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<td>Lin, M; Li, S; Whinston, A</td>
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Dynamic Innovation in a Two-Sided Platform

Abstract

We are interested in a two-sided platform, in which dynamic innovation plays a role in stimulating consumer demand that also drives firms’ incentive to innovate. By explicitly modeling the price competition within the two-sided market, we study ways consumers’ platform fee interacts with firms’ pricing strategies on the platform. Our framework also characterizes a dynamic R&D race and solves the stationary Markov equilibrium using computation methods. We find that by charging consumers a fee, the platform is not necessarily better off, because firms may subsidize this cost by lowering their prices in the market, which leads to lower transaction revenues and innovation rate. Platform’s revenues may also suffer if it shares firms’ transaction revenues. Surprisingly, despite the platform fee, consumer welfare improves as a result of lower prices. However, these effects are not monotonic, and shifts in the opposite direction occur when firms switch to different pricing strategies, because consumers’ platform fee also mitigates price competition between low- and high-quality firms.

1 Introduction

The two-sided market literature has explored problems in network externality, platform’s allocation of cost between two sides of the market, and social efficiencies, using static settings [13] [3] [10]. However, the recent explosion of two-sided app markets and other platform has been heavily driven by innovative products. For example, the success of Apple’s App Store1 and Google Android Market2 for mobile devices such as smartphones and wireless tablets (e.g., the iPad), largely owes to the novelty, variety, and quality of the apps created for them. Market competition as well as innovation race play increasingly important roles [1] [7] in “bringing both sides on board” aside from platform pricing.

The market structure of two-sided markets exhibits far more complexity than what has been assumed in past studies. While consumers purchase the mobile devices, the app developers pay a fee in order to obtain the software development kit (SDK) for coding apps. These costs are the prices for enrolling on the platform. The market for apps has matured significantly since the first introduction of iPhone. Paid apps now vary greatly in their quality and prices. Consumers also have abundant information to help them assess app quality, and have different budget for purchasing apps.3 App developers face innovation decisions of whether to obtain the SDK and the level of effort to invest to create an attractive app. Consumers also continuously shop for newer apps as more get listed in the app store.

Consumers are then interested in more than the mere size of the developer network size, since the rate at which quality apps are introduced to the app market for their platform plays a major part in their experience. App developers’ incentives to innovate are also motivated by consumers demand for newer apps and market profitability. Such externality driven by continuous innovation has not been well researched to our best knowledge. We are interested in tying the innovation race among the app developers with the microfoundation for the price competition within the two-sided market. Through modeling a dynamic problem and computing for the stationary Markov equilibrium, we offer understanding on ways the platform fee (charged by the platform to the consumers) impacts the economy of the market created by the platform.

When consumers pay a fee to use or purchase a platform device, such cost may lower the prices of goods sold on the platform, and, interestingly, mitigate price competition in the market in the meantime. The forces of competition cause firms to subsidize consumers’ platform fee by lowering prices. In particular, the higher quality firms cut their prices more substantially than the lower quality firms, which find leverage and compete less aggressively. Consequently, consumer welfare improves despite the result that the equilibrium innovation rate suf-

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3Some would only stay with the free apps, while others are willing to buy more expensive one.
fers. The latter effect is due to the reduction in firms’ revenues, which serve as the main innovation incentive. Lastly, while the platform gains from higher fees charged to the consumers, it actually experiences overall revenue loss as well if it obtains a share from the revenues of market transactions.

While results due to price cuts in the platform market appear undesirable besides increasing consumer welfare, we also discover that the mitigated price competition may generate a boost over time. The level of consumers’ platform fee indirectly determines the market structure of the platform market – significant increase of such cost can lead to drastically different equilibrium pricing strategies. As the lower quality firms perceive sufficient leverage created by consumers’ platform fee, equilibrium prices of all firms will shift upward, resulting in a less competitive market. In this case, firm revenues, equilibrium innovation rate, and platform revenues all have an upward shift.

The rest of the paper is organized as follows. We review related literature in Section 2. Section 3 describes our framework including the microfoundation which characterize the price competition of different qualities of goods in the platform market and its equilibrium, and the dynamic innovation race among potential entrants. We then explain our computation approach and discuss the findings in detail in Section 4. Lastly, we conclude.

