The University of Hong Kong

A Continuum Modeling Approach to Congestion Management of Transportation System in an Urban City

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Introduction

- Continuum transportation system in an urban city
- Model formulation and solution algorithm
- Some applications
- Conclusions and further works

Continuum Transportation System

Two approaches for modeling traffic equilibrium problems

- Discrete modeling approach

  Patriksson (1994); Gendreau and Marcotte (2002); Lee (2003); etc.

- Continuum modeling approach

  - Idealized city configuration (e.g. circular city)

    Lam and Newell (1967); Zitron (1967); D'Este (1987); Wong (1994); etc.

  - General city configuration

    Beckmann (1952), one of the earliest works in this area; Wardrop (1971); Williams and Ortuzar (1976); Puu (1977); Buckley (1979); Dafermos (1980); Sasaki et al. (1990); Yang et al. (1994); Wong (1998) etc.

Continuum Transportation System

Basic assumptions in continuum modeling approach

- The difference in land use pattern and network configuration between adjacent areas is relatively small, compared with the variation over the entire city
- The characteristics of the land use and network can be represented by piecewise smooth mathematical functions (Vaugham, 1987)
- The interrelation among these mathematical functions are governed by some appropriate forms of differential or integral equations
- The user equilibrium conditions that are commonly adopted in discrete network modeling can be generalized to the continuum case in a two-dimensional plane

