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TECHNICAL NOTE

Relating the maximum radial stress on pile shaft to pile base resistance

J. YANG* and F. MU†

An approximate analytic relationship is developed between the maximum radial stress on the shaft of a displacement pile in sand and the base resistance of the pile. Using the cavity expansion analogy, together with a confined failure mechanism, the ratio between the two quantities, defined as a factor \( S_t \), is established as a function of the friction angle, shear stiffness, compressibility and mean effective stress of the sand near the pile tip. It is shown that, given otherwise identical input parameters, the value of \( S_t \) will decrease with increasing friction angle, and with decreasing mean stress level. It also tends to decrease with an increase in relative density. It is predicted that \( S_t \) has typical values between 0·03 and 0·05, in broad agreement with the range of empirically derived values in the literature. The relationship also predicts that \( S_t \) may take much higher values (≈ 0·1) for piles installed in dense sand or in highly compressible sand. Because of the analytical nature, the established relationship provides useful insights into the mechanisms involved and important implications for design practice.

KEYWORDS: bearing capacity; compressibility; piles; sands; stiffness

INTRODUCTION

The shaft resistance of displacement piles in sand has been an area of great uncertainty, and thus of considerable interest, in foundation design. Recent experiments with instrumented model piles in the field (Lehane et al., 1993; Chow, 1997), through measurement of radial effective stresses acting on the pile shaft, have significantly improved understanding of shaft friction characteristics. This has allowed the development of new design approaches with increased rationality (Randolph et al., 1994; Jardine et al., 2005). These new approaches, while presented in different forms, share two important considerations (Fig. 1): (a) a maximum shaft friction, associated with a maximum radial effective stress, exists in the vicinity of the pile tip; and (b) a degradation of the maximum shaft friction or the maximum radial effective stress will occur as the pile tip advances further. The physical basis for friction degradation has been revealed by the aforementioned model tests in the field, and later by model tests on the centrifuge (Klotz & Coop, 2001; White & Lehane, 2004). With respect to the maximum shaft friction, however, the factors governing its characteristics remain unclear. This may be due to the complexity of pile–soil interactions in the highly stressed zone near the tip, and to the lack of reliable data in this zone.

In light of the work of Vesic (1970) and Fleming et al. (1992), Randolph et al. (1994) made a good proposal relating the maximum shaft friction, \( \tau_{max} \), to the pile base resistance, \( q_b \):

\[
\frac{\tau_{max}}{q_b} = S_t \tan \delta
\]

(1)

where \( \delta \) is the interface friction angle between the pile and the soil, and \( S_t \) is the ratio between the maximum radial effective stress \( \sigma_{r,\text{max}} \) and the base resistance (Fig. 1). This can be shown by assuming that the Coulomb failure criterion applies:

\[
\sigma_{r,\text{max}} = S_t q_b
\]

(2)

Fleming et al. (1992) suggested a constant value 0·02 for \( S_t \). Later, Randolph et al. (1994) proposed an exponential expression relating \( S_t \) to the friction angle of the sand near the tip, \( \phi \), as:

\[
S_t = a \exp (-bt \tan \phi)
\]

(3)

where \( a \) and \( b \) are two parameters requiring back-analysis of pile test results. They suggested that \( a = 2 \) and \( b = 7 \), and predicted that \( S_t \) values are between 0·02 and 0·05 for a range of friction angles (27 – 33°).
Because of the empirical nature, the two parameters in equation (3) do not bear physical meanings, and their values depend on the database used as well as on the interpretations. Keeping in mind the physical process involved in pile installation, a reasonable postulation made here is that $S_t$ should be closely linked with the soil properties near the pile tip, such as shearing resistance, stiffness, relative density and confining stress level. In this respect, an expression for $S_t$ that is able to account for these key factors in a rational manner is much preferred, since it may provide insights into the problem, and lead to improved understanding. This is the motivation of the present study.

MODELLING

The spherical cavity expansion analogy has been well accepted for analysis of piles and penetrometers (e.g. Vesic, 1972; Yu & Houlsby, 1991; Yasufuku & Hyde, 1995). A comparison of various failure patterns (Yang et al., 2005) suggests that the confined local failure mechanism, shown in Fig. 2, can provide a fairly reasonable prediction of the end-bearing capacity of displacement piles in sand. It is assumed that the limit pressure acts on the spherical surface $AC$, and that $ACF$ forms part of the wedge under the pile, with the angle $\psi$ equal to $(\pi/4 + \phi/2)$.