2 Literature review

Among the studies on platform economy, a line of interesting works have looked at the pricing problem of two-sided markets. Rochet and Tirole examine the factors that determine how platforms allocate the charges to the two sides of the market [12]. They find that characteristics of buyers, such as degree of multithoming that generates a high surplus, lead to more a favorable pricing structure [12]. Armstrong presents three models of different types of competition between platforms based on whether agents join one or several platforms [3]. The fee structures of two-sided markets are studied by Caillaud and Jullien in [4], where the one-time registration fee and transaction prices could be introduced. They derive the impact that different fee structures have on platform competition and equilibrium prices [4].

Other interesting questions on two-sided market have also been explored. Hagiu [10] identifies a fundamental economic welfare trade off between two-sided open platforms and proprietary platforms, and shows that under certain circumstances proprietary platforms can be more socially desirable than open platforms. Derdenger conducts an empirical investigation of the issue of technological tie in the video game industry using structural estimation and finds that tying increases the console price competition [8]. All studies on two-sided markets thus far, to our best knowledge, are limited by static modeling, including structural estimation techniques in the empirical work [8]. To gain a deeper understanding from another angle, We aim to take into consideration inter-temporal elements that orchestrate the dynamic innovation process and demand on two sides of the market, in a system consisted of a platform, app developers, and consumers.

Our interest coincides with the two-sided market; but we offer a more elaborate analysis by building a framework that endogenizes the consumer network size by the vertically differentiated product market where sellers compete in price, and endogenizes the potential sellers’ innovation decisions. Combined with analytical solutions, we use computation to explore a number of experiments addressing the issues of impact of platform pricing on innovation, platform revenue due to innovation, and the market structure of the two-sided market. This setup extends from a previous work by Lin et al. [11] by incorporating platform prices and considering a two-sided market, whereas [11] focuses on the relationship between income inequality and innovation without the intervention of an intermediary in the market.

Our study also contribute to the body of literature on innovation in industrial organization. Segal and Whinston provide a dynamic model for analyzing antitrust policy and innovation [14], and assume that a successful innovator enters the market, receives an entrant’s profit in the first period, and then becomes the monopolist if another innovation enters the market. We offer an additional dimension by endogenizing the market structure. Without assuming a monopolistic market, we consider a vertically differentiated market where incumbents’ profits are determined by the price competition. Moreover, several interesting studies have investigated simultaneously the effect of subsidies on innovative and imitative technologies using growth models [15] [5]. These works consider horizontally differentiated innovative goods [9] [15] [5], whereas we treat generations of innovations as vertically differentiated based on the setting in [16]. The two different perspectives allow for a more in-depth understanding of the role of R&D subsidies, which in our case is a reduction in the platform price for innovating sellers.
3 The model

In our dynamic problem, each discrete period has the discount factor $\beta \in (0, 1)$. In each period, there exist two groups of firms differing in their objectives and actions. The incumbent firms compete on price in the product market, into which the innovations are introduced as the latest generation or the highest quality good; the potential entrants are the firms making innovation decisions in the R&D race. This section presents the model setup for the price competition in each period and analyze firms’ pricing strategies and market segmentation based on consumers’ preferences and platform prices. In Section 3.4, we will analyze the firm’s innovation decisions in the infinite horizon: The innovators, prior to successfully innovating and entering the product market, choose whether to enter the R&D race and, if so, determine the equilibrium level of innovation effort.

In Section 4, we solve for the stationary Markov perfect equilibria of the dynamic game. In each period, consumers observe the price set by the platform, $p_b$, and firms that produce vertically differentiated, substitute goods as a result of the innovation race, described in Section 3.4. Denote $k = 1, ..., n$ as the index for product quality, where a higher $k$ represents a higher quality.

3.1 Consumers

The setup here follows that of Shaked and Sutton [16]. Consumers are heterogeneous in their budget for purchasing products on the two-sided market. Denote a consumer’s budget by the random variable $z$, which follows the uniform distribution: $z \sim U[z, \bar{z}]$.

In each period, consumers observe the price set by the platform, $p_b$, and firms that produce vertically differentiated, substitute goods as a result of the innovation race, described in Section 3.4. Denote $k = 1, ..., n$ as the index for product quality, where a higher $k$ represents a higher quality.

