \Delta\bar{\sigma}_{cs}(t)/\bar{E}_{cs}(t, t_0) + \varepsilon_{cs}(t, t_0) \quad (4)$$

where the strain components $\bar{\varepsilon}_{cc}(t)$ and $\bar{\varepsilon}_{cs}(t)$ are respectively

$$\bar{\varepsilon}_{cc}(t) = [1 + \varphi_c(t, t_0)][\sigma_c(t_0)/E_c(t_0)] + \Delta\bar{\sigma}_{cc}(t)/\bar{E}_{cc}(t, t_0) \quad (5)$$

$$\bar{\varepsilon}_{cs}(t) = \Delta\bar{\sigma}_{cs}(t)/\bar{E}_{cs}(t, t_0) + \varepsilon_{cs}(t, t_0) \quad (6)$$

The AAEM $\bar{E}_c(t, t_0)$ is given in equation (2b) and the SAEM is given in terms of the shrinkage coefficient χ_{cs} as

$$\bar{E}_{cs}(t, t_0) = E_c(t_0)/[1 + \chi_{cs}(t, t_0)\varphi(t, t_0)] \quad (7)$$

where the shrinkage coefficient χ_{cs} can be written in terms of stressing function $s(t, t_0)$, shrinkage strain $\varepsilon_{cs}(t, t_0)$, Young's modulus $E_c(t_0)$ at the age of loading t_0 and concrete creep coefficient $\varphi(t, t_0)$ as

$$\chi_{cs}(t, t_0) = -[E_c(t_0)\varepsilon_{cs}(t, t_0)]/[s(t, t_0)\varphi(t, t_0)] - 1/\varphi(t, t_0) \quad (8)$$

The ageing coefficient χ_{cc} and χ_{cs} can be calculated by a step-by-step numerical procedure.

2.4. Single-step method for composite beams

The single-step method for composite beams is developed by adopting the finite element method in conjunction with AAEM and SAEM in two parts. Firstly, in order to obtain the creep stress increment $\Delta\bar{\sigma}_{cc}$, by noting the initial strain $\varepsilon_c(t_0) = \sigma_c(t_0)/E_c(t_0)$ and incremental strain $\Delta\bar{\varepsilon}_{cc}(t) = \bar{\varepsilon}_{cc}(t) - \varepsilon_c(t_0)$, further re-arrangement of equation (5) gives

$$\Delta\bar{\sigma}_{cc}(t) = \bar{E}_c(t, t_0)(\Delta\bar{\varepsilon}_{cc}(t) - \varepsilon_c(t_0)\varphi(t, t_0)) \quad (9)$$

For general application to concrete elements, equation (9) can be rewritten in matrix form in terms of the age-adjusted elasticity matrix $[\bar{D}_c(t, t_0)]$ (i.e. constitutive matrix) as

$$\{\Delta\bar{\sigma}_{cc}(t)\} = [\bar{D}_c(t, t_0)] [\{\Delta\bar{\varepsilon}_{cc}(t)\} - \{\varepsilon_c(t_0)\}\varphi(t, t_0)] \quad (10)$$

The relationship between strain vector $\{\varepsilon_c\}$ of a concrete element and the local nodal displacement vector $\{u_c\}$ can be expressed as

$$\{\varepsilon_c\} = \{du/dx\} = [B_c]\{u_c\} \quad (11)$$

in terms of the strain matrix $[B_c]$ which can be easily derived by the shape functions adopted in the kind of element. Then, following the traditional formulation of finite element method and neglecting body forces, the incremental load vector $\{\Delta q_{cc}^e\}$ from time t_0 to t can be obtained by integrating over the volume V^e as

$$\begin{aligned} \{\Delta q_{cc}^e\} &= \iiint_{V^e} [B_c]^T \{\Delta\bar{\sigma}_{cc}(t)\} dV \\ &= \left(\iiint_{V^e} [B_c]^T [\bar{D}_c(t, t_0)] [B_c] dV \right) \{\Delta u_{cc}\} - \iiint_{V^e} [B_c]^T [\bar{D}_c(t, t_0)] \varphi(t, t_0) dV \end{aligned} \quad (12a)$$

$$\{\Delta q_{cc}^e\} = [\bar{k}_{cc}(t, t_0)] \{\Delta u_{cc}\} + \{\Delta f_{cc}(t, t_0)\} \quad (12b)$$

where $\{\Delta u_{cc}\}$ is the incremental creep displacement vector, $[\bar{k}_{cc}(t, t_0)]$ is the stiffness matrix and $\{\Delta f_{cc}(t, t_0)\}$ is the incremental load vector due to effects of concrete creep and external loads which can be formulated by the AAEM.

