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Modeling Electrically Small Structures in Layered Medium with Augmented EFIE Method

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Abstract—Electrically small structures embedded in a planarly layered medium are modeled by the augmented electric field integral equation (EFIE) method in this paper. By separating charge as extra unknown list, and enforcing the current continuity equation, an augmented EFIE (A-EFIE) can be setup. The matrix-friendly formulation of layered medium Green’s function is applied and the frequency scaling of the impedance matrix in the moment method is analyzed when the frequency tends to zero. Rank deficiency and the charge neutrality enforcement in the moment method is also discussed in detail. Numerical results show that the low frequency breakdown of electrically small structures embedded in a layered medium can be effectively remedied by this A-EFIE method.

I. INTRODUCTION

The traditional electric field integral equation (EFIE) method usually suffers from a low-frequency breakdown problem, when the structure under consideration is much smaller compared with the wavelength. Various methods have been developed to solve this problem during the last few years. The loop-tree or loop-star decomposition [1], [2] was proposed to separate the current into a solenoidal part and a non-solenoidal counterpart. Frequency normalization can then be easily applied to make the matrix system well-conditioned [3]. The Calderón preconditioner is another effective method to improve the spectrum of the EFIE operator [4]. In a later implementation, the Buffa-Christiansen basis function [5] is applied to make the preconditioner multiplicative [6]. Recently, the idea of separating current and charge to construct a stable formulation has been studied. In the current and charge integral equation (CCIE) method [7], the equation system is manipulated to be of the second kind by adding the charge as the extra unknowns. Alternatively in the separated potential integral equation (SPIE) method [8], the scalar potential other than charge is included as the unknowns, where resistive loss and dielectric loss are introduced to make the condition number bounded when the frequency is low. In the augmented electric field integral equation (A-EFIE) method [9], [10], the similar idea of separating current and charge is applied. Different from the CCIE, the current continuity condition is enforced explicitly and a proper frequency scaling is developed, to make the new matrix to be a generalized saddle point system [11].

In real engineering applications, the structures are usually embedded in a layered medium, as for the microstrip antennas and integrated circuits. In this case, the layered medium Green’s function can be applied to reduce the number of unknowns by considering the layered medium as the background. In this paper, the A-EFIE method is extended to the layered medium problems. The matrix-friendly layered medium Green’s function is first briefly reviewed, followed by the frequency scaling analysis of the impedance matrix in the moment method. The rank deficiency and the necessity of enforcing charge neutrality condition is also discussed in detail. Numerical results are presented to validate this method.

II. FORMULATION

A. Matrix-Friendly Layered Medium Green’s Function in EFIE

In the matrix-friendly formula of the layered medium Green’s function, the impedance matrix in moment method can be expressed as [12],

$$
\mathbf{Z} = i\omega\mu_m \{ \mathbf{Z}^{ss} + \mathbf{Z}^{zz} + \mathbf{Z}^{z1} + \mathbf{Z}^{z2} + \mathbf{Z}^{d} \}
$$

(1)

where

$$
[\mathbf{Z}^{ss}]_{ji} = \langle f_j(r), g_{ss}(r, r') \rangle
$$

(2)

$$
[\mathbf{Z}^{zz}]_{ji} = \langle \tilde{z} \cdot f_j(r), g_{zz}(r, r') \rangle
$$

(3)

$$
[\mathbf{Z}^{z1}]_{ji} = -\langle \tilde{z} \cdot f_j(r), g_{z1}(r, r'), \nabla' \cdot f_i(r') \rangle
$$

(4)

$$
[\mathbf{Z}^{z2}]_{ji} = -\langle \nabla \cdot f_j(r), g_{zz}(r, r'), \tilde{z} \cdot f_i(r') \rangle
$$

(5)

$$
[\mathbf{Z}^{d}]_{ji} = \langle \nabla \cdot f_j(r), g_{d}(r, r'), \nabla' \cdot f_i(r') \rangle
$$