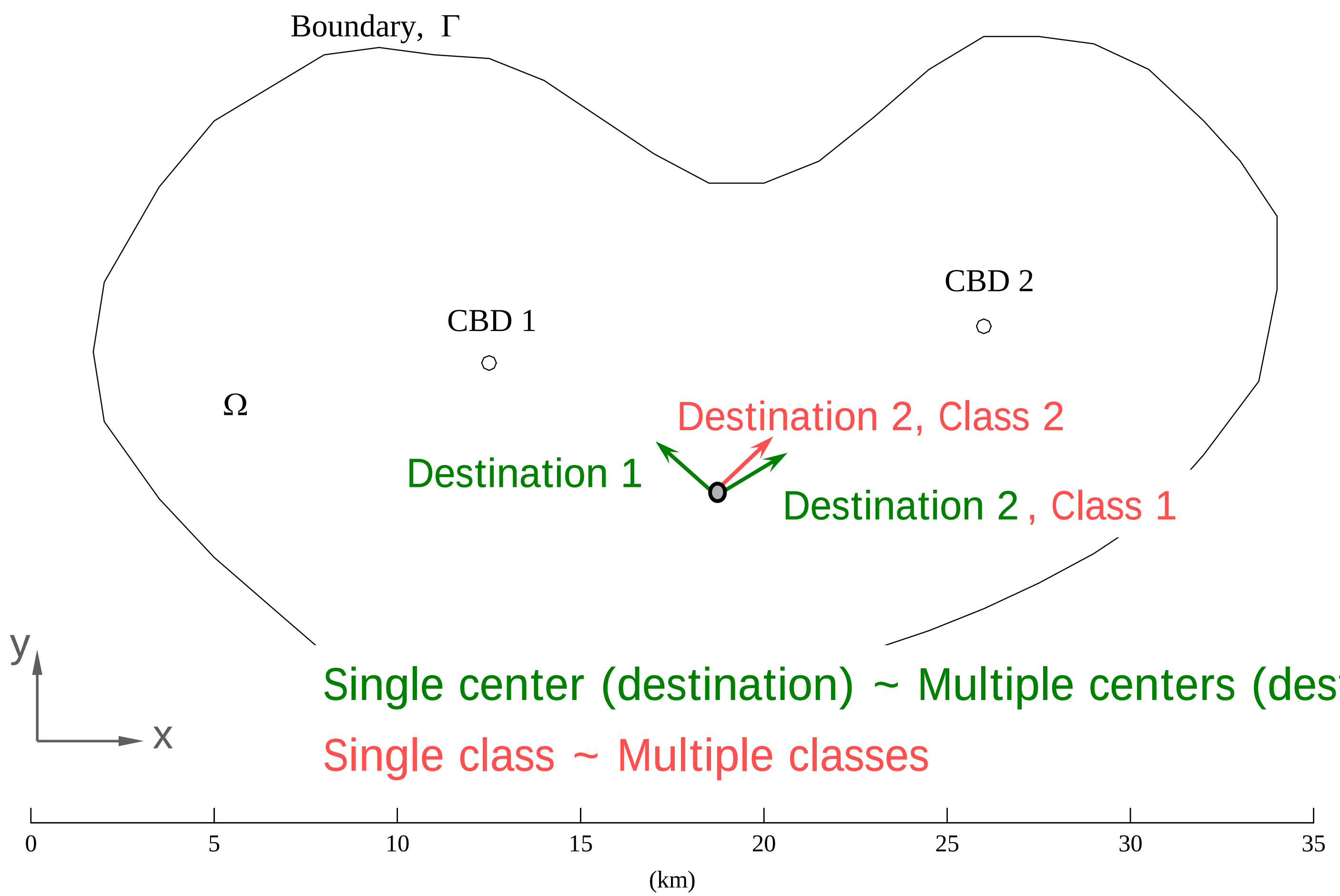
Continuum Transportation System

Rationale for using continuum modeling approach

- For the initial phase of planning and modeling in broad-scale regional studies, the focus may be on the general trend and pattern of the distribution and travel choice of users, and their changes in response to policy changes at the macroscopic level
- Insufficient data for setting up the “dense” transportation network for detailed analysis
- Conceptual plan for the catchment regions of competing facilities (such as airports, ports), locations of cordons for congestion charging, location of an additional CBD, strategic expansion of transportation system in different parts of the city, etc.
- Travel demand is inherently continuously spread over the city