Secondly, noting the incremental strain $\Delta \bar{\varepsilon}_{cs}(t) = \bar{\varepsilon}_{cs}(t) - \bar{\varepsilon}_{cs}(t_0)$ and the zero initial strain condition $\bar{\varepsilon}_{cs}(t_0) = 0$, the shrinkage stress increment $\Delta \bar{\sigma}_{cs}(t)$ can be obtained by rearranging and modifying equation (6) in the matrix form as

$$\{\Delta \bar{\sigma}_{cs}(t)\} = [\bar{D}_{cs}(t, t_0)](\{\Delta \bar{\varepsilon}_{cs}(t)\} - \{\varepsilon_{cs}(t, t_0)\}) \quad (13)$$

where the constitutive matrix $[\bar{D}_{cs}(t, t_0)]$ is the shrinkage-adjusted elasticity matrix. Similarly, the incremental load vector $\{\Delta q_{cs}^e\}$ from time t_0 to t due to effects of concrete shrinkage and its interaction with creep can be derived by conventional finite element technique as

$$\{\Delta q_{cs}^e\} = [\bar{k}_{cs}(t, t_0)]\{\Delta u_{cs}\} + \{\Delta f_{cs}(t, t_0)\} \quad (14)$$

where $\{\Delta u_{cs}\}$ is the incremental creep displacement vector, $[\bar{k}_{cs}(t, t_0)]$ is the stiffness matrix and $\{\Delta f_{cs}(t, t_0)\}$ is the incremental load vector due to concrete shrinkage formulated using SAEM.

To sum up, the single-step method to analyze the long-term performance of composite structures can be implemented as follows:

- (a) Discretise the concrete slab and steel girder into elements properly connected together at the interface to assure compatibility.
- (b) Calculate the local stiffness matrices $[k_c]$ and $[k_s]$, and load vectors $\{q_c^e\}$ and $\{q_s^e\}$ of the concrete and steel elements respectively at time t_0 . Then transform them to the global system and assemble them to form the global stiffness matrix $[K]$ and load vector $\{Q\}$. The initial displacement vector $\{U\}$ can be obtained by solving the equation $[K]\{U\} = \{Q\}$.
- (c) In the first part of solution, calculated the stiffness matrix $[\bar{k}_{cc}(t, t_0)]$ and incremental load vector $\{\Delta f_{cc}(t, t_0)\}$ at time t using the AAEM matrix $[\bar{D}_c(t, t_0)]$ due to effects of concrete creep and external loads. Transform the local matrices and vectors to the global system. Then assemble them to form the global stiffness matrix $[\bar{K}_{cc}]$ and incremental load vector $\{\Delta Q_{cc}\}$. Solve the

incremental displacement vector $\{\Delta U_{cc}\}$ due to creep from $[\bar{K}_{cc}]\{\Delta U_{cc}\} = \{\Delta Q_{cc}\}$, and determine the incremental internal forces.

(d) In the second part of solution, calculate the stiffness matrix $[\bar{k}_{cs}(t, t_0)]$ and incremental load vector $\{\Delta f_{cs}(t, t_0)\}$ due to concrete shrinkage and its interaction with creep. Transform the local matrices and vectors to the global system. Then assemble them to form the global stiffness matrix $[\bar{K}_{cs}]$ and incremental load vector $\{\Delta Q_{cs}\}$. Solve the incremental displacement vector $\{\Delta U_{cs}\}$ due to shrinkage from $[\bar{K}_{cs}]\{\Delta U_{cs}\} = \{\Delta Q_{cs}\}$, and determine the incremental internal forces.

(e) Combine the results from (b), (c) and (d) above to give the structural response at time t .

3. CASE STUDY

Figure 1 shows the hypothetical simply supported composite beam of Dezi et al. (1993) with the properties shown in Table 1, which is used to validate the proposed method. The concrete slab and steel girder in each segment are assumed to be weightless and modelled by two beam elements connected by rigid arms. The code MC-90 (CEB 1993) is adopted to describe the rheological behaviour of concrete creep and shrinkage for the study.

Table 1: Properties of the composite structure

Symbol	Value	Description
A_c	270,000mm ²	Cross-sectional area of concrete slab ($b_c = 1800\text{mm}$, $h_c = 150\text{mm}$)
A_s	12,500mm ²	Cross-sectional area of steel girder ($h_s = 600\text{mm}$)
d_0	375mm	Distance between centroids of concrete slab and steel girder
E_c	33.62GPa	Young's modulus of concrete slab
E_s	200GPa	Young's modulus of steel girder
f_{ck}	30MPa	Cylinder compressive strength of concrete
I_c	550,625×10 ⁴ mm ⁴	Moment of inertia of concrete slab
I_s	750×10 ⁶ mm ⁴	Moment of inertia of steel girder
l	12m	Span length of beam
Δl	0.5m	Length of each segment
P	25 kN/m	Uniformly distributed load
t_0	28 days	Age of concrete at loading
t_f	25,550 days	Age of concrete at the time considered (i.e. 70 years)
t_s	28 days	Age of concrete at the beginning of shrinkage
RH	80%	Relative humidity of ambient environment

The stresses at the top and bottom flanges of steel girder at the mid-span section are calculated for the time considered taking into account effects of concrete ageing and creep with or without shrinkage. The results at time t_f obtained from the proposed method are compared with those of Dezi et al. (1993) in Table 2, which shows good agreement. The present stress results are slightly higher than those of the general method, which are in turn slightly higher than those of the AAEM method. The present single-step method can therefore be considered as an efficient method for composite structures which takes proper account of the development of shrinkage.

