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A Novel Implementation of Discrete Complex Image Method for Layered Medium Green’s Function

Yong-Pin P. Chen, Student Member, IEEE, Weng Cho Chew, Fellow, IEEE, and Lijun Jiang, Member, IEEE

Abstract—A novel implementation of discrete complex image method (DCIM) based on the Sommerfeld branch cut is proposed to accurately capture the far-field behavior of the layered medium Green’s function as a complement to the traditional DCIM. By contour deformation, the Green’s function can be naturally decomposed into branch-cut integration (radiation modes) and pole contributions (guided modes). For branch-cut integration, matrix pencil method is applied, and the alternative Sommerfeld identity in terms of \( k_\| \) integration is utilized to get a closed-form solution. The guided modes are accounted for with a pole-searching algorithm. Both one-branch-cut and two-branch-cut cases are studied. Several numerical results are presented to validate this method.

Index Terms—Branch cut, discrete complex image method (DCIM), Green’s function, layered medium, pole.

I. INTRODUCTION

T
HE LAYERED medium Green’s function [1] plays an important role in the integral equation formulation for a broad class of applications, such as modeling of printed antennas and circuits, object detection, and remote sensing. Since the Green’s function consists of an infinite oscillatory integral, which typically has no closed-form solution, many studies have been carried out to expedite the calculation. Function approximation in the spectral domain is one of the most popular methods. In this method, the integration kernel is first approximated by certain “simple functions,” and the integral is then evaluated in a closed form by applying relevant integration identities. Though lots of function approximation techniques are available from a numerical analysis point of view, those candidates with closed-form identities of the infinite integrals in our context can finally be utilized. This leads to the following methods: the complex discrete complex image method (DCIM) [2] (based on the complex exponential functions), the rational function fitting technique (RFFM) [3] (based on the rational fraction functions), or their combination [4].

The popular DCIM has unpredictable errors when the interaction is in the far-field region \( \rho \gg 0 \). The original sampling path cannot effectively capture certain singularities, which correspond to the guided mode or surface waves and lateral waves physically. Several efforts have been made to remedy this problem. A two-level approximation [5] was proposed to separate the sharp-transition region from the smooth-to-varying region, with higher sampling rate in the former part to capture the singularities. In [6], the surface-wave poles are extracted explicitly for a general multilayer medium before applying the DCIM, which is also suggested in [2]. Other attempts are made to deform the sampling path to carry more pole singularities. For instance, in [7], a direct DCIM was developed to push the sampling path closer to the poles and rely on the matrix pencil method [8] itself to take care of the singularities. Meanwhile, spatial error control is another big issue in DCIM, and recent progress can be found in [9].

Though the pole singularities can be captured successfully by the above methods, the branch-point singularities are still not considered. These singularities contribute to the lateral wave when the source and observation points are at the interface of the layers [10], [15]. Recently, a three-level DCIM [11] was proposed to bring the sampling path closer to the branch point to capture more information of this singularity. In the RFFM, the continuous spectrum in the far field is also considered based on a vertical path and asymptotic analysis [12]. In this letter, we propose an alternative implementation of the DCIM to capture these singularities. Other than introducing extra segments of the sampling path to approach the singularities, we simply deform the sampling path to the Sommerfeld branch cut (SBC) [10], [15] when \( \rho \) is relatively large. The matrix pencil method is applied to approximate the function along the SBC, which can be mapped into the real axis in the \( k_\| \) plane. The pole contributions are accounted for by applying a robust pole-searching algorithm [13]. A microstrip structure with one branch cut and a general layered medium with two branch cuts are both discussed. Numerical results are demonstrated to validate this new implementation.