Model formulation and solution algorithm

The modeled city

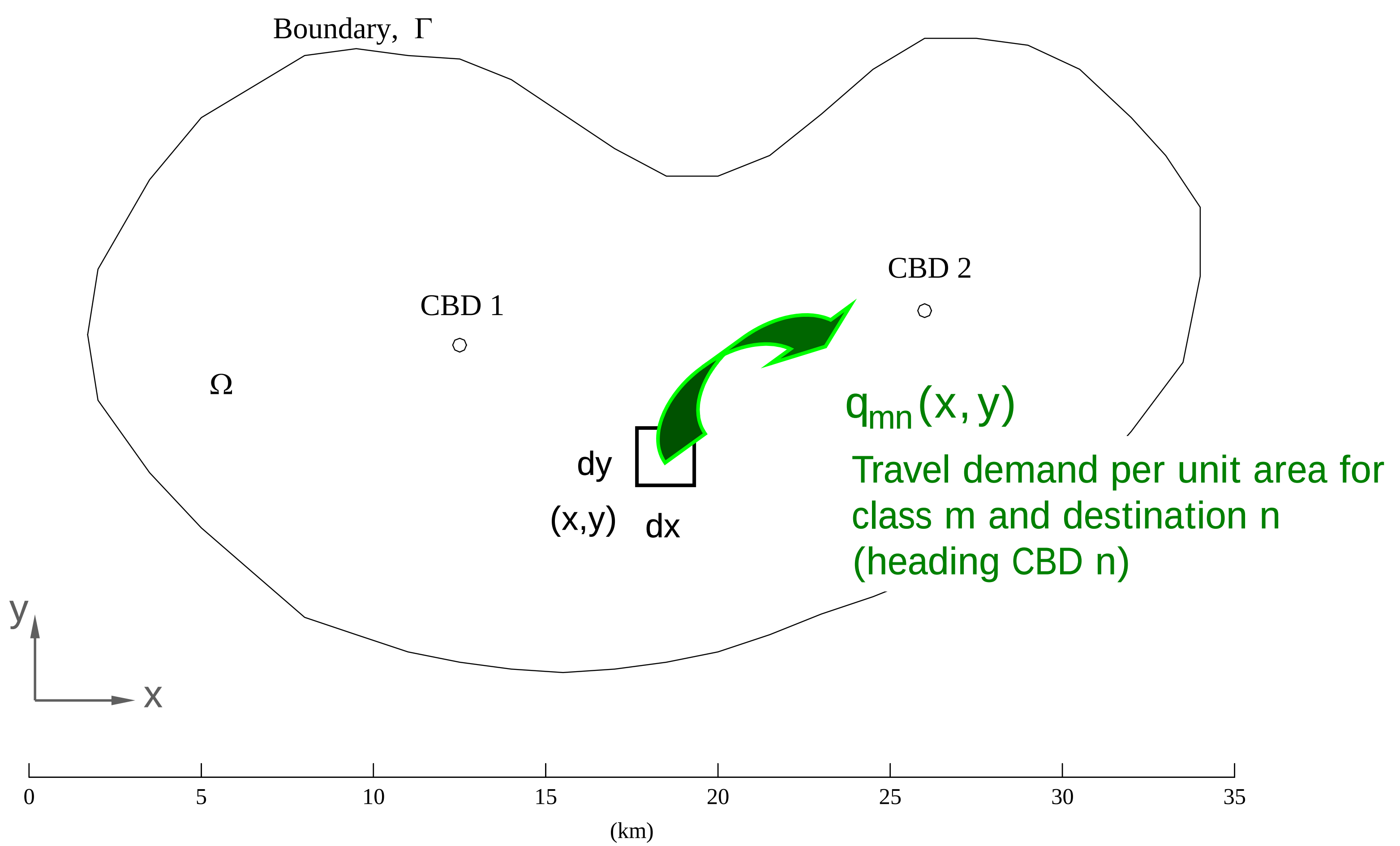


Single center (destination) ~ Multiple centers (destinations)

Single class ~ Multiple classes

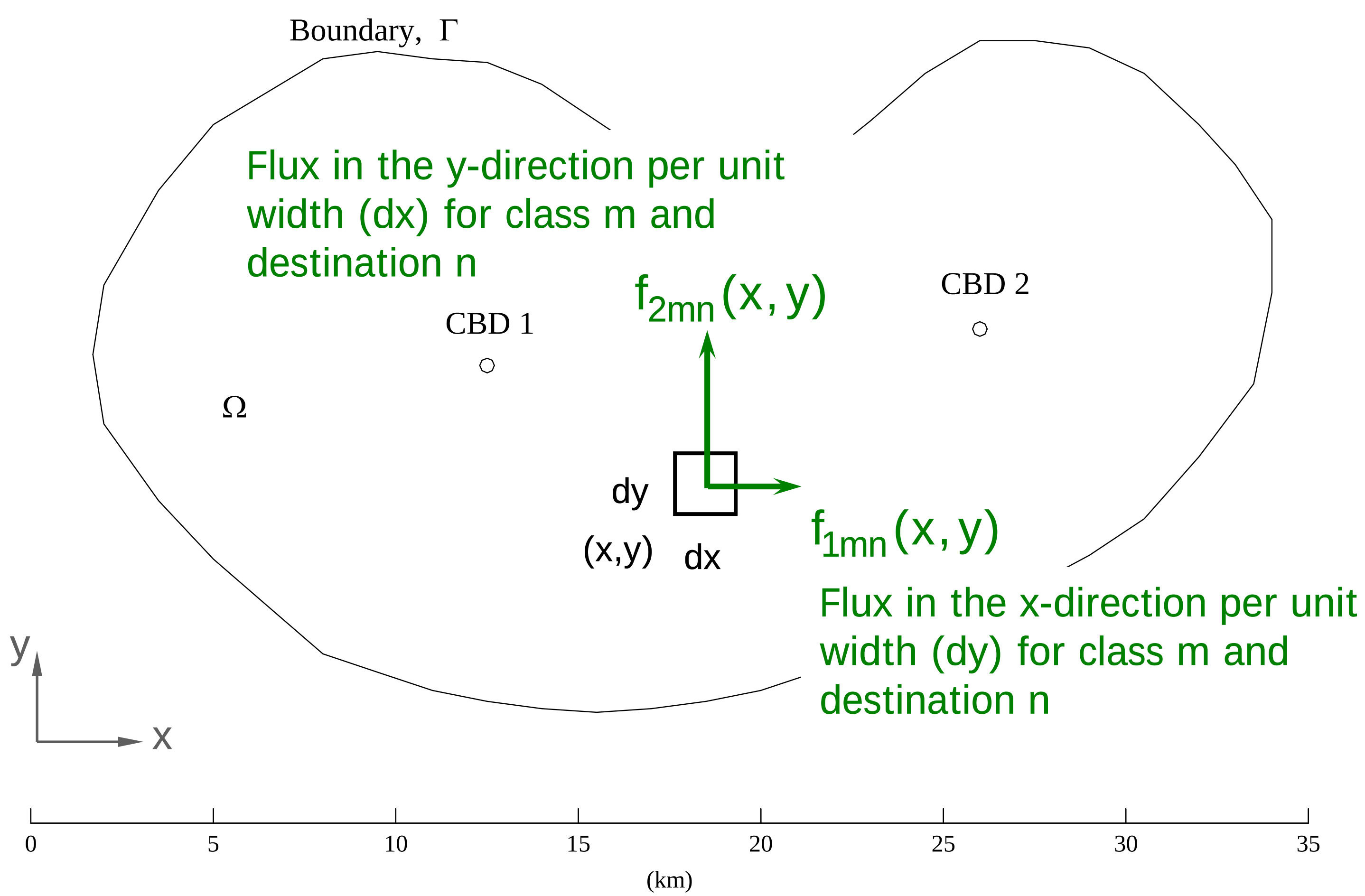
Model formulation and solution algorithm

