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Ordering and Pricing Decisions in a Closed-loop Supply Chain with Fuzzy Demand

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Abstract—This paper investigates ordering and pricing decisions in a closed-loop supply chain with fuzzy demand. In this paper, the market demand is characterized as a fuzzy variable and two settings, decentralized channel and centralized channel, are considered. Based on game theory and fuzzy theory, the optimal ordering decision and the optimal recovery prices are given for each setting. The factors that impact the optimal ordering decision and the optimal recovery prices are also found. Some characteristics of the optimal decisions are discussed from the view of management.

Keywords—supply chain management; closed-loop supply chain; ordering; pricing; fuzzy demand

I. INTRODUCTION

Closed loop supply chain (CLSC) has gained an extensive importance today. A CLSC consists of both the forward supply chain, and the reverse supply chain. The forward supply chain essentially involves the movement of products from the upstream suppliers to the downstream customers. The reverse supply chain involves the movement of used products from the customer to the upstream supply chain [1]. Over the past decades, CLSC has become a critical research topic in operations research and management area. Many analytical and quantitative approaches can be found in various problems about CLSC, such as production planning [2], inventory control [3], supply chain network design [4], and so on.

There also has been a lot of research work concerned on ordering and pricing decisions in CLSC. Mitra [5] determined the optimal prices when the availability of discarded products, price and quality affect the demand of remanufactured products. Guide et al. [6] determined the optimal product acquisition price and remanufactured product sale prices when an original equipment manufacturer influences the quality, quantity, and timing of product returns via quality-dependent price incentives. Vadde et al. [7] considered pricing decisions in a multi-criteria environment for product recovery facilities. Bakal and Akcali [8] investigated a pricing optimization problem for an auto-motive dismantling and parts remanufacturing firm which has the power to determine the acquisition price of the end-of-life vehicles (ELVs) and the selling price of remanufactured parts. Both the supply quantity of ELVs and the demand of remanufactured parts are assumed to be linear price-sensitive, and mathematical models to optimize the two prices are developed. These research made great contributions to the management of CLSC. However, most of the existing literature investigated the subject in deterministic environment or in stochastic environment. In order to make effective supply chain strategies, we cannot ignore the uncertainties that happen in the real world. In many cases, probability distributions of some random variables are difficult or impossible to obtain with lack of historical data. Some uncertainties have to be given or estimated by the experts based on their experiences and judgements. Therefore, it is reasonable to suppose that these uncertainties are fuzzy variables. However, there is still no research on ordering and pricing decisions for a closed-loop supply chain in fuzzy environment.

To overcome this limitation, this paper investigates the optimal ordering and pricing decisions in a closed-loop supply chain with fuzzy demand. First we derive the optimal decisions for a decentralized channel. In the decentralized channel, the manufacturer and the retailer work independently. The problem is described as a two-echelon manufacturer-Stg process. Then the model is extended to the centralized channel case. In the centralized channel, the manufacturer and the retailer make cooperation, which can also be regarded as an integrated-firm. Based on game theory and fuzzy theory, the optimal ordering decision and the optimal recovery prices are given for each setting. The factors that impact the optimal ordering decision and the optimal recovery prices are founded. Some characteristics of the optimal decisions are discussed from the view of management. Our main contribution is to develop the optimal decisions in the CLSC with fuzzy demand. The purpose of describing the demand as a fuzzy variable is to contribute new insight into decision making in CLSC.

The rest of this paper is organized as follows. Relevant preliminaries are reviewed in Section II. Problem statement is presented in Section III. Main conclusions on supply chain models with fuzzy demand are given in Section IV. Section V concludes this paper.
II. PRELIMINARIES

Definition 2.1 [9] Let ξ be a fuzzy variable. The expected value of ξ is defined by

\[ E(\xi) = \frac{E_x(\xi) + E^*(\xi)}{2} = \int_0^1 d_1(\lambda) + d_2(\lambda) d\lambda \]

where \([E_x(\xi), E^*(\xi)]\) is the interval-valued expectation, and \([d_1(\lambda), d_2(\lambda)]\) is the λ-level set of ξ.

