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Tragedy of the Commons in Online Social Search

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Abstract—The overall performance of an Online Social Search (OSS) system may suffer due to users’ non-cooperation, as in the “tragedy of the commons” problem in social science. In this paper, we study this non-cooperation problem. We propose an analytical model that captures the behavior of OSS nodes, and, from a gaming-strategy point of view, analyze various strategies an individual node can utilize to allocate its awareness capacity. Based on this we derive the Pareto inefficiency in terms of the system cost. We also propose an incentive scheme which can lead selfish nodes to the “social optimal” state of the whole system. Extensive simulations show that the strategy with our proposed incentive mechanism outperforms other strategies in terms of the system cost and the search success rate. To the best of our knowledge, this is the first study of the tragedy-of-the-commons problem in OSS.

I. INTRODUCTION

Online social search (OSS) refers to information search utilizing the underlying network structure of online social networks (OSN), and it has received attention in both research [1] and actual applications [2]. In a typical OSS system, a person looking for information sends the question to his selected contacts. When a user receives a question on which he is an expert, he responds to the questioner. He can also forward it to his selected contacts. In this way, the question is passed on in the social network. Finally, the questioner may be presented with a great number of potential respondents.

In economics, a cost is an alternative that is given up as a result of a decision [3]. As for OSS, cost can be interpreted in different senses. For example, it can refer to the communication effort required by a node to become aware of the expertise of other nodes, and it can also refer to OSS nodes’ privacy exposure [4]. Naturally, each participating node wishes to minimize its own cost. Our previous work studied the performance of OSS [5] [6], but certain issues of the basic cooperative mechanism among OSS nodes are still unaddressed. Tragedy of the commons [7] is one such issue. The phenomenon of tragedy of the commons is a kind of non-cooperativeness, in particular, a situation in which multiple rational individuals inevitably exhaust a limited public resource even though it is contrary to their long-term goals. In OSS, for example, if every node spends less effort on communicating with others in order to incur a lower cost, it is highly possible that a question will be forwarded to a person who is not expert on it, and thus the forwarding path for this referral session could be very long and be forwarded to a person who is not expert on it, and thus the node spends less effort on communicating with others in order contrary to their long-term goals. In OSS, for example, if every inevitably exhaust a limited public resource even though it is

II. MODEL

We consider an OSN as an undirected graph \( G(V,E) \), where \( V \) is the set of nodes (OSN users) and \( E \subseteq V \times V \) is the set of edges (social ties) in the network. Let \( n = |V| \). We also denote by \( N_u \subseteq V \) the set of neighbors of Node \( u \), and \( d_u = |N_u| \). Since a node maintains its local social network, we equip the nodes with the intelligence of awareness. Denote by \( S_u(v,i) \) Node \( u \)'s awareness of its neighbor \( v \), with respect to Question \( i \). We study the most general case and classify nodes’ awareness into three types. In other words, we divide a node’s neighbors into three possible sets, namely, experts \((S_u(v,i) = 1)\), non-experts \((S_u(v,i) = -1)\), and unknown \((S_u(v,i) = 0)\). Let \( R = \{r_1,r_2,\ldots,r_m\} \) and \( E = \{e_1,e_2,\ldots,e_m\} \) be the distribution of nodes’ awareness level and the expert density distribution over \( m \) posed questions, respectively. \( c = \sum_{i=1}^{m} r_i \), which represents the average capacity of a node’s awareness towards another node. The detailed description of the awareness level, the expert density, and the OSS referral strategy is presented in [5].

Definition 1. Node \( u \)'s individual awareness level of its neighbors’ expertise in Question \( i \) is defined as

\[
r_{ui} = \frac{\delta_{ui}}{d_u},
\]

where \( \delta_{ui} = |\{v \in N_u | S_u(v,i) \neq 0\}| \).

Besides we set a new constraint
\[ \sum_{i=1}^{m} r_{ui} = c_u, \]
which represents the capacity [4] of Node \( u \)’s awareness towards its neighbors.

Denote by \( m_u \), the number of questions posed by Node \( u \) in the system, and \( \sum_{u \in V} m_u = m \). Suppose the number of questions in the system is very large, which means that an individual node requires a large capacity (\( c_u \)) to gain enough awareness of its neighbors. In addition, a rational node may avoid asking too many questions in order not to strain the willingness of possible responders. Thus we can assume that the individual capacity is larger than the number of questions a node raises to the network, that is, \( c_u > m_u \).