Travel demand



Model formulation and solution algorithm

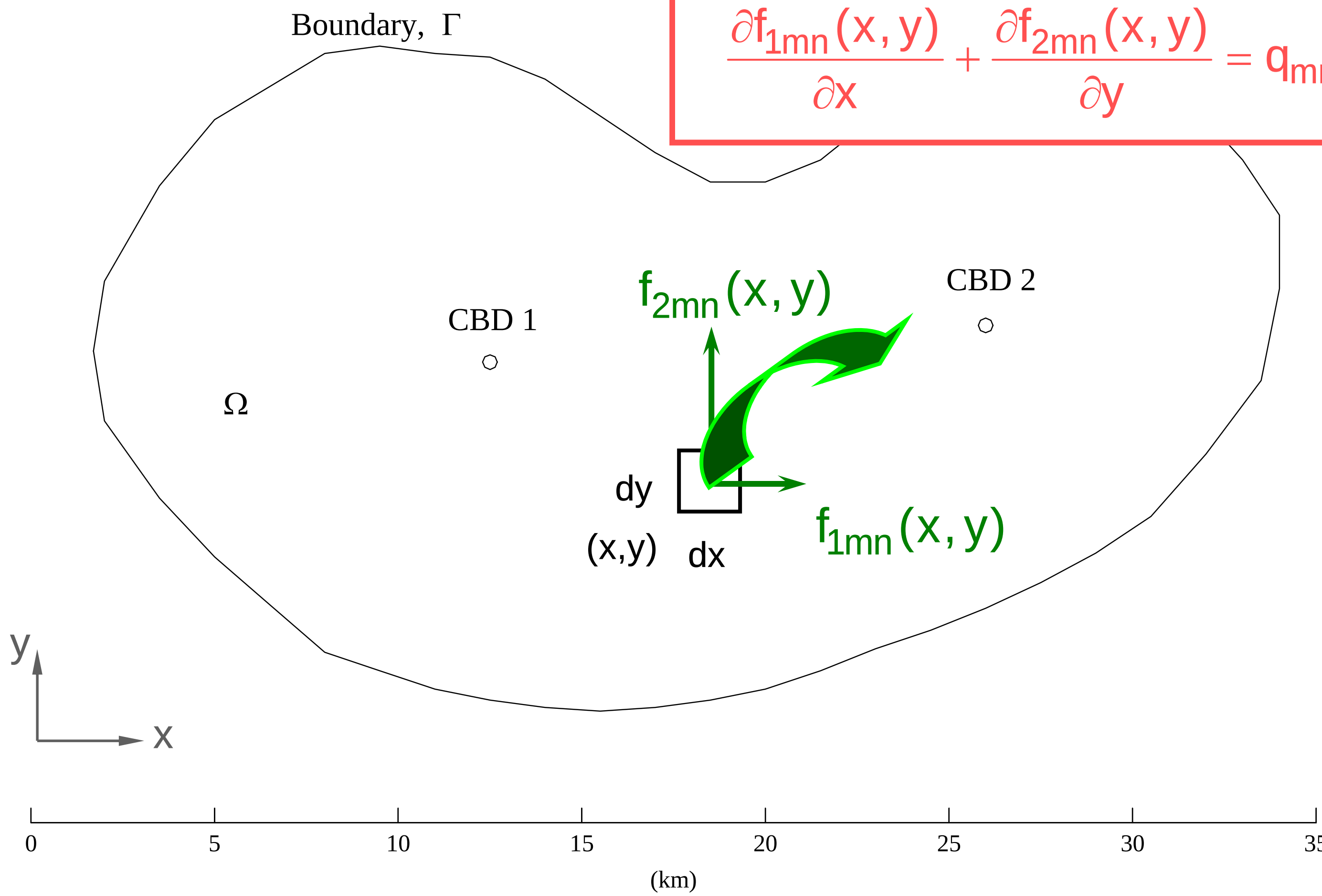
Traffic flows



Model formulation and solution algorithm