II. FORMULATION

The impulse response observed at \( \mathbf{r} \) in layer \( n \) of a dipole located at \( \mathbf{r}' \) in layer \( m \) can be represented by the following dyadic Green’s function [10], [15]:

\[
\mathbf{G}(\mathbf{r}, \mathbf{r}') = \left( \nabla \times \hat{\mathbf{z}} \right) \left( \nabla' \times \hat{\mathbf{z}}' \right) g_{TE}^{\mathbf{TM}}(\mathbf{r}, \mathbf{r}') + \frac{1}{k_{\text{SBC}}^2} \left( \nabla \times \hat{\mathbf{z}} \right) \left( \nabla' \times \hat{\mathbf{z}}' \right) g_{TM}^{\mathbf{TM}}(\mathbf{r}, \mathbf{r}')
\]

(1)

where \( k_{\text{SBC}}^2 = \omega^2 \varepsilon_\| \mu_\| \). The \( g_{TE}^{\mathbf{TM}}(\mathbf{r}, \mathbf{r}') \) is expressed as a Sommerfeld integral

\[
g(\mathbf{r}, \mathbf{r}') = \frac{i}{8\pi} \int_{-\infty}^{\infty} \frac{dk_p}{k_m k_p} \mathcal{H}_0^{(1)}(k_p \rho) F(k_p, \mathbf{z}', \mathbf{z}')
\]

(2)

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where $F(k_{\rho \alpha}, z, z')$ is the propagation factor \[ 10, \ 15 \], $k_{mn} = \sqrt{k_m^2 - k_p^2}$, and $H_0^{(1)}(k_{\rho \rho})$ is the first-kind Hankel function of order 0 (we assume that the time convention is $e^{-i\omega t}$). The dyadic Green’s function can be cast into several scalar Green’s functions in the electric field integral equation (EFIE) formulation \[ 14 \]. Different from the mixed-potential expression \[ 1 \], only the zeroth-order Hankel function is involved in this approach, which makes the DCIM implementation straightforward. Only the following two Green’s functions are taken as illustrative examples here, and the complete information can be found in \[ 14 \]:

\[
g_{ao}(r, r') = k_p^2 g_{TE}(r, r') \tag{3}
\]

\[
g_{o}(r, r') = \frac{\partial_{z'} g_{TM}(r, r')}{k_m^2} - g_{TM}(r, r') \tag{4}
\]

where the partial derivative with respect to $z$ and $z'$ can be easily implemented in the spectral domain, namely, $\partial_{z'} = \pm ik_{mn}$ and $\partial_{z} = \pm ik_{mn}$ where the signs are determined by the relative positions of the source and observation points.

**A. Traditional DCIM**

The Green’s function can be expressed as an infinite integral of the following type:

\[
g(\rho) = \frac{i}{8\pi} \int_{-\infty}^{+\infty} dk_p \frac{k_p}{k_z} H_0^{(1)}(k_{\rho \rho}) \tilde{g}(k_{\rho}). \tag{5}
\]

If the integration kernel can be approximated by a series of complex exponentials

\[
\tilde{g}(k_{\rho}) = \sum_{i=1}^{M} a_i e^{ik_{z}b_i}, \tag{6}
\]

by applying the Sommerfeld identity \[ 10, \ 15 \]

\[
e^{-ik_{\rho}r} = \frac{i}{2} \int_{-\infty}^{+\infty} dk_p \frac{k_p}{k_z} H_0^{(1)}(k_{\rho \rho}) e^{ik_{z}z}, \quad r = \sqrt{\rho^2 + z^2} \tag{7}
\]

the infinite integral can be calculated in a closed form

\[
g(\rho) = \sum_{i=1}^{M} a_i \frac{e^{ik_{\rho}r_i}}{4\pi r_i}, \quad r_i = \sqrt{\rho^2 + b_i^2}. \tag{8}
\]

The complex exponential series can be obtained by, for example, the matrix pencil method \[ 8 \], which approximates a function with real argument by

\[
y(t) = \sum_{i=1}^{M} R_i e^{S_i t}. \tag{9}
\]

The mapping of the real variable $t$ to $k_{z}$ is given by \[ 5 \]

\[
k_{z} = k \left[ t + \left( 1 - \frac{t}{T_0} \right) \right], \quad 0 \leq t \leq T_0 \tag{10}
\]

where $a_i$ and $b_i$ in \[ 8 \] can be obtained by

\[
b_i = \frac{i S_i T_0}{k(1 - iT_0)} \quad a_i = R_i e^{-i b_i}. \tag{11}
\]

The far-field prediction of the traditional DCIM is poor, and various remedies have been proposed \[ 5 \]–\[ 7 \], \[ 11 \].