It can be observed that in the case of homogeneous settings of \( d \) (the number of a node’s neighbors), the awareness level on Question \( i \) is the arithmetic average of the individual awareness levels of nodes in the system, that is
\[ r_i = \frac{\sum_{u \in V} \delta_{ui}}{nd} = \frac{1}{n} \sum_{u \in V} r_{ui}. \]

Further we can obtain
\[ c = \sum_{i=1}^{m} r_i = \frac{1}{n} \sum_{i=1}^{m} \sum_{u \in V} r_{ui} = \frac{1}{n} \sum_{u \in V} \sum_{i=1}^{m} r_{ui} = \frac{1}{n} \sum_{u \in V} c_u. \]

Based on the assumption of \( c_u \), we can see that \( c > \frac{m}{n} \).

**Definition 2.** The cost of a referral session for Question \( i \), denoted by \( C(i) \), is defined as the expected number of nodes Question \( i \) has visited upon termination of this referral session. We further denote by \( C \) the system cost, \( C = \frac{1}{n} \sum_{i=1}^{m} C(i) \), i.e., the average cost of a referral session in the system. Given the distribution of awareness level and expert density, we can calculate the system cost by the following formula derived in [4]
\[ C(E, R) = \frac{1}{m} \cdot \sum_{i=1}^{m} \frac{1}{(1 + r_i d) \epsilon_i}. \]

As in [4] in which the cost reflects the exposure degree of nodes’ privacy, \( C \) also measures the network resource (e.g., communication cost incurred by users of OSS who contact their neighbors) consumption from a networking point of view, and we have the following theorem.

**Theorem 1** [4]. The optimal distribution of nodes’ awareness level that minimizes the system cost satisfies
\[ r_i = \left( \frac{m}{d} + c \right) \cdot \frac{1}{\sum_{j=1}^{m} \sqrt{\frac{1}{\epsilon_j}}} - \frac{1}{d}, \quad i = 1, \ldots, m \]  \hspace{1cm} (3)

### III. PARETO INEFFICIENCY

In this section, we study the problem of Pareto inefficiency in terms of the system cost because of nodes’ selfish behavior. We first describe the problem under the assumption of homogeneous setting of \( d \), and then we discuss possible approaches to mitigate the inefficiency.

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### A. Tragedy of the commons

The phenomenon of the “tragedy of the commons” [7] is a situation in which multiple rational individuals, acting independently to pursue their own interest, inevitably exhaust a public limited resource even though it is contrary to their long-term goals. The concept is first proposed and studied in social science since it occurs in many natural and social systems (e.g., a common pasture overgrazed by selfish herdsman and the failure of negotiation among nations on global environmental pollution issues). In a more general and game-theoretic perspective, the tragedy of the commons refers to a dilemma where the Nash equilibrium achieved in the system may turn out to be inefficient overall because of the selfish actions of individual players in the game. In the field of communication and computer networks, [9], [10] and [8] respectively studied the problem of noncooperative routing, load balancing, and flow control as examples of strong Pareto inefficiency. [8] further proposed a general framework of strongly Pareto-inefficient Nash equilibria. While previous work focused on the Pareto inefficiency in traditional transportation and communication networks, here we address this question in the OSN scenario. To study this problem in more details, we have to introduce the definition of Pareto efficiency first.

[Pareto efficiency] [8] Denote by \( \lambda \) the strategy profile of the noncooperative game and \( C \) the set of feasible values of \( \lambda \). Let \( U_i(\lambda) \) denote the utility function of player \( i \). Then, in the case where \( \forall i, U_i(\lambda^*) \geq U_i(\lambda) \) and \( \exists i, U_i(\lambda^*) > U_i(\lambda) \), \( \lambda^* \) is Pareto superior to \( \lambda \). In the case where there exists \( \lambda \in C \) s.t. \( \lambda \) is Pareto superior to \( \lambda \), \( \lambda \) is **Pareto efficient**. In the case where \( \forall i, U_i(\lambda^*) > U_i(\lambda) \), we say that \( \lambda^* \) is strongly Pareto superior to \( \lambda \). In the case where there exists \( \lambda \in C \) s.t. \( \lambda \) is strongly Pareto superior to \( \lambda \), we say that \( \lambda \) is strongly Pareto efficient.