The consumers are utility maximizing:

$$\max U(z, k) = u_k \cdot z$$

where $u_k = e^{\alpha \pi_M(k)}$ following [6]. Here $u_0 < u_1 < \ldots < u_n$, and $\pi_M(k)$ denotes the probability of an innovation being introduced in the current period. This captures the externality created by innovation, such that when innovations occur at a higher rate on a platform, consumers derive higher utility, which leads to higher profitability for those firms who successfully innovate. The functional form of $\pi_M(k)$ is described by Eq. (25) in Section 3.4.

Each consumer’s utility is defined by the utility for consuming a certain quality good weighted by the consumer’s remaining budget. Let $C_k$ be the relative utility difference between products $k$ and $k-1$, and $C_k > 1$:

$$C_k = \frac{u_k}{u_k - u_{k-1}} = \frac{e^{\alpha \pi_M(k)}}{e^{\alpha \pi_M(k)} - 1} = C.$$

Define $z_k$ as the indifference budget level, such that the consumer with budget $z_k$ is indifferent between products $k$ and $k-1$ at their respective prices. So,

$$U(z_k - p_k - p_b, k) = U(z_k - p_{k-1} - p_b, k - 1).$$

And note that if a consumer chooses to not purchase, she does not pay the platform price $p_b$; thus, her utility would be $U(z, 0) = u_0 * z$. From here, we derive:

$$z_1 = (p_1 + p_b)C_1,$$

$$z_k = p_{k-1}(1 - C_k) + p_kC_k + p_b.$$ (2)

Then, consumers with taste $z > z_k$ have the preference order $(k, p_k) \succ (k-1, p_{k-1})$.

3.2 Market structure

Let the product cost be zero, the profit, or revenue, of the $k$-th firm is:

$$R_1 = p_1(z_2 - z), \text{ if } z_1 \leq z$$

$$R_1 = p_1(z_2 - z_1), \text{ if } z_1 > z$$

$$R_k = p_k(z_{k+1} - z_k), \text{ if } 1 < k < n$$

$$R_n = p_n(\bar{z} - z_n).$$ (6)

If $n$ firms share the market and compete in price, the first-order conditions (FOCs) are as follows:

For $k = 1$,

$$z_2 - z - p_1(C_2 - 1) = 0, \text{ if } z_1 \leq z$$

$$z_2 - z_1 - p_1[(C_2 - 1) + C_1] = 0, \text{ if } z_1 > z.$$ (7)

For $k = 2, ..., n - 1$,

$$z_{k+1} - z_k - p_k[(C_{k+1} - 1) + C_k] = 0$$

(9)

For $k = n$,

$$\bar{z} - z_n - p_nC_n = 0.$$ (10)

Lemma 1. Let $\bar{z} < 4z - 3p_b$, then in any Nash equilibrium, at most two firms (of quality $n$ and $n-1$)
obtain a positive market shares.

Proof. Assume there are more than two firms in equilibrium, we can rewrite the FOCs using Eq. (1) and (2) as,

\[
z_{k+1} - 2z_k - p_k(C_{k+1} - 1) - p_{k-1}(C_k - 1) + p_b = 0
\]

\[
\bar{z} - 2z_n - p_{n-1}(C_n - 1) + p_b = 0.
\]

We can then get \(\bar{z} + p_b > 2z_n\) and \(z_{k+1} + p_b > 2z_k\), which yields \(2z_n + 2p_b > 4z_{n-1}\), and then \(\bar{z} + 3p_b > 4z_{n-1}\). By assumption, we have \(\bar{z} < 4z - 3p_b\), so \(4\bar{z} > \bar{z} + 3p_b\). Therefore, \(z_{n-1} < \bar{z}\), implying that the top two firms cover the market. \(\square\)

Let us now define,

\[
V = \frac{u_2 - u_0}{u_2 - u_1} = \frac{C_2 - 1}{C_1} + 1. \quad (11)
\]

Applying Eq. (1) and (2), we have

\[
p_1 = \frac{z_1}{C_1} - p_b \quad (12)
\]

\[
p_2 = \frac{z_2 + z_1(V - 1)}{C_2} - p_b \quad (13)
\]