**B. DCIM Based on the Sommerfeld Branch Cut**

When the transverse distance is large, we deform the integration path to the Sommerfeld branch cut.

1) One-Branch-Cut Case: If the layered medium is backed by a perfect electric conductor (PEC) ground plane, there is only one branch cut associated with the top layer \[ 10, \ 15 \]. A typical microstrip structure falls into this case, as shown in Fig. 1. Due to the Cauchy’s theorem and Jordan’s lemma, the integral defined along the Sommerfeld integration path (SIP) is equivalent to the path integral along the Sommerfeld branch cut (SBC) and some pole contributions, as shown in Fig. 2.

Based on the deformed path, the Green’s function can be expressed as a superposition of the following two terms:

\[
g = g_{\text{branch}} + g_{\text{pole}} \tag{12}
\]

where

\[
g_{\text{branch}} = \frac{i}{8\pi} \int_{\text{SBC}} dk_p \frac{k_p}{k_{NZ}} H_0^{(1)}(k_{\rho \rho}) \tilde{g}(k_{\rho}) \tag{13}
\]

\[
g_{\text{pole}} = -\frac{1}{4} \sum_{q} k_{p\rho q} H_0^{(1)}(k_{p\rho q}) \text{Res}[\tilde{g}(k_{p\rho q})] \tag{14}
\]

where $q$ is the number of poles and $\text{Res}[\tilde{g}(k_{p\rho q})]$ is the residue of the kernel. The locations of the poles and relevant residues can be obtained by a robust pole-searching algorithm \[ 13 \]. Pole contributions are represented by the Hankel function, which has the following asymptotic behavior:

\[
H_0^{(1)}(k_{\rho \rho}) \sim \sqrt{\frac{2}{\pi k_{\rho \rho}} e^{ik_{\rho \rho} - \frac{k_{\rho \rho}^2}{2}}} \quad (k_{\rho \rho} \rightarrow \infty). \tag{15}
\]
If $k_p$ is real, we have

$$g_{\text{pole}} \sim \sqrt{1/\rho}$$  \hspace{1cm} (16)

The $g_{\text{branch}}$ represents the radiation modes from the branch cut integration, which can be obtained in closed form by the new DCIM. By transforming the variable from $k_p$ to $k_{NZ}$, (13) becomes

$$g_{\text{branch}} = \frac{i}{8\pi} \int_{-\infty}^{+\infty} dk_{NZ} H_0^{(1)}(k_{NZ}k_p)\tilde{g}(k_p)$$ \hspace{1cm} (17)

with

$$dk_{NZ} = \frac{k_p}{k_{NZ}} dk_p.$$ \hspace{1cm} (18)

In order to use (9) to approximate the kernel of (17), we can let

$$k_{NZ} = t - \frac{T_0}{2}, \quad 0 \leq t \leq T_0.$$ \hspace{1cm} (19)

Then, $a_t$ and $b_t$ can be obtained similarly from $S_t$ and $R_t$,

$$b_t = \frac{S_t}{i} a_t = R_t e^{i T_0/2}.$$ \hspace{1cm} (20)

Once it is approximated by complex exponentials, we can apply the alternative Sommerfeld identity in terms of $k_{NZ}$ integration [10], [15] to get a closed-form solution of $g_{\text{branch}}$

$$\frac{e^{ikr}}{r} = \frac{i}{2} \int_{-\infty}^{+\infty} dk_{NZ} H_0^{(1)}(k_{NZ}k_p)e^{ik_{NZ}z}, \quad r = \sqrt{x^2 + z^2}.$$ \hspace{1cm} (21)

The radiation modes include spatial wave and the lateral wave, which have the following asymptotic behavior, respectively:

$$g_{\text{spatial}} \sim 1/\rho$$ \hspace{1cm} (22)

$$g_{\text{lateral}} \sim 1/\rho^2.$$ \hspace{1cm} (23)

From (16), (22), and (23), we can see that usually the surface wave dominates in the far field. However, at the interface, the spatial waves of the primary term and the secondary term cancel each other and the lateral wave can be observed, if there are no pole contribution for certain cases. This cancellation in DCIM was first analyzed in detail in [11].