Table 2: Final stresses (MPa) at top and bottom flanges of steel girder at mid-span section

Methods	Without shrinkage effects		With shrinkage effects	
	Top flange	Bottom flange	Top flange	Bottom flange
General (Dezi et al. 1993)	-23.3	130.5	-68.9	143.3
AAEM (Dezi et al. 1993)	-23.1	130.4	-67.5	143.0
Proposed method	-24.4	130.8	-70.0	143.6

The time-dependent behaviour of this composite beam is further investigated to obtain the deflections and stresses of components for three scenarios, namely:

- (a) Case 1: the initial state at the time of loading t_0 ;
- (b) Case 2: the hypothetical state at time t_f taking into account concrete ageing and creep only; and
- (c) Case 3: the state at time t_f taking into account concrete ageing, creep and shrinkage.

The stresses at the top and bottom fibres of the concrete slab and steel girder are shown in Table 3, while the stress distributions over the section at mid-span are shown in Figure 3. Table 3 shows that the steel girder mainly takes tensile stresses except for a small portion near the top, while the concrete slab takes compressive stresses. Stresses at the mid-span section are redistributed with the passage of time under the effects of concrete ageing, creep and shrinkage, as shown in Figure 3. In parallel with the tendency of the concrete slab to shorten due to creep and shrinkage, the top part of steel girder gradually takes up more compression. The stress differences of Cases 2 and 3 at the interface show that concrete shrinkage and its interaction with creep have substantial influence on stress distribution due to the restraint of shear connectors. Figure 4 shows that the beam deflections increase with time. Compared with the mid-span deflection in Case 1, those of Cases 2 and 3 have increased by 26.7% and 86.7% respectively. This shows that concrete shrinkage is an important factor in the long-term structural behaviour.

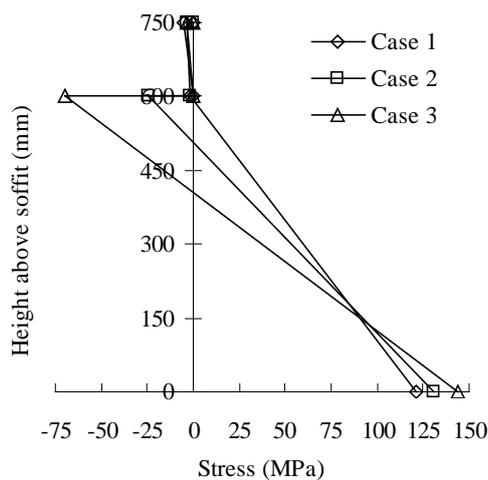


Figure 3: Stresses at mid-span.

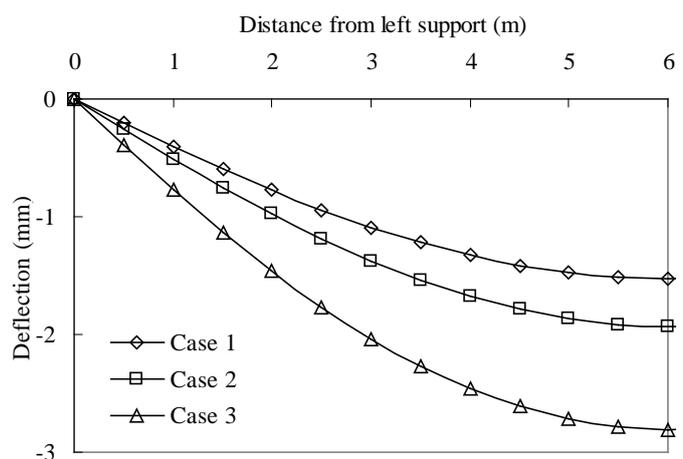


Figure 4: Deflections along the beam.

Table 3: Stresses (MPa) at top and bottom fibres of each component

Component	Stress	Case 1	Case 2	Case 3
Steel girder	Top	-1.8	-24.4	-24.4
	Bottom	121.1	130.8	130.8
Concrete slab	Top	-5.3	-3.4	-3.2
	Bottom	-0.3	-1.5	-0.2

4. CONCLUSIONS

This paper presents an efficient single-step method for analyzing long-term behaviour of steel-concrete composite structures taking into effects of concrete ageing, creep and shrinkage by using the AAEM and SAEM. The finite element model is built up by dividing the structure into segments and modelling the concrete slab and steel girder separately. Spring elements can also be introduced to model the composite action such as full interaction, slip or uplift at the interface. A numerical example is given to validate the proposed method.

5. ACKNOWLEDGEMENTS

The authors acknowledge the support from the Research Grants Council of the Hong Kong Special Administrative Region, China (RGC Project No. HKU 7102/08E).

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