Rewriting the FOCs yields, for firm 1

\[
z_2 = \bar{z} + z_1(V - 1) - p_b(C_2 - 1), \text{ if } z_1 \leq \bar{z} \quad (14)
\]

\[
z_2 = z_1(V + 1) - p_b(C_2 - 1 + C_1), \text{ if } z_1 \geq \bar{z} \quad (15)
\]

And for firm 2,

\[
\bar{z} - 2z_2 = z_1(V - 1) - p_bC_2. \quad (16)
\]

**Lemma 2.** If \(2\bar{z} - p_b(3C_2 - 2) \leq \bar{z} \leq 4\bar{z} - 3p_b\), then a unique equilibrium exists.

**Proof.** Since \(\bar{z} \leq 2\bar{z} - p_b(3C_2 - 2)\), the decreasing function (16) crosses Figure 1 above \(\bar{z} - p_b(C_2 - 1)\) at \(z_1 = 0\). Thus, two firms’ first-order conditions cross at one point. To verify that this is indeed an equilibrium, we can show that both firms’ revenue functions are concave in each firm’s own price holding the competitor’s price constant. Therefore, given \(2\bar{z} - p_b(3C_2 - 2) \leq \bar{z} \leq 4\bar{z} - 3p_b\), a unique equilibrium exists. \(\square\)

In Region I, firms’ equilibrium prices and revenues are,

\[
p_1^* = \frac{[\bar{z} - 2\bar{z} + p_b(3C_2 - 2)]}{3C_1(V - 1)} - p_b \quad (17)
\]

\[
p_2^* = \frac{1}{3C_2} [2\bar{z} - \bar{z} - p_b] \quad (18)
\]

\[
R_1^* = \frac{p_1^*}{3} [\bar{z} - 2\bar{z} + p_b] \quad (19)
\]

\[
R_2^* = \frac{1}{9C_2} [2\bar{z} - \bar{z} - p_b]^2 \quad (20)
\]

In Region II, the equilibrium results are,

\[
p_1^* = \frac{\bar{z}}{C_1} - p_b \quad (21)
\]

\[
p_2^* = \frac{1}{2C_2} [\bar{z} + z(V - 1) - p_bC_2] \quad (22)
\]

**Figure 1: Firm 1’s First Order Conditions**

Figure 1 plots firm 1’s FOCs showing different regions. Different ranges of \(V\) values will result in a certain market segmentation, in which market may not be covered. Firm 2’s FOC is a decreasing function of \(z_1\). The point where it intersects firm 1’s FOCs in figure 1 is the equilibrium. The resulting closed-form expression for two firms’ equilibrium prices and profits can be derived. And these equilibrium profits are the innovation rewards of the innovators, which we discuss in the following section.

**3.3 Equilibrium revenues**

Value of \(V\) determines the equilibrium regions, shown in the table below:

<table>
<thead>
<tr>
<th>Region</th>
<th>(V\geq\frac{1}{3\bar{z}}[\bar{z} + \bar{z} + p_b(3C_2 - 2)])</th>
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<tbody>
<tr>
<td>Region I</td>
<td>(\bar{z} + z + p_b(3C_2 - 2)) ≥ (V) ≥ (\frac{1}{3\bar{z}}[\bar{z} - \bar{z} + p_b(3C_2 - 2 + 2C_1)])</td>
</tr>
<tr>
<td>Region III</td>
<td>(\frac{1}{3\bar{z}}[\bar{z} - \bar{z} + p_b(3C_2 - 2 + 2C_1)]) ≥ (V)</td>
</tr>
</tbody>
</table>

**Lemma 2.** If \(2\bar{z} - p_b(3C_2 - 2) \leq \bar{z} \leq 4\bar{z} - 3p_b\), then a unique equilibrium exists.

**Proof.** Since \(\bar{z} \geq 2\bar{z} - p_b(3C_2 - 2)\), the decreasing function (16) crosses Figure 1 above \(\bar{z} - p_b(C_2 - 1)\) at \(z_1 = 0\). Thus, two firms’ first-order conditions cross at one point. To verify that this is indeed an equilibrium, we can show that both firms’ revenue functions are concave in each firm’s own price holding the competitor’s price constant. Therefore, given \(2\bar{z} - p_b(3C_2 - 2) \leq \bar{z} \leq 4\bar{z} - 3p_b\), a unique equilibrium exists. \(\square\)
\[ R_1^* = \frac{p_i^*}{2} [z - \bar{z}(V + 1) + p_bC_2] \quad (23) \]

\[ R_2^* = \frac{1}{4C_2} [z + \bar{z}(V - 1) - p_bC_2]^2 \quad (24) \]