Proposition 2.1 For linear operations of closed interval, by classical extension principle, we have the following conclusions:

(a) \([a, b] + [c, d] = [a + c, b + d];\)
(b) \(\alpha[a, b] = [\alpha a, \alpha b]\) if \(\alpha \geq 0, \alpha \in R;\)
(c) \(\alpha[a, b] = [ab, \alpha a]\) if \(\alpha < 0, \alpha \in R.\)

Proposition 2.2 [10] The triangular fuzzy number \(\xi = (a, b, c)\) as shown in Figure 1 can be described by the following membership function \(\mu_\xi(x)\)

\[ \mu_\xi(x) = \begin{cases} 
L(x) & \text{if } a \leq x \leq b \\
R(x) & \text{if } b < x \leq c \\
0 & \text{else}
\end{cases} \]

where \(L(x)\) and \(R(x)\) are the increasing and decreasing continuous functions, respectively. The \(\lambda\)-level set of \(\tilde{D}\) can be denoted as \(D_\lambda=[L^{-1}(\lambda), R^{-1}(\lambda)], \lambda \in [0, 1]\), which is a closed interval on real number set \(R.\)

![Figure 1: The membership function of \(\xi\)](image)

III. PROBLEM STATEMENT

This paper considers a closed-loop supply chain consisting of a manufacturer and a retailer. The retailer engages in the collection of used products. In the forward supply chain, the manufacturer wholesales a product to the retailer, who in turn retails it to the customer. In the reverse supply chain, the retailer buys a used product from the customer and remanufactures it to the manufacturer for remanufacturing. In this paper, we assume that there is no distinction between a remanufactured and a manufactured product and the demand is a triangular fuzzy number, i.e., \(\tilde{D} = (d_L, d_M, d_R).\) The problem can be described as Figure 2.

![Figure 2: Illustration of flow in a closed-loop supply chain](image)

The following notations are used throughout this paper.

- \(w\) Unit wholesale price,
- \(p\) Unit retail price,
- \(p_m\) Unit price of a used product from the retailer to the manufacturer (decision variable),
- \(p_r\) Unit price of a used product from the customer to the retailer (decision variable),
- \(p_s\) Unit price of a used product from the customer to the manufacturer (decision variable),
- \(Q_d\) Retailer’s order quantity in the decentralized channel (decision variable),
- \(Q_s\) Retailer’s order quantity in the centralized channel (decision variable),
- \(\tilde{D}\) The amount of used products collected from customers,
- \(c_n\) Unit cost of manufacturing of original products,
- \(c_r\) Unit cost of remanufacturing,
- \(s\) Retailer’s unit salvage value,
- \(u\) Retailer’s unit shortage value,
- \(\alpha\) Remanufacturing rate (0 ≤ \(\alpha\) < 1),
- \(r\) Marginal profit rate for collecting used products,
- \(\hat{R}\) Demand for the new product in the market

In this paper, we assume that \(\hat{R}\) takes on the exponential form, i.e., \(\hat{R} = \frac{b p_r k}{p_m}\), where \(b\) is a constant and \(k\) denotes the elasticity of the price.

IV. SUPPLY CHAIN MODELS WITH REMANUFACTURING

In this section, two settings are considered: decentralized channel and centralized channel.

A. Decentralized Channel

In the decentralized channel, the manufacturer and the retailer act to maximize their individual profits. In this case, a Stackelberg game occurs between them. We assume that the manufacturer behaving as a Stackelberg leader dominates the CLSC, and the retailer is a follower. The retailer’s problem and the manufacturer’s problem are as follows:
1) Retailer’s problem: If the quantity that the retailer orders is $Q_d$, the sales volume, holding and shortage quantity for the retailer can be denoted as $\min\{Q_d, D\}$, $\max\{Q_d - D, 0\}$ and $\max\{D - Q_d, 0\}$ respectively. Consequently, the total profit for the retailer can be expressed as follows:

$$
\pi_r = p \min\{Q_d, D\} + s \max\{Q_d - D, 0\} - wQ_d + R(p_m - p_r).
$$

(1)

In this paper, we assume that $p$ is decided by the market. Thus, for given $w$ and $p_m$, the retailer’s decision problem is to find $Q_d$ and $p_r$ to maximize $E(\pi_r)$, which can be expressed as

$$
\max_{Q_d, p_r} E[\pi_r].
$$

(2)

For this optimal problem, we have the following theorems.