Informally speaking, Pareto efficient situations are those in which it is impossible to make one person better off without necessarily making someone else worse off. That means the so called “Pareto improvement” cannot be further achieved.

**Theorem 2.** To achieve the Pareto efficient situation of the system cost, all nodes in the system are required to contribute up to the capacity of their awareness towards their neighbors.

**Proof:** [4] has deduced the expressions of the minimum value of privacy exposure degree. Since a Pareto efficient strategy profile represents the “social optimal” state in the network, we can use it to calculate the system cost, that is
\[ C_{\text{min}} = \frac{1}{m(m + dc)} \cdot \left( \sum_{i=1}^{m} \sqrt{\frac{1}{\epsilon_i}} \right)^2. \]

Suppose that certain nodes do not contribute their awareness up to the capacity, namely, \( \sum_{i=1}^{m} r_i^* = c^* < c \). Denote by \( R^* = [r_1^*, r_2^*, \ldots, r_m^*] \) the optimal distribution of nodes’ awareness level defined by Eqn. (3). Combining with Eqn.
we obtain
\[ C_{\text{min}}^* = \frac{1}{m(m + d e^*)} \left( \sum_{i=1}^{m} \sqrt{\frac{\Gamma_i}{e_i}} \right)^2 > C_{\text{min}}. \]

We can see that the system with \( R^* \) incurs higher cost, which is Pareto inefficient.

From Theorem 2 we can see that to achieve the minimum system cost, each node is required to gain a thorough knowledge about the expertise of its neighbors in order to avoid blind search. However, in a realistic network scenario, a node needs to communicate with its neighbors to obtain knowledge about their expertise, which may incur certain communication overhead. For example, in a wireless network scenario, a mobile node may consume its battery power as well as wireless bandwidth to communicate with other nodes. Denote by \( CE(i) \) the communication effort of Node \( i \). Suppose Node 1 through Node \( i - 1 \) have raised a total of \( k - 1 \) questions (i.e., \( \sum_{j=1}^{k-1} m_j = k - 1 \) ) and Node \( i \) poses Question \( k, k + 1, \ldots, k + m_i - 1 \) (totally \( m_i \) questions) to the system. Considering both the question forwarding cost (or privacy exposure degree [4]) and the communication effort, combining (1) and (2), we define the utility of an individual node as,

\[ U_i = -\sum_{s=k}^{k+m_i-1} C(s) - CE(i) \]

(5) so that the Nash equilibrium should be in the form \((0, 0, \ldots, r_{i,k}, r_{i,k+1}, \ldots, r_{i,k+m_i-1}, \ldots, 0)\), where Question \( k, k + 1, \ldots, k + m_i - 1 \) are \( m_i \) questions Node \( i \) poses on the system.

Proof: From Lemma 1 we can see that the Nash equilibrium of the game exists. The next step is to find the corresponding strategy. Suppose that Node \( i \) chooses a different action \( \lambda_i = (r_{i,1}, r_{i,2}, \ldots, r_{i,m_i}) \), s.t. \( \exists j \neq k, k + 1, \ldots, k + m_i - 1, r_{ij} \neq 0 \), from the utility function (5) we can see that Node \( i \) can obtain a better payoff by setting \( r_{ij} = 0 \). Thus \( \lambda_i \) is a strictly dominated action and cannot be used in any Nash equilibrium. Thus the strategy which constitutes the Nash equilibrium should be in the form \((0, 0, \ldots, r_{i,k}, r_{i,k+1}, \ldots, r_{i,k+m_i-1}, \ldots, 0)\).

From Theorem 3, in Nash equilibrium, we can simplify the utility function of Node \( i \) (5) so that

\[ U_i = -\sum_{j=k}^{k+m_i-1} \frac{1}{(1 + \frac{d}{n} r_{ij}) e_j} + D \cdot r_{ij} \] (6)

We assume \( d \cdot r_{ij} = \delta_{ij} \ll n \), which holds in large OSN and allows us to do the approximation on (6), that is

\[ U_i = -\sum_{j=k}^{k+m_i-1} \frac{1}{e_j} (1 - \frac{d}{n} r_{ij}) + D \cdot r_{ij} \]

(7)