(6)

with the Green’s function components as

$$
g_{ss}(r, r') = k_{ji}^{2TE}(r, r')
$$

(7)

$$
g_{zz}(r, r') = k_{mn}^{2TM}(r, r') - \partial_z \partial'_{z'} g^{TE}(r, r')
$$

(8)

$$
g_{z1}(r, r') = \frac{\mu_n}{\mu_m} \partial_z g^{TM}(r, r') + \partial'_{z'} g^{TE}(r, r')
$$

(9)

$$
g_{z2}(r, r') = \frac{\epsilon_m}{\epsilon_n} \partial'_z g^{TM}(r, r') + \partial'_{z'} g^{TE}(r, r')
$$

(10)

$$
g_{d}(r, r') = \frac{\partial_z \partial'_{z'}}{k_{nm}} g^{TM}(r, r') - g^{TE}(r, r')
$$

(11)
B. Frequency Scaling

For general cases, namely \( \epsilon_i \neq \epsilon_j \) and \( \mu_i \neq \mu_j \), when \( \omega \to 0 \), \( k_{iz} \to ik_{0z} \), if the layered medium is lossless, the frequency scaling of the Fresnel reflection coefficient is

\[
R_{ij} = \frac{p_j k_{iz} - p_i k_{jz}}{p_j k_{iz} + p_i k_{jz}} = \frac{p_j - p_i}{p_j + p_i} \sim O(\omega^0)
\]  

(12)

where \( p = \mu \) for TE wave and \( p = \epsilon \) for TM wave. Then the frequency scaling for other quantities are:

\[
\begin{array}{l}
\bar{R}_{ij} \sim O(\omega^0), \quad \bar{M}_m \sim O(\omega^0), \quad \bar{T}_{mn} \sim O(\omega^0), \quad \bar{F} \sim O(\omega^0)
\end{array}
\]  

(13)

where \( \bar{R}, \bar{M}, \bar{T} \) and \( \bar{F} \) can be found in [13]. Finally, the frequency scaling for the matrix element in (2)-(6) is

\[
Z^{ss} \sim O(\omega^0), \quad \bar{Z}^{zz} \sim O(\omega^0), \quad \bar{Z}^{zi} \sim O(\omega^0)
\]  

\[
\bar{Z}^{z \phi} \sim O(\omega^{-2})
\]  

(14)

(15)

The matrix can be separated into two parts, according to the scaling properties,

\[
\bar{Z} = \bar{Z}^A + \bar{Z}^S = ik_0 \eta_0 \bar{A} + \frac{\eta_0}{ik_0} \bar{S}
\]  

(15)

where

\[
\bar{A} = \mu_{nr} [Z^{ss} + Z^{z^2} + Z^{zi} + Z^{z \phi}] \sim O(\omega^0)
\]  

\[
\bar{S} = -\frac{k_0^2 \epsilon_{nr} Z^{\phi}}{\epsilon_0} \sim O(\omega^0)
\]  

(16)

(17)

C. Augmented EFIE (A-EFIE)

In the above derivation, the RWG basis function is applied [14], which is defined on two adjacent triangular patch pairs,

\[
f_i(r) = \begin{cases} 
\frac{1}{2A_t}, & r \in T_i^+ \\
-\frac{1}{2A_t}, & r \in T_i^- \\
0, & \text{otherwise}
\end{cases}
\]  

(18)

Note that the function has been normalized by the edge length. The surface divergence of the RWG basis can be further expressed as,

\[
\nabla_s \cdot f_i(r) = \begin{cases} 
\frac{1}{A_t}, & r \in T_i^+ \\
-\frac{1}{A_t}, & r \in T_i^- \\
0, & \text{otherwise}
\end{cases}
\]  

(19)

When \( \omega \to 0 \), the \( \bar{Z}^A \) gradually loses its accuracy due to the finite precision of machine, making the system singular. To remedy it, a pulse basis function \( p(r) \) defined on each patch is introduced

\[
p_i(r) = \begin{cases} 
\frac{1}{A_t}, & r \in T_i \\
0, & \text{otherwise}
\end{cases}
\]  