Theorem 1 In the decentralized channel, for given $w$ and $p_m$, the optimal $Q_d$ is as follows:

$$
Q_d^* = \begin{cases} 
L^{-1}[\frac{2(p+u-w)}{p+u-s}], & \text{if } p + u \leq 2w - s \\
R^{-1}[\frac{2(w-s)}{p+u-s}], & \text{if } p + u > 2w - s 
\end{cases}
$$

(3)

Proof: We discuss this optimal problem by the following two cases.

1) Case 1 $d_L \leq Q_d \leq d_M$

$$
\begin{align*}
(\min \{Q_d, \hat{D}\})_\lambda &= \begin{cases} 
[L^{-1}(\lambda), Q_d], & 0 \leq \lambda \leq L(Q_d) \\
[Q_d, Q_d], & L(Q_d) < \lambda \leq 1
\end{cases} \\
(\max \{Q_d - \hat{D}, 0\})_\lambda &= \begin{cases} 
[0, Q_d - L^{-1}(\lambda)], & 0 \leq \lambda \leq L(Q_d) \\
[0, 0], & L(Q_d) < \lambda \leq 1
\end{cases} \\
(\min \{\hat{D} - Q_d, 0\})_\lambda &= \begin{cases} 
[0, R^{-1}(\lambda) - Q_d], & 0 \leq \lambda \leq L(Q_d) \\
[L^{-1}(\lambda) - Q_d, R^{-1}(\lambda) - Q_d], & L(Q_d) < \lambda \leq 1
\end{cases}
\end{align*}
$$

According to proposition 2.1, we know

if $0 \leq \lambda \leq L(Q_d)$

$$
(\pi_r)_\lambda = [pL^{-1}(\lambda) - uR^{-1}(\lambda) + (u - w)Q_d + R(p_m - p_r),
-sL^{-1}(\lambda) + (p + s - w)Q_d + R(p_m - p_r)].
$$

(4)

if $L(Q_d) < \lambda \leq 1$

$$
(\pi_r)_\lambda = [(p + u - w)Q_d - uR^{-1}(\lambda) + R(p_m - p_r),
(p + u - w)Q_d - uL^{-1}(\lambda) + R(p_m - p_r)].
$$

(5)

Then, according to definition 2.1, we get

$$
E(\pi_r) = \frac{1}{2} \int_0^{L(Q_d)} [(p - s)L^{-1}(\lambda) - uR^{-1}(\lambda)]d\lambda
+ \frac{1}{2} \int_{L(Q_d)}^1 uL^{-1}(\lambda) + R^{-1}(\lambda)]d\lambda
+ \frac{1}{2}(s - p - u)Q_dL(Q_d) + (p + u - w)Q_d
+ R(p_m - p_r)
$$

(6)

The first and second derivatives of the retailer’s expected profit $E(\pi_r)$ with respect to $Q_d$, respectively, are

$$
dE(\pi_r)/dQ_d = \frac{1}{2}(s - p - u)L(Q_d) + (p + u - w)
$$

and

$$
d^2E(\pi_r)/dQ_d^2 = 1/2(dL(Q_d)/dQ_d - (s - p - u)).
$$

where $L(Q_d)$ is an increasing function of $Q_d$, which means that $dL(Q_d)/dQ_d > 0$. Since $s < p$, we get $d^2E(\pi_r)/dQ_d^2 < 0$. In this case, we can conclude that $E(\pi_r)$ is concave with respect to $Q_d$. By setting $d^2E(\pi_r)/dQ_d^2$ to zero, we get $L[Q_d] = \frac{2(p+u-w)}{p+u-s}$. It is obvious that $\frac{2(p+u-w)}{p+u-s} > 0$. Thus, when $\frac{2(p+u-w)}{p+u-s} < 1$, that is $p + u \leq 2w - s$, we can determine the profit-maximizing $Q_d^*$ as follows:

$$
Q_d = L^{-1}\left[\frac{(p+u-w)}{p+u-s}\right].
$$

(7)