In this paper, we omit the case under Region III. We first focus on analyzing the scenario when the market is covered — the feasible \( \bar{z} \) values such that the equilibrium occurs in Regions I or II.\(^4\)

### 3.4 Innovating firms

This section describes the innovation race and firms’ innovation decisions. Potential entrants make decisions in three stages: 1) Entry to the innovation race – firms choose whether to innovate by taking into account the platform price, \( f \), which is also the fixed innovation cost, and the expected value of innovating; 2) Innovation effort – firms choose the level of R&D, which affects their probability of successful innovation and hence the probability of market entry; 3) In case of market entry, firms choose their prices, which are described in the previous section. Our setup follows the framework developed by Segal and Whinston [14] with the extension of heterogeneity of innovation costs across firms.

There exist \( M \) firms that are potential entrants. These firms compete in the R&D race. They have to pay the cost of innovation, which is \( c(\phi_i) \). Assume the cost function \( c(\cdot) \) is convex. \( \phi_i \in (0,1) \) is the innovation rate of firm \( i \), and also firm \( i \)'s probability of creating a new product.

Multiple innovators may succeed in developing new products. However, only one of these innovations is sufficiently competitive to be listed in the two-sided market. This successful firm then becomes an incumbent producing the currently highest quality product in the market. We use the simultaneous entry and exit setup; thus, the lowest quality incumbent is pushed out of the market when a new firm enters. The innovation model connects to the market structure analysis at this point, as the post-innovation rents are characterized by the equilibrium market structure.

Let \( \pi(\phi^*_L) \) denote the probability of at least one firm successfully creating a new product, where \( \phi^*_L \in [0,1]^N \) describes the innovation efforts of all potential entrants. Because in each period only one of these firms is granted a patent and enters the market, the probability of actually obtaining the patent is denoted by \( \lambda(\phi, \phi_-) \), where \( \phi_- \in [0,1]^{N-1} \) is the innovation efforts of the rest of the innovators. In a symmetric equilibrium, firms with the same draw will make the same decision. Thus, we only consider whether both low- and high-cost firms choose to innovate. Both \( \pi(\phi^*_L) \) and \( \lambda(\phi, \phi_-) \) have different formulations when either all the firms innovate or only one type of firms innovate. Thus, we analyze these formulations case by case. Also note that the number of innovating firms also affects consumers’ utility through network externality.

The probability of at least one firm successfully creating a new product among \( M \) potential entrants is denoted by \( \pi_M(\phi^*_L) \), where:

\[ \pi_M(\phi^*_L) = 1 - (1 - \phi_-)^M. \quad (25) \]

Among \( M \) potential entrants, conditional on successful innovation, the probability of entering the market for any one firm is denoted by \( r_M(\phi_-) \):

\[ r_M(\phi_-) = \frac{1 - (1 - \psi_-)^N}{\psi_- \cdot N}. \quad (26) \]

The probability of obtaining a patent for this firm is \( \lambda_M(\phi, \phi_-) \):

\[ \lambda_M(\phi, \phi_-) = \phi r_M(\phi_-). \quad (27) \]

Following [14] we use the dynamic programming approach to formulate this problem and look for the stationary Markov perfect equilibria. The value functions of the innovating firms are listed below:

\[ V^0(\phi_-) = \max\{0, -f + V^E(\phi_-)\}; \quad (28) \]

\[ V^E(\phi_-) = \max_{\phi} \{\lambda(\phi, \phi_-) V_f^I \} \]

\[ + (1 - \lambda(\phi, \phi_-)) \beta V^0(\phi_-) - c(\phi)\}; \quad (29) \]

\[ V^I_i(\phi_-) = R_i + \beta \pi(\phi^*_{L}) V^I_{i-1}(\phi_-) + \beta(1 - \pi(\phi^*_L)) V^I_i(\phi_-), \quad (30) \]

\[ i = 2, \ldots, J; \]

\[ V^I_1(\phi_-) = R_1 + \beta \pi(\phi^*_L) V^0(\phi_-) + \beta(1 - \pi(\phi^*_L)) V^I_1(\phi_-). \quad (31) \]