(20)

If we define the following matrix,

\[
[p]_{ij} = -\frac{k_0^2}{\epsilon_{nr}} (p_j(r), g_\phi(r, r'), p_i(r'))
\]  

(21)

the \( \bar{S} \) in (17) can be expressed in terms of \( \bar{P} \),

\[
\bar{S} = \bar{D}^T \cdot \bar{P} \cdot \bar{D}
\]  

(22)

where the incidence matrix \( \bar{D} \) relates the domain of the RWG basis and the patch basis,

\[
[\bar{D}]_{ji} = \begin{cases} 
1, & \text{Patch } j \text{ is the positive part of RWG } i \\
-1, & \text{Patch } j \text{ is the negative part of RWG } i \\
0, & \text{otherwise}
\end{cases}
\]  

(23)

Due to the current continuity equation, we have

\[
\bar{D} \cdot J = i k_0 c_0 \rho
\]  

(24)

where \( c_0 \) is the light speed in vacuum and \( \rho \) is the charge unknowns. Finally, the A-EFIE can be setup,

\[
\begin{bmatrix} \bar{A} & \bar{D}^T \cdot \bar{P} \\ \bar{D} & -i k_0 \bar{I} \end{bmatrix}, \begin{bmatrix} ik_0 J \\ c_0 \rho \end{bmatrix} = \begin{bmatrix} \eta_0^{-1} V \\ 0 \end{bmatrix}
\]  

(25)

This equation is the generalized saddle point system with the lower right block nearly equals to zero when \( \omega \to 0 \) and various methods can be applied to solve this problem efficiently [11].

Dielectric loss and conductor loss can be introduced to alleviate the low frequency breakdown in free space [8]. For a structure embedded in a layered medium, if we introduce dielectric loss to each layer, since the equivalent permittivity is

\[
\bar{\varepsilon}_r = \varepsilon_r + \frac{i \sigma}{\omega \varepsilon_0}
\]  

(26)

the frequency scaling of the \( \bar{Z}_0 \) becomes

\[
\bar{Z}^\phi \sim O(\omega^{-1})
\]  

(27)

so the scalar potential matrix becomes

\[
\bar{S} \sim O(\omega^1)
\]  

(28)

The A-EFIE can then take the alternative form

\[
\begin{bmatrix} \bar{A} (ik_0)^{-1} \bar{D}^T \cdot \bar{P} \\ \bar{D} \end{bmatrix}, \begin{bmatrix} J \\ c_0 \rho \end{bmatrix} = \begin{bmatrix} (ik_0 \eta_0)^{-1} V \\ 0 \end{bmatrix}
\]  

(29)

D. Charge Neutrality Issue

In general cases, rank deficiency exists in the A-EFIE matrix, due to the definition of the incidence matrix \( \bar{D} \). It is shown that \( k_0^2 \) is an eigenvalue of the A-EFIE and it tends to zero when the frequency goes to DC [10]. The deflation method [15] can be applied to remove the smallest eigenvalue, for example, in the CCIE formula [7]. Motivated by the basis rearrangement in the loop-tree decomposition [3], we can also apply the charge neutrality enforcement to remedy this problem. This is driven by the physical observation of the problem, and can be easily extended to different layered medium problems. If the layered medium is backed by a conducting ground plane, the ground will act as a “charge bath” and absorbs the extra charge of the structure. Situations should be distinguished whether there is a via connected to the ground in some parts of the structure.

If the structure is not connected to the “charge bath”, the total charge is always neutral. This condition shall be enforced at low frequencies to make the system with full rank. The
Fig. 1. The geometrical structure of the loop inductor embedded in a seven-layer medium, unit: mm. The central layer is a magnetic material.