2) Case 2 $d_M \leq Q_d \leq d_R$

$$
\begin{align*}
(\min \{Q_d, \hat{D}\})_\lambda &= \begin{cases} 
[L^{-1}(\lambda), Q_d], & 0 \leq \lambda \leq R(Q_d) \\
[Q_d, Q_d], & L(Q_d) < \lambda \leq 1
\end{cases} \\
(\max \{Q_d - \hat{D}, 0\})_\lambda &= \begin{cases} 
[0, Q_d - L^{-1}(\lambda)], & 0 \leq \lambda \leq R(Q_d) \\
[Q_d - R^{-1}(\lambda), Q_d - L^{-1}(\lambda)], & R(Q_d) < \lambda \leq 1
\end{cases} \\
(\min \{\hat{D} - Q_d, 0\})_\lambda &= \begin{cases} 
[0, R^{-1}(\lambda) - Q_d], & 0 \leq \lambda \leq R(Q_d) \\
[0, 0], & R(Q_d) < \lambda \leq 1
\end{cases}
\end{align*}
$$

According to proposition 2.1, we get

if $0 \leq \lambda \leq R(Q_d)$

$$
(\pi_r)_\lambda = [pL^{-1}(\lambda) - uR^{-1}(\lambda) + (u - w)Q_d + R(p_m - p_r),
(p + s - w)Q_d - sL^{-1}(\lambda) + R(p_m - p_r)].
$$

(8)

if $R(Q_d) < \lambda \leq 1$

$$
(\pi_r)_\lambda = [pL^{-1}(\lambda) - sR^{-1}(\lambda) + (s - w)Q_d + R(p_m - p_r),
prR^{-1}(\lambda) - sL^{-1}(\lambda) + (s - w)Q_d + R(p_m - p_r)].
$$

(9)
2) Manufacturer’s problem: The total profit for the manufacturer can be expressed as follows:

\[
\pi_m = (w - c_n)Q_d + (c_n - c_r)\alpha \bar{R} - p_m \bar{R}. \tag{13}
\]

In this paper, we assume that \( w \) is exogenous. Thus, the manufacturer’s decision problem is to find \( p_m \) to maximize \( E(\pi_m) \), which can be expressed as

\[
\max_{p_m} E[\pi_m]. \tag{14}
\]

For this optimal problem, we have the following theorem.

**Theorem 3** In the decentralized channel, the optimal unit recovery price to manufacturer is as follows:

\[
p_m^* = \frac{\alpha k(c_n - c_r)}{k + 1}.
\]

The optimal recovery price is an increasing function of \((c_n - c_r)\).

Proof: The first and second derivatives of the manufacturer’s profit \( \pi_m \) with respect to \( p_m \), respectively, are

\[
\frac{d\pi_m}{dp_m} = b(1 - r)^k p_m^{k-1}[\alpha k(c_n - c_r) - (k + 1)p_m]
\]

and

\[
\frac{d^2\pi_m}{dp_m^2} = kb(1 - r)^k p_m^{k-2}[\alpha k(1 - c_n) - (k + 1)p_m].
\]

Since \( \alpha k(1 - c_n) - (k + 1)p_m \leq \alpha k(1 - c_n) - (k + 1)p_m < 0 \), we get \( \frac{d^2\pi_m}{dp_m^2} < 0 \), which means that \( \pi_m \) is concave with respect to \( p_m \). By setting \( \frac{d\pi_m}{dp_m} \) to zero, we get

\[
p_m^* = \frac{\alpha k(c_n - c_r)}{k + 1}.
\]