Suppose \( D - \frac{d}{n e_j} > 0 \), then the maximum of (7) is achieved when \( r_{ij} = 1, j = k, k + 1, \ldots, k + m_i - 1 \) (note that we have assumed that \( c_i > m_i \)). Denote the awareness level in Question \( j \), individual awareness level of Node \( i \), the system’s awareness level under the “selfish” strategies of nodes in the system as \( \hat{r}_j, \hat{c}_i, \hat{c} \), respectively, we can further obtain that

\[ \hat{r}_j = \frac{1}{n} \sum_{u \in V} r_{uj} = \frac{1}{n}, \quad j = 1, 2, \ldots, m, \]

\[ \hat{c}_i = \sum_{j=1}^{m} r_{ij} = m_i, \quad i = 1, 2, \ldots, n, \]

\[ \hat{c} = \frac{1}{n} \sum_{i=1}^{n} c_i = \frac{m}{n} \]

According to our previous assumption on \( c_i \), we can see that in the Nash equilibrium of the game among nodes in OSS, all nodes choose to reserve their ability of knowing their neighbors. Based on Theorem 2, the minimum system cost cannot be achieved, which means that the outcome of the game is Pareto inefficient.

B. Incentive mechanism

Classical economic theory states that in an economic system, it may not be sufficient to make a change aimed at improving economic efficiency while ensuring that nobody is harmed merely with the “invisible hand”, some specific
incentive, for example, compensation from third parties, may be required. This gives us the first clue on proposing a mechanism which can solve the tragedy of the commons.

Assume Node $i$ allocates its awareness level following the rule similar to the inverse square root law in (3), that is

$$ r_{ij} = \left( \frac{m}{d} + c_i \right) \cdot \frac{\sqrt{c_j}}{\sum_{k=1}^{m} \sqrt{c_k}} - \frac{1}{d}, \tag{8} $$

we can derive that $r_j = \frac{1}{n} \sum_{i=1}^{n} r_{ij} = \left( \frac{m}{d} + c \right) \cdot \frac{\sqrt{c_j}}{\sum_{k=1}^{m} \sqrt{c_k}} - \frac{1}{d},$

which corresponds to the optimal distribution of $r_j$ according to (3). An obvious mechanism for the system operator is to give a large enough incentive to encourage nodes to follow the above distribution law, because proper incentives can guarantee that the optimal state of individual nodes is also optimal for the whole system.

**Proposition 1.** Suppose Node $i$ raises Question $k$, $k + 1$, ..., $k + m_i - 1$. Under the mechanism that the system operator will compensate all the communication effort ($C_E(i)$) of Node $i$ plus some bonus $B_i > 0$ if and only if Node $i$ allocates its individual awareness level $(r_{i1}, r_{i2}, \ldots, r_{im})$ following rule (8), the equilibrium strategy of Node $i$ should follow the instruction of the operator if and only if $B_i$ satisfies the following expression

$$ B_i \geq \sum_{s=k}^{k+m_i-1} (C_i(s) - \hat{C}(s)) - D \cdot m_i, $$

where $C_i(s)$ and $\hat{C}(s)$ stand for the cost of referral sessions for questions raised by Node $i$ under the strategies of following system operator’s instructions and the “selfish” action according to Theorem 3, respectively.

**Proof:** Denote by $\lambda_i = (r_{i1}, r_{i2}, \ldots, r_{im})$ the action of Node $i$ following operator’s instructions, and $\hat{\lambda}_i = (\hat{r}_{i1}, \hat{r}_{i2}, \ldots, \hat{r}_{im})$ as the “selfish” action of Node $i$ according to Theorem 1. Combining (5) and the mechanism proposed in Proposition 1, we can calculate the utility functions under two strategies

$$ U_i = -\sum_{s=k}^{k+m_i-1} C_i(s) + B_i $$

$$ U_i = -\sum_{s=k}^{k+m_i-1} \frac{1}{(m + dc)\sqrt{c_s}} + B_i $$

$$ \hat{U}_i = -\sum_{s=k}^{k+m_i-1} \hat{C}(s) - D \cdot m_i $$

$$ \hat{U}_i = -\sum_{s=k}^{k+m_i-1} \frac{1}{(1 + \frac{d}{n})e_s} - D \cdot m_i $$

From (9) and (10) we can easily observe that if $B_i \geq \sum_{s=k}^{k+m_i-1} (C_i(s) - \hat{C}(s)) - D \cdot m_i$, the “selfish” strategy of Node $i$ is always strictly dominated by following the operator’s rule. Combining with Theorem 3 we can further conclude that the equilibrium strategy of Node $i$ is to follow the operator’s instruction.