At the limit state the cavity has a radius $R_u$ and the plastic zone extends to a radius $R_p$, beyond which the soil mass remains in a state of elastic equilibrium. By combining the equilibrium condition, equation (4), with the Coulomb yield criterion, equation (5), the following is obtained

$$\frac{\partial \sigma_r'}{\partial r} + 2 \frac{\sigma_r' - \sigma_\phi'}{r} = 0$$

$$\sigma_r'(1 - \sin \phi) = (1 + \sin \phi)\sigma_\phi'$$

where $\sigma_r'$ and $\sigma_\phi'$ are the radial and circumferential stress components. The radial stress in the plastic zone, $\sigma_{r,p}$, can be derived as

$$\sigma_{r,p} = \frac{p_u}{r} \left( \frac{R_u}{r} \right)^4 \sin \phi / (1 + \sin \phi)$$

where $r$ is a radial distance varying from $R_u$ to $R_p$.

Instrumented pile tests have indicated that the maximum radial stress occurs in a zone close to the pile tip. Jardine et al. (2005) assume that the maximum radial stress is at the position $4D$ ($D$ is pile diameter) above the pile tip, whereas Lehane et al. (2005) assume that it is at $2D$ above the pile tip. Given this uncertainty, and considering that the sand close to the pile tip is at failure, a reasonable assumption...
made here is that the maximum radial stress acts at some distance from the pile tip in the plastic zone (point E shown in Fig. 2). The distance (CE) is taken as \( \lambda D \), where \( \lambda \) is a proportional factor to be discussed later. Without involving the complicated conversion from spherical to cylindrical stress components, the maximum radial stress on the pile is approximately estimated here as

\[
\sigma'_{\text{r,max}} = \sigma_{\text{tp}} \cos \theta \tag{7}
\]

For most practical cases of interest the above expression provides a reasonable level of accuracy: the difference is within about 5% compared with the more complicated expression. Making reference to the triangle OCE, and denoting the angle formed by CO and CE as \( \beta \), one has

\[
\cos \theta = \sin \beta \frac{R_u}{OE} \tag{8}
\]

where OE, representing the radial distance \( r \), can be determined as

\[
OE = r = \sqrt{(\lambda D)^2 + R_u^2 - 2\lambda DR_u \cos \beta} \tag{9}
\]

Based on equations (6)–(9), and noting that \( \beta = (\pi/2 + \phi) \), the maximum radial stress can be established in the form

\[
\sigma'_{\text{r,max}} = \frac{p_u \cos \phi}{\sqrt{(\lambda D)^2 + R_u^2 - 2\lambda DR_u \cos[\phi + (\pi/2)]}} \left[ 4 \sin \phi/(1 + \sin \phi) \right]^{1/2} \tag{10}
\]

Now, introducing the relationship between cavity pressure \( p_u \) and pile base resistance \( q_b \) as (Yasufuku & Hyde, 1995)

\[
q_b = \frac{p_u}{1 - \sin \phi} \tag{11}
\]

and noting that the radius of the cavity is given as (Yang, 2006)

\[
R_u = D \cos \phi \tag{12}
\]

equation (10) can be further written as

\[
\sigma'_{\text{r,max}} = q_b (1 - \sin \phi) \cos \phi \times \left( 4\lambda^2 \cos^2 \phi + 2\lambda \sin 2\phi + 1 \right)^{-2 \sin \phi/(1 + \sin \phi) - 1/2} \tag{13}
\]

It then follows that

\[
S_i = \frac{\sigma'_{\text{r,max}}}{q_b} = (1 - \sin \phi) \cos \phi \times \left( 4\lambda^2 \cos^2 \phi + 2\lambda \sin 2\phi + 1 \right)^{-2 \sin \phi/(1 + \sin \phi) - 1/2} \tag{14}
\]

With equation (14), a preliminary evaluation of \( S_i \) can be made, as shown in Table 1 for various combinations of \( \phi \) and \( \lambda \). Given a range of friction angles (29°–35°), the value of \( S_i \) varies from 0.06 to 0.04 for \( \lambda = 1 \) and from 0.02 to 0.01 for \( \lambda = 2 \). It is striking that the predicted values are in broad agreement with the range of empirically derived values in the literature. Having noted the influence of the parameter \( \lambda \) and the existing uncertainty with the location of the maximum radial stress, a rational consideration taken here is to average \( S_i \) over the plastic zone (i.e. from point C to point J in Fig. 2) such that

\[
S_i = \frac{1}{I_{\text{FU}}} \int_{0}^{I_{\text{FU}}} S d(\lambda D) \tag{15}
\]

where \( I_{\text{FU}} = FM \) denotes the upper limit of the influence zone (Yang, 2006)

\[
I_{\text{FU}} = \frac{D}{2} \left( \frac{\zeta^2}{\cos^2 \phi} - 1 - \tan \phi \right) \tag{16}
\]

in which

\[
\zeta = \frac{R_b}{R_u} = \sqrt[3]{\frac{I_i}{1 + I_i \Delta}} \tag{17a}
\]

and

\[
I_i = \frac{G}{\rho_0 \tan \phi} \tag{17b}
\]