B. Centralized Channel

In the centralized channel, the manufacturer and the retailer cooperate with each other and behave as an integrated firm. The total profit for the system can be expressed as follows:

\[
\pi_s = p_{\min}\{Q_s, \bar{D}\} + \max\{Q_s - \bar{D}, 0\} - u_{\max}\{\bar{D} - Q_s, 0\} - c_nQ_s + \alpha \bar{R}(c_n - c_r) - p_s \bar{R}. \tag{15}
\]

The problem for the system is to find \( Q_s \) and \( p_s \) to maximize \( E(\pi_s) \), which can be expressed as

\[
\max_{p_s} E(\pi_s). \tag{16}
\]

For this optimal problem, we have the following theorems.

**Theorem 4** In the centralized channel, the optimal production \( Q_s \) and the optimal unit recovery price \( p_s \) are as follows:

\[
Q_s^* = \begin{cases} 
L^{-1}\left[\frac{2(w - u)}{p + u - s}\right], & \text{if } p + u \leq 2c_n - s \\
R^{-1}\left[\frac{2(c_n - s)}{p + u - s}\right], & \text{if } p + u > 2c_n - s
\end{cases}
\]

\[
p_s^* = \frac{\alpha k(c_n - c_r)}{k + 1}.
\]

The proof is similar to Theorem 1.

**Theorem 5** In the centralized channel, the optimal \( p_s \) is
an increasing function with respect to \( k, \alpha \) and \( (c_n - c_r) \), respectively.

**Proof:** As mentioned in Theorem 4, \( p_s^* = \frac{\alpha k(c_n - c_r)}{k+1} \).
It is clear that \( p_s^* \) is an increasing function with respect to \( \alpha \) and \( (c_n - c_r) \), respectively. Since \( p_s^* = \frac{\alpha k(c_n - c_r)}{k+1} = \alpha(c_n - c_r) - \frac{\alpha(c_n - c_r)}{k+1} \), we can observe that \( p_s^* \) is an increasing function of \( k \).

C. Numerical example

In order to illustrate our model, let us consider the case that the fuzzy demand \( D = (450, 500, 600) \), \( p = 8 \), \( w = 5 \), \( s = 2 \), \( u = 1 \), \( c_n = 4 \), \( c_r = 2.5 \), \( \alpha = 0.7 \), \( k = 1.5 \), \( b = 2 \).
By Theorem 1, we can get the retailer’s optimal order quantity in the decentralized channel \( Q_d^* = 514 \). By Theorem 2, we can get the optimal unit recovery price to the retailer \( p_s^* = 0.252 \). By Theorem 3, we can get the the optimal unit recovery price to the manufacturer \( p_m^* = 0.42 \). By Theorem 4, we can get the the optimal order quantity in the centralized channel \( Q_s^* = 542 \) and the optimal unit recovery price \( p_s = 0.63 \).

The comparison and the detailed analysis of the results in the two settings will be reported in the future.

V. Conclusion

In this paper, the problem associated with ordering and pricing decisions in a closed-loop with fuzzy demand has been investigated for two settings: decentralized channel and centralized channel. By using game theory and fuzzy theory, we have derived the optimal ordering decision and the optimal pricing decisions. From the optimal decisions, we have observed that: (1) the unit recovery price from the customer to the retailer is related to the remanufacturing rate \( \alpha \), the elasticity of the price \( k \), and the difference of the unit cost of manufacturing of original products \( c_n \) and the unit cost of remanufacturing \( c_r \), no matter in the decentralized channel or in the centralized channel; (2) the marginal profit rate \( r \) for collecting used products to the retailer in the decentralized channel is related to the elasticity of the price \( k \).

It is worth pointing out that there are other uncertainties in systems, such as the quantity of returned used products and the remanufacturing rate. The problem associated with ordering and pricing decisions in a closed-loop supply chain with these kinds of uncertainties can become a new topic in further research, and we also can extend the result of this paper to the scene that contains multi-retailers.

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