Conservation of flows

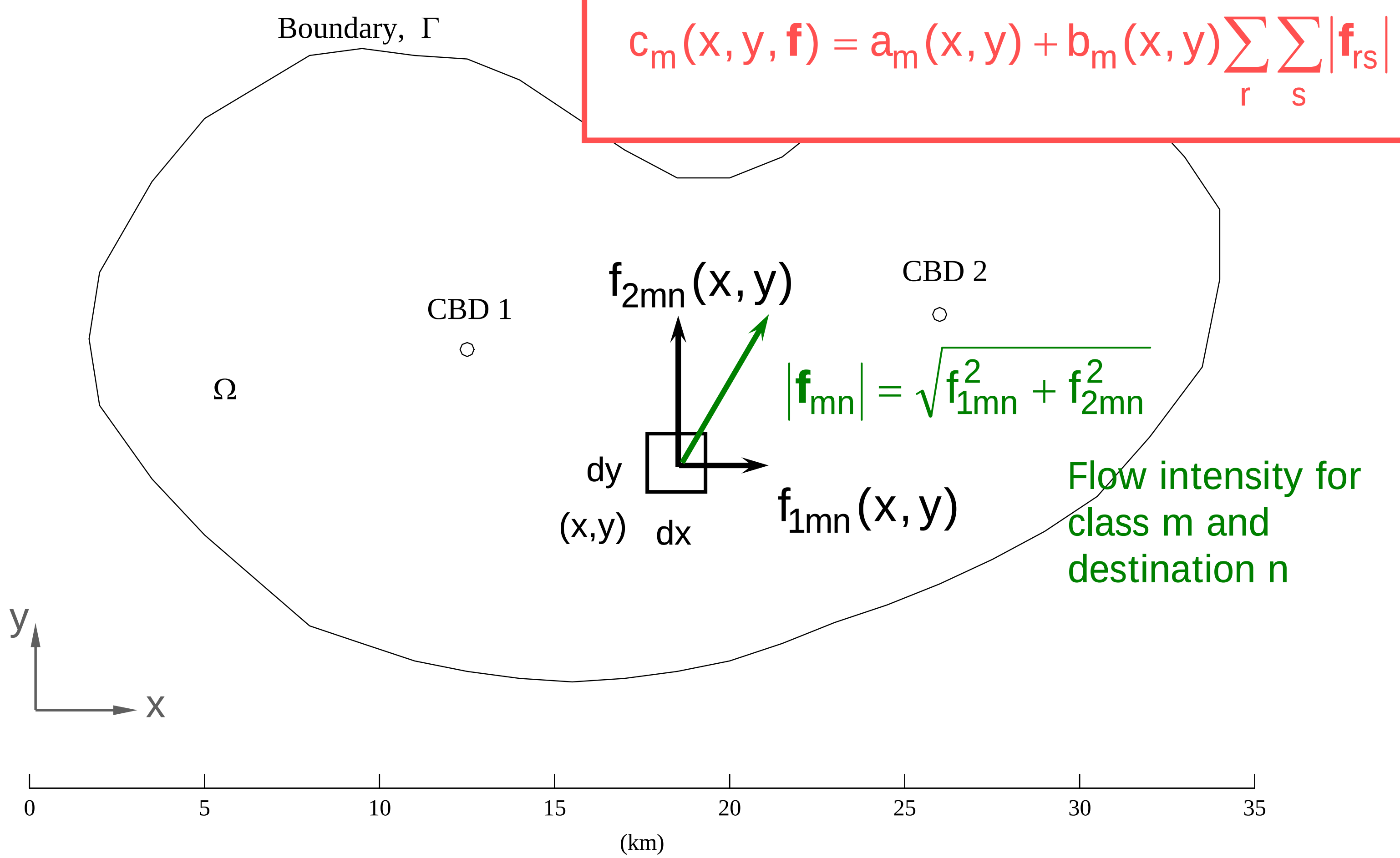
$$\frac{\partial f_{1mn}(x,y)}{\partial x} + \frac{\partial f_{2mn}(x,y)}{\partial y} = q_{mn}(x,y)$$