Here \( I_i \) is known as the rigidity index, \( \Delta \) is the average volumetric strain in the plastic zone, \( G \) is the shear modulus, and \( \rho_0 \) is the mean effective stress at the pile tip. With equations (14)–(16), the factor \( S_i \) is finally given as (with the upper bar removed for convenience)

\[
S_i = \frac{1}{\chi} \int_{0}^{\chi} \frac{d(\lambda D)}{d(\lambda D)} \left( 4\lambda^2 \cos^2 \phi + 2\lambda \sin 2\phi + 1 \right)^{-2 \sin \phi/(1 + \sin \phi) - 1/2} \tag{18}
\]

where \( \chi = I_{\text{FU}}/D \) is a dimensionless parameter.

The expression established above makes it possible to investigate how \( S_i \) varies with key soil properties. Such an investigation is of considerable interest, in that it can help identify the governing factors for the relationship between the maximum radial stress and the pile base resistance, and thereby provide useful design implications, as will be discussed in the next section.

### PREDICTION AND DISCUSSION

Figure 3 presents calculated \( S_i \) values as a function of the friction angle for piles in medium dense sand (\( D_3 = 50\% \))

<table>
<thead>
<tr>
<th>( \lambda )</th>
<th>( \phi )</th>
<th>25°</th>
<th>27°</th>
<th>29°</th>
<th>31°</th>
<th>33°</th>
<th>35°</th>
<th>37°</th>
<th>39°</th>
<th>41°</th>
<th>43°</th>
<th>45°</th>
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<tr>
<td>1</td>
<td>0.0762</td>
<td>0.0675</td>
<td>0.0599</td>
<td>0.0533</td>
<td>0.0475</td>
<td>0.0424</td>
<td>0.0379</td>
<td>0.0339</td>
<td>0.0304</td>
<td>0.0272</td>
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</tr>
<tr>
<td>2</td>
<td>0.0233</td>
<td>0.0202</td>
<td>0.0176</td>
<td>0.0154</td>
<td>0.0136</td>
<td>0.0120</td>
<td>0.0107</td>
<td>0.0095</td>
<td>0.0085</td>
<td>0.0076</td>
<td>0.0069</td>
<td></td>
</tr>
<tr>
<td>3</td>
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<td>0.0091</td>
<td>0.0078</td>
<td>0.0068</td>
<td>0.0059</td>
<td>0.0052</td>
<td>0.0046</td>
<td>0.0040</td>
<td>0.0036</td>
<td>0.0032</td>
<td>0.0029</td>
<td></td>
</tr>
</tbody>
</table>

Note: All values are calculated using equation (14).
and dense sand (\(D_r = 80\%\)). For each case three mean stress levels (100, 300 and 500 kPa) are taken into consideration, roughly representing stress levels for piles with different embedded lengths. In producing these data, the shear modulus of sand has been estimated using the correlation of Lo Presti (1987)

\[
\frac{G}{p_0} = 400 \exp \left(0.7D_r\right) \left(\frac{\sigma_0}{p_0}\right)^{0.5}
\]  

where \(\sigma_0\) is the mean effective confining stress, \(p_0\) is a reference pressure (100 kPa), and \(D_r\) is relative density. The average volumetric strain, \(\Delta\), has been estimated using the empirical correlation of Yasufuku et al. (2001)

\[
\Delta = 50\left(l_i\right)^{-1.8}
\]  

There are several features that are worth noting. First, the value of \(S_t\) always decreases with increasing friction angle. This trend is consistent with that predicted using the proposal of Randolph et al. (1994), but the new proposal gives a much lower reduction rate. Second, for a given friction angle and relative density, \(S_t\) tends to increase with increasing stress level or penetration depth. Third, for a given friction angle and stress level, \(S_t\) tends to reduce with increasing relative density.

An alternative comparison of the \(S_t\) values from the new and existing proposals is presented in Figs 4 and 5. It is seen that, at a low friction angle (\(\phi = 25^\circ\)), the proposal of Randolph et al. (1994) always gives the highest value (0.08) and the proposal of Fleming et al. (1992) gives the smallest (0.02), with the prediction from the new proposal being in between (0.05–0.07). If the friction angle becomes higher (\(\phi = 35^\circ\)), \(S_t\) has values between 0.03 and 0.05, whereas the proposal of Randolph et al. (1994) gives a value as low as 0.015.

From a practical point of view, the above comparison may help partly explain why the method of Randolph et al.