\( V^0(\phi_-) \) is the value function of potential entrants at the start of each stage game; \( V^E(\phi_-) \) is the value function of entrants in the R&D race; and \( V^I_i(\phi_-) \) and \( V^I_1(\phi_-) \) are the value functions for incumbents producing product quality \( i \) and the lowest quality product before exiting, respectively. We show that the dynamic programming problem described by Eq. (28) through (31) satisfies the Black-
well sufficient conditions; thus, it has a unique fixed point in a bounded space.

**Lemma 3.** The dynamic programming problem characterized by Eq. (28) through (31) has a unique fixed point.

**Proof.** We need to show that the problem defined by Eq. (28) through (31) satisfies Blackwell sufficient conditions. The problem of interest can be written as follows:

\[
V^0(\phi) = \max \{0, -f + V^E(\phi)\}
\]

\[
V^E(\phi) = \max \left\{ \frac{\lambda}{1 - \beta(1 - \pi)} R_2 + \frac{\lambda \beta \pi}{(1 - \beta(1 - \pi))^2} R_1 - c(\phi) + \beta V^0(\phi) \right\}
\]

where \(\tilde{\beta} \equiv \left( \frac{\beta \pi}{1 - \beta(1 - \pi)} \right)^2 \lambda + (1 - \lambda) \beta\).

We are interested in the stationary Markov Equilibria (see Segal and Whinston (2007) for a similar model). Thus two sets of strategies are compared. In the first case, the potential entrant always enters the R&D race. In the second case, the potential entrant never enters the race. In the former case, the dynamic programming problem can be written as follows:

\[
V^0(\phi) = \max (-f + \frac{\lambda}{1 - \beta(1 - \pi)} R_2 + \frac{\lambda \beta \pi}{(1 - \beta(1 - \pi))^2} R_1 - c(\phi) + \beta V^0(\phi) )\]

(32)

We need to prove the above dynamic programming problem satisfies the Blackwell’s sufficient conditions. Let \(X = X_1 \times \ldots \times X_M\), \(X_i = [0, 1], i \in \{1, \ldots, M\}\). B(X) is a space of bounded functions \(f : X \rightarrow R\). Define the operator \(T : B(X) \rightarrow B(X)\) as follows:

\[
Tv(\phi) = \max \phi \tilde{R}(\phi) + \beta v(\phi)
\]

Where \(\tilde{R}(\phi) = -f + \frac{\lambda}{1 - \beta(1 - \pi)} R_2 + \frac{\lambda \beta \pi}{(1 - \beta(1 - \pi))^2} R_1 - c(\phi)\).

1) monotonicity.
2) Discounting. \(\forall x \in X\):

\[
T(v + a)(x) = \max _\phi \tilde{R}(\phi) + \beta (v(x) + a) = \max \tilde{R}(\phi) + \beta v(x) + \tilde{a}
\]

(33)

We also try a large set of initial guesses to check whether there may exist multiple equilibria. We then evaluate the distance between the derived innovation rate and the original guess and update the \(\phi\) to find the solutions of these equations. We also try a large set of initial guesses to check whether there may exist multiple equilibria. Our computation results are robust under different initial guesses.

**4 Computational findings**

**4.1 Parameterization**

Because of the complexity of the dynamic problem, we use computation methods to find the numerical solutions to the problem described by Eq. (28) through (31). In particular, given \(\phi\), we solve this problem using the value function iteration method and derive the policy function \(\phi(\phi)\).

We then evaluate the distance between the derived innovation rate \(\phi(\phi)\) and the original guess and update the \(\phi\) to find the solutions of these equations. We also try a large set of initial guesses to check whether there may exist multiple equilibria. Our computation results are robust under different initial guesses.

The discount rate \(\beta = 0.95\) implies the annual interest rate is approximately 5%. \(a\) in the utility function is 2.2. The income of consumers follows a uniform distribution \(U[z, z]\). The upper bound of taste shock \(z\) is 4.2, while the lower bound of the taste shock \(z\) is 1.7. The price that firms pay to the platform, \(f\), is set to 0.01. As for the functional form of innovation cost \(c(\cdot)\), we follow Aghion, et al.’s model and use quadratic form, \(c(\epsilon) = \epsilon \phi^2\) [2].