Fig. 2. The condition number versus frequency for the rectangular loop. The condition number is unbounded when decreasing the frequency. Charge neutrality enforcement (CNE) makes the condition number constant.

forward and backward transform matrices can be introduced to fulfill the charge neutrality enforcement [10],

$$\rho_r = \bar{F} \cdot \rho, \quad \rho = \bar{B} \cdot \rho_r$$  \hspace{1cm} (30)

where $\rho_r$ is the reduced charge unknowns and the $\bar{I}$ is the reduced identity matrix. The final A-EFIE system becomes:

$$\begin{bmatrix} A & D^T \cdot P \cdot B \\ F \cdot D & k_0^2 \bar{I} \end{bmatrix} \begin{bmatrix} ik_0 J \\ c_0 \rho_r \end{bmatrix} = \begin{bmatrix} \eta_0^{-1} V \\ 0 \end{bmatrix}$$  \hspace{1cm} (31)

A rectangular loop embedded in a seven-layer medium is shown in Fig. 1, with its layer parameters specified in the figure. The condition numbers versus frequency are demonstrated in Fig. 2. We can see that without charge neutrality enforcement, the condition number is unbounded when decreasing the frequency; it increases in the order of $1/k_0^2$ because of the right lower block. The eigenvalue distribution of the A-EFIE matrix at $f = 1\, \text{Hz}$ is shown in Fig. 3. After the charge neutrality enforcement, the smallest eigenvalue has been removed away from the origin. We also observe that when frequency increases, however, the lower-right block is a identity matrix scaled by $k_0^2$, thus the lower block is no longer singular and such enforcement is no longer necessary.

If the structure is connected to the ground plane, the charge neutrality condition cannot be guaranteed since the “charge bath” absorbs the extra charge. The incidence matrix $\bar{D}$ is no longer singular. In this situation, no special treatment is needed since the A-EFIE system is with full rank. The forward and backward transform matrices become the identity matrix.

$$\bar{F} = \bar{B} = \bar{I}$$  \hspace{1cm} (32)

For a structure with $s$ independent surfaces, each with $p_k$ triangular patches, $i_k$ inner edges and $g_k$ ground edges, $k = 1, 2, ..., s$, if there are $m$ surfaces connected to the ground plane, the total number of unknowns is

$$N = \sum_{k=1}^{s} (i_k + g_k + p_k) - m$$  \hspace{1cm} (33)

One should note that though the number of unknowns in A-EFIE increases much compared to the loop-tree decomposition, where the number of unknowns is the same as the number of RWG basis, the memory requirement increases marginally since all transformation matrices such as $\bar{D}$, $\bar{F}$ and $\bar{B}$ are all sparse and consume marginal memory when an iterative solver is applied.

### III. Numerical Results

Half loops shown in Fig. 4 are embedded in a six-layer medium, which has a conducting ground plane as the bottom layer. The layer parameters, dimension and position of this structure are shown in Fig. 5 and Fig. 6. A delta gap excitation is applied at the central arms shown in Fig. 4. The input reactance is calculated and shown in Fig. 7, compared with the one from traditional EFIE. We can observe that the EFIE and A-EFIE agree well when at mid-frequencies. As the frequency decreases, the EFIE quickly breaks down while the A-EFIE is always stable down to 1Hz.

### IV. Conclusion

Electrically small structures embedded in a planarly layered medium are modeled by the augmented EFIE (A-EFIE) method in this paper. The frequency scaling of the matrix system from the matrix-friendly formula of layered medium Green’s function is analyzed, for both lossless and lossy
Fig. 4. The 3D plot of the geometrical model. A delta gap excitation is applied. The dimension will be scaled by a factor of $10^{-3}$ in the following computation.

Fig. 5. Side view of the half loops embedded in a six-layer medium (including the ground plane), $yoz$ plane. The central layer is a magnetic material. Unit: mm.

cases, and an A-EFIE for layered medium Green’s function is proposed. Charge neutrality issue is also discussed for different cases. Numerical results demonstrate the effectiveness of the A-EFIE in dealing with layered medium problems.

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Fig. 6. Side view of the half loops embedded in a six-layer medium (including the ground plane), $zox$ plane. The central layer is a magnetic material. Unit: mm.

Fig. 7. The input reactance versus frequency. The EFIE breaks down when frequency decreases, while the A-EFIE is stable.

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