Fig. 3. Predicted \(S_t\) values for various states of sand: (a) \(D_r = 50\%\); (b) \(D_r = 80\%\)

Fig. 4. Values of \(S_t\) estimated with different proposals for various states of sand (stress level = 100 kPa): (a) \(\phi = 25^\circ\); (b) \(\phi = 31^\circ\); (c) \(\phi = 35^\circ\)
involving the use of $S_t$ has led to significant underestimates of shaft resistance (see Fig. 6). The largest measured-to-predicted ratio, 2.12, is with a driven pile of 6.7 m; (b) $\phi = 31^\circ$; (c) $\phi = 35^\circ$.

Fig. 5. Values of $S_t$ estimated with different proposals for various states of sand (relative density $=80\%$): (a) $\phi = 25^\circ$; (b) $\phi = 31^\circ$; (c) $\phi = 35^\circ$.

Fig. 6. Measured-to-calculated shaft resistance for a database of pile load tests (after Randolph et al., 1994)

$S_t$. In doing this, the following correlation for sands containing about 15–30% fines (Lo Presti, 1987) is adopted.

$$\frac{G}{\sigma_k} = 75 \exp \left(0.7D \right) \left( \frac{\alpha}{\sigma_k} \right)^{0.5}$$

(21)

The newly calculated $S_t$ values are presented in Fig. 7, together with the results obtained previously for higher stiffness for comparison. A marked feature is that the reduction of stiffness leads to greater $S_t$ values: for a range of angles (30–35$^\circ$), $S_t$ can become as large as 0.1. Such a large value was indeed recorded in recent model tests (Gavin & Gallagher, 2005). Another point of interest is that, when the stiffness is reduced, the influence of stress level and friction angle on the value of $S_t$ tends to be more profound. Since the existing proposal for $S_t$ does not account for the factors of stiffness and stress level, the comparison in Fig. 7 implies that, given otherwise identical input, greater underestimation of shaft resistance might be produced for piles in silty sands than in clean sands. Interestingly, there is a clue in this respect in the case studies reported by Randolph et al. (1994): quite large measured-to-predicted values (2.02 and 1.99) were also obtained for two piles driven in a sand deposit containing a significant amount of silt (Mansur & Kaufman, 1956).

Lastly, it should be noted that in the cavity expansion analysis, the shear stiffness affects the rigidity index – an indicator for the average volumetric strain of the sand near the pile tip. In this context, an alternative view of the effect of stiffness is as follows: when the sand becomes more compressible, $S_t$ tends to take larger values, suggesting that care should be taken about possible underestimation of shaft capacity in highly compressible sand.

CONCLUSIONS

The main findings and design implications derived from this study are summarised as follows.

(a) The value of $S_t$ is dependent on several factors – the friction angle, shear stiffness, compressibility, relative density and mean stress level of the sand near the pile tip – and these factors are interrelated.

(b) Given otherwise identical parameters, $S_t$ tends to decrease with an increase in friction angle, and with a decrease in mean stress level. It also tends to decrease with increasing relative density.

(c) For a typical range of parameters, $S_t$ has values varying from 0.03 to 0.05. It may take higher values (~0.1) if the stiffness of sand becomes significantly low.
Fig. 7. Effect of stiffness on $S_t$ values (solid lines, high stiffness; broken lines, low stiffness): (a) $D_s = 50\%$; (b) $D_s = 80\%$

(d) The existing proposals generally provide lower values for $S_t$ (0.015–0.03), which may contribute to the observed underestimation of shaft resistance, particularly for piles installed in dense sand or in highly compressible sand.

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NOTATION

- $a$, $b$: empirical parameters
- $D$: pile diameter
- $D_s$: relative density
- $D_{FU}$: upper limit of influence zone
- $G$: shear modulus
- $I_1$: rigidity index
- $p_0$: reference pressure
- $p_e$: cavity pressure
- $p_0$: mean effective stress at pile tip
- $q_b$: pile base resistance
- $R_b$: plastic zone radius at limit state
- $R_s$: cavity radius at limit state
- $r$: radial distance
- $S_t$: ratio between $\sigma_{t,max}$ and $q_b$
- $\beta$: angle formed by two lines CO and CE (Fig. 2)
- $\Delta$: average volumetric strain in plastic zone
- $\delta$: interface friction angle between pile and soil
- $\zeta$: ratio of $R_b$ to $R_s$
- $\lambda$: proportional factor
- $\sigma_{t,max}$: maximum radial effective stress
- $\sigma_{t,p}$: radial stress in plastic zone
- $\sigma_{b}$: circumferential stress component
- $\sigma_0$: mean effective confining stress
- $\tau_{max}$: maximum shaft friction
- $\phi$: friction angle of sand near pile tip
- $\chi$: dimensionless parameter
- $\psi$: angle of soil wedge below pile tip

REFERENCES