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5We use the “fsolve!” function in Matlab to solve this system of equations.
\( \epsilon = 12 \) We also assume the number of potential entrants is 10 each period. We set these numbers relatively small to reduce the computation load.

### 4.2 Results

In all the following numerical experiments, we have found values of \( p_b \) that cover both equilibrium regions. Equilibrium Region I depicts a highly competitive market, where the quality gap between the two firms is relatively narrow. Thus, the low quality firm prices below the valuation of the lowest-budget consumer. In Region II, the quality gap between the firms is wider, thus the low quality firm only prices low enough to cover the market resulting in higher equilibrium prices from both firms.

Figure 1 shows that increasing \( p_b \) lowers the cutoff of the adjacent regions. The additional premium \( p_b \) makes the low quality firm the preferred choice for consumer who would otherwise choose the high quality firm if \( p_b \) is not imposed; therefore, as \( p_b \) increases, the low quality firm more easily forgoes the price cutting strategy – price competition is mitigated and two firms are more easily engaged in Region II pricing strategy.

Referring back to Eq. (12) and (13), we can see that although \( p_b \) is charged to consumers by the platform, it appears to reduce the prices of the transacted goods paid by consumers to the firms. However, it is not clear that firms actually bear (even a part of) \( p_b \) for consumers in such equilibrium, because values of \( C_1 \), \( C_2 \), and \( V \) are endogenous in the innovation rate. In other words, if \( p_b \) affects the equilibrium innovation—which it does as discussed in the following—then values of the other parameters will also be different; hence, the values of \( p_1 \) and \( p_2 \) in equilibrium may or may not be lower as \( p_b \) increases.

We first investigate the changing price gaps of \( p_b \), \( p_1^* \), and \( p_2^* \) in the stationary Markov equilibrium, where \( p_1^* \) and \( p_2^* \) are determined such that innovating firms are optimizing their innovation rates at different levels of \( p_b \) and consumers’ purchase decisions are based on \( p_b \) and the optimal innovation rates. Figure 2 shows three price gaps from Region I to Region II (a jump is observed between the two regions around \( p_b = 0.15 \)).

In Region I, while the high quality firm indeed subsidizes part of \( p_b \), the low quality firm not only does not compensate consumers for \( p_b \), it actually increases \( p_2 \) further as well. Increasing \( p_b \) gives the low quality firm leverage by shifting demand from the high quality firm to the low quality firm. As a result, the price competition is mitigated, as the high quality firm attempts to “pay back” to its consumers while the low quality firm lifts up its price level.

**Figure 2: Price gaps**

When \( p_b \) reaches a point, two firms both experience a price surge due to change of pricing strategy in entering Region II, where quality levels are sufficiently wide such that low quality firm only price low enough to cover the market. In this region, both \( p_1^* \) and \( p_2^* \) decrease as \( p_b \) goes up, meaning the low quality firm also subsidizes consumers for \( p_b \) as the low quality firm continues to do so. In this competitive setting, increasing \( p_b \) poses a threat to the low quality firm as well, which can easily lose consumers who would decide to not purchase at all. As the graph shows, the high quality firm still subsidizes more than the low quality firm because of demand shift from the former to the latter.

In terms of firm revenues, increases in \( p_b \) benefit the low quality firm at the expense of the high quality firm in both regions. The low quality firm receives higher revenue gain in Region I compared to Region II due to the increase in both equilibrium price and demand.

**Figure 3: Low- and High-Quality Firm Revenues**
$p_b$ indirectly impacts the equilibrium innovation rate through the revenue effect shown in Figure 3. Higher $p_b$ implies that for successful entrants initial revenues as the high quality firm is reduced but later revenues as the low quality firm (when another innovator enters) will increase. As Figure 4 indicates, the overall effect of revenue changes on the innovation incentives is negative. Thus, higher fees imposed by the platform on consumers may reduce the innovation rate, because the market created by the platform transfers sufficient level of this fee onto firms, which in turn diminishes the innovation prize.