2) Two-Branch-Cut Case: For a general layered medium, there are two branch cuts associated with the top and bottom layers, where radiation modes can be supported. In this case, the path and possible poles are shown in Fig. 3.

In this case, (5) becomes

$$g = g_{\text{branch},N} + g_{\text{branch},1} + g_{\text{pole}}$$ \hspace{1cm} (24)

where $g_{\text{branch},N}$ and $g_{\text{pole}}$ are similar to those in (14) and (17), while $g_{\text{branch},1}$ has the form of

$$g_{\text{branch},1} = \frac{i}{8\pi} \int_{-\infty}^{+\infty} dk_{NZ} H_0^{(1)}(k_{NZ}k_p)\tilde{g}(k_p).$$ \hspace{1cm} (25)

III. NUMERICAL RESULTS

Several numerical results are presented in this section. The microstrip shown in Fig. 1 is first studied. The working frequency is set to be $f = 3$ GHz, and the source point and observation point are at the interface between the air and dielectric substrate. We apply the traditional DCIM for small $\rho$ and switch it to the new implementation when $\rho$ is large. The transition region can be set in $10^1 < k_0\rho < 10^4$. In the following examples, we set the transition point at $k_0\rho = 10^2$. For this microstrip problem, only one real TM pole is found. The $g_{\text{sn}}$ and $g_{\rho}$ are calculated in Figs. 4 and 5; both agree well with those from numerical integration. In Fig. 4, since there is no TE
pole, we can observe that the asymptotic behavior of $g_{\hat{r}}$ is $1/p^2$, which represents the lateral wave. It agrees with the results by the three-level DCIM in [11], except for a constant due to the definition of the Green’s function. It is also reported in [11] that the popular two-level DCIM cannot correctly capture the branch-point contribution in this case. In Fig. 5, since $g_{\hat{r}}$ contains both TE and TM waves, the pole contribution dominates in the far field, which is of $1/\sqrt{p}$. To validate the two-branch-cut case, a three-layer model with lossy material shown in Fig. 6 is studied. The working frequency is $f = 1.5$ GHz. Both $g_{\hat{r}}$ and $g_{\hat{b}}$ are calculated and compared to the numerical integration results, as are shown in Figs. 7 and 8. Again, good agreement can be observed. In this case, the poles are general complex numbers away from the real axis, so their contributions decay quickly when $p$ is large. The lateral wave dominates in the far field with the asymptotic behavior of $1/p^2$, as shown in Figs. 7 and 8.

IV. Conclusion

A novel implementation of the discrete complex image method based on Sommerfeld branch cut is proposed to improve the far-field prediction of the layered medium Green’s function. By contour deformation, the Green’s function can be naturally decomposed into the radiation modes and guided modes. The guided modes can be obtained by a robust pole-searching algorithm, and the radiation modes can be calculated in a closed form so that the evaluation can be made efficient compared to the direct numerical integration. For small $p$ interaction, we simply switch back to the traditional DCIM to capture the near field. One should note that in this new implementation, when $p$ becomes small, the length of sampling path in spectral domain increases, and it becomes harder to approximate the kernel. Efforts can be made to improve this DCIM in the near field, such as extracting the asymptotic behavior analytically. At the same time, for cases when poles are very close to the branch cut, the accuracy of function approximation may be affected and more careful treatment of the poles is necessary. Such efforts shall also be carried out in the future to improve this DCIM.

REFERENCES