**IV. EVALUATION**

In this section, we study the effect of node’s awareness distribution on the system cost and average individual utility. In the simulation, 20 questions are randomly posed, with $e = 5 \times 10^{-4}, 1 \times 10^{-3}, \ldots, 1 \times 10^{-2}$ (20 values with increment of $5 \times 10^{-4}$). Since we assume that a single node’s awareness capacity should be more than the number of questions it asks, to simplify the simulation, we set an extra constraint that each node in the system can ask no more than one question. Also we set the price coefficient in (5) to 2. For comparison, we utilize four different allocation strategies of individual awareness level $r_i$, namely, the “selfish” strategy according to Theorem 3 (referred to as Selfish distribution), “optimal” strategy according to (8) (referred to as Optimal distribution), allocating the individual awareness level randomly subject to the capacity constraint (referred to as RAFC strategy) and subject to a predetermined value which is less than the capacity (referred to as RALC strategy). The results are obtained as averages of 1000 simulation runs. Figure 1 shows the average individual utilities under the four individual awareness level allocations based on a $k$-regular graph of $1 \times 10^4$ nodes. The x-axis represents the average individual awareness capacity $c$. We can see that the average individual utility with the Selfish allocation of $r_i$ is the highest among the four allocation strategies. This verifies Theorem 3 that the selfish strategy constitutes the Nash equilibrium of the game among nodes since this action is the best action for an individual node.

Figure 2 presents the simulation results of system cost under different individual awareness level distributions. We observe that the Optimal allocation incurs the lowest system cost among all four distributions. This verifies Theorem 1 that the Optimal distribution minimizes the system cost. In addition, we observe that although the system cost obtained under the RAFC strategy is higher than the Optimal allocation, it is still less than the Selfish and RALC strategy, which matches Theorem 2 that says all nodes in the system are required to contribute up to their awareness capacity towards their neighbors to achieve the Pareto efficient situation in terms of the system cost. We also notice that the average performances under the strategy of Optimal and RAFC improve as $c$ increases, while the system costs under the Selfish and RALC strategy remain almost the same. Intuitively, the more familiar a node is about its neighbors, the less hops it takes for a question to find the right expert. However, if every node in the network chooses to reserve its awareness ability, the performance of the system cannot improve even with a higher awareness capacity.

Inspired by [5], we also study the average search success rate of referral sessions for the four awareness level distr-
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...nodes in the system should contribute all their individual rate under RAFC and RALC also substantiate our assertion of average search success rate. The result of search success causes the deterioration of the overall performance in terms of questions. The ignorance of all other nodes in the system choose to spend no effort to gain knowledge on these unrelated (i.e. 100%) of its neighbors’ expertise, all the other nodes a single node can get the highest possible awareness level at first sight, because a node who utilizes the selfish strategy of the Selfish allocation. The result is quite counter-intuitive which seems to guarantee a high likelihood for the question to reach a right expert. A possible explanation is that although a single node can get the highest possible awareness level (i.e. 100%) of its neighbors’ expertise, all the other nodes choose to spend no effort to gain knowledge on these unrelated questions. The ignorance of all other nodes in the system causes the deterioration of the overall performance in terms of average search success rate. The result of search success rate under RAFC and RALC also substantiate our assertion that nodes in the system should contribute all their individual awareness capacities and cooperate with each other to achieve a satisfactory search success rate.

V. CONCLUSION

We study the tragedy-of-the-commons problem in online social search. We propose an analytical model to capture the behavior of OSS nodes. From a gaming strategy point of view, we derive the Pareto inefficiency in terms of the system cost due to the non-cooperation among OSS nodes. To solve this inefficiency, we propose an incentive scheme under which the optimal state of individual nodes is also optimal for the whole system. We also evaluate various awareness allocation strategies a node can utilize. The results of extensive simulations validate our analysis, and further show that the strategy under our proposed incentive mechanism outperform other strategies in terms of the system cost and the search success rate. In this paper, we describe a centralized scheme, i.e., the system operator has to provide the participating nodes with the incentives. In the future, we would like to develop a distributed mechanism where the incentives are provided by other nodes in the system [12]. In addition, we will further evaluate the system based on real-world OSNs.

Fig. 1. Comparison of individual utilities under different awareness allocation strategies

Fig. 2. Comparison of system costs under different awareness allocation strategies

Fig. 3. Comparison of search success rates under different awareness allocation strategies.

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