![Figure 4: Innovation Rate and Probability of Entry](image1)

**Figure 4: Innovation Rate and Probability of Entry**

When innovating firms innovate at a lower intensity due to $p_b$, the probability for introduction of an innovation is also adversely affected. This implies that the platform may experience a slower market turnover rate – successful innovators enjoy longer market incumbency at lower revenues due to $p_b$, and consumers obtain less novel products over time. Consumers’ preference is then affected by such market turn over rate. Figure 5 explores the effect of $p_b$ on consumer welfare, as a result of the impact of $p_b$ on other factors in this innovation process.

![Figure 5: Consumer Welfare and Platform Revenues](image2)

**Figure 5: Consumer Welfare and Platform Revenues**

Consumers face two opposing effects from increasing $p_b$. As discussed earlier, higher $p_b$ leads to possible price subsidies born by firms in the market, hence lower product prices. However, the resulting less rapidly innovative market (lower probability of product entry) diminishes consumers’ utility. Figure 5 shows that the positive effect dominates the negative effect in both Region I and II. Consumers benefit more from lower prices than the loss due to reduced innovation rate. However, note that there is a significant drop from Region I to II, because $p_b$ also mitigates firms’ price competition and causes the transition to Region II where equilibrium prices are much higher. Therefore, as an unexpected finding, platform’s fee, $p_b$, for consumers actually improves consumer welfare, but only within each region.

Lastly, we shift focus on the effect of $p_b$ on platform’s own revenues. We suppose platform obtains revenues from the fee $p_b$ imposed on consumers, the fixed cost $f$ charged on innovating firms, and a share of the revenues from the transactions in the market (Eq. (19) (20) (23) and (24)). We use the ratio 30/70, as in platform collects 30% and firms collect 70% of the equilibrium revenues, since that is the common split used by Amazon for Kindle books, Apple and Google for Android apps. While increasing $p_b$ clearly adds to platform’s revenues, the downside is the reduction in innovation rate as well as lower market revenues, which platform obtains 30% of. As a result, the platform’s revenues are actually decreasing in $p_b$ for both regions; however, similar to previous findings, a jump occurs at the transition from Region I to Region II, due to improved revenues in Region II.

5 Conclusion

In this paper, we characterize the network externality of a two-sided platform by connecting firms’ innovation with consumers’ utility. In effect, we study the interaction of fees imposed by the platform and the pricing of goods inside the platform market. Our framework introduces a dynamic problem that describes the innovation race of the innovating app developers and explicitly models the price competition among incumbents that produce vertically differentiated apps. Contradictory to conventional wisdom, we find that increasing consumers’ platform fee leads to higher consumer welfare, because such cost may be subsidized by firms on the platform as a result of competition. Consequently, firms’ revenues are reduced, which further impacts the equilibrium innovation rate and
platform revenues negatively. However, increasing consumers’ platform fee also dampens the platform market competition, as the low quality producer finds a leverage from the demand shift; such dynamics will eventually lead to a different equilibrium of a less competitive market, where consumers’ welfare may suffer from elevated prices. The innovation rate and revenues then benefit from such equilibrium switch.

Our work suggests that membership pricing for two sides of a platform should take into consideration its effect on the economy of the market created by the platform, especially when revenues of the transactions are split between the platform and its members. The cost of consumer platform fee or purchase of the platform hardware device restricts consumers’ budget for purchases in the market. Prices of market goods, such as Kindle-version books and smartphone or iPad apps, are then discounted driven by market competition. Such evidence is commonly observed in related industries. For example, Kindle-version books are priced at a substantially lower price than hardcopy books.

Our results also indicate that consumer platform fee also serves as a device to mitigate competition by providing the lower quality firms some leverage in competition with the higher quality firms. Intuitively, as consumer income shifts down, or overall prices shift up, consumers would resort to lower quality goods that are more affordable. Increasing consumer platform fee has a similar effect. The higher quality firms then subsidizes such costs for consumer to a higher degree than the lower quality firms (if they subsidize at all). Substantial reduction in competition intensity may eventually lead to a drastic change in firms’ pricing strategy, which results in considerably higher prices, benefits platform and firms’ revenues, but lowers consumer welfare. Such connection offers the implication that platform’s pricing of fees may indirectly determine the market structure, under which goods are transacted.

References