Model formulation and solution algorithm

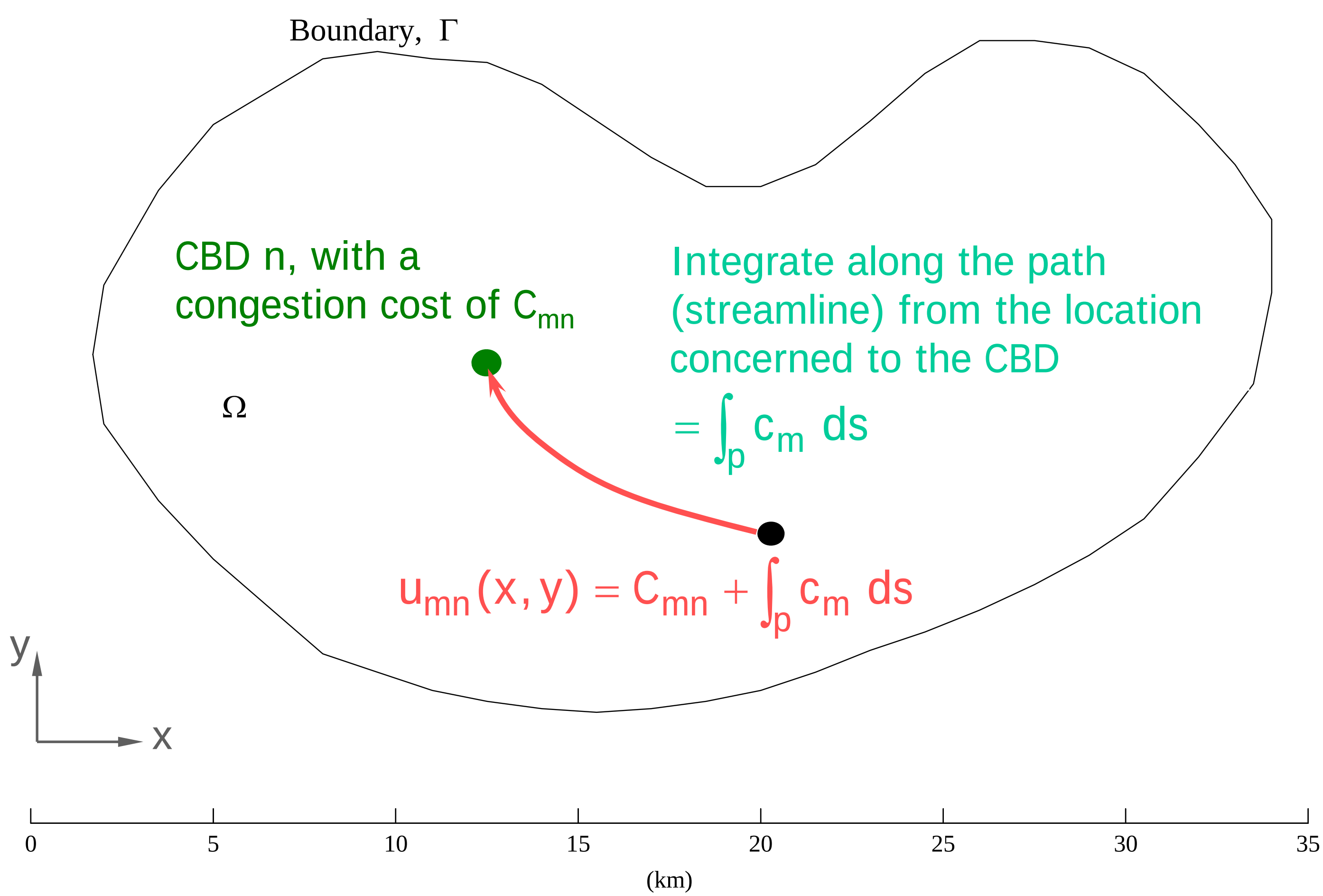
Local cost function

$$c_m(x, y, \mathbf{f}) = a_m(x, y) + b_m(x, y)\sum_r \sum_s |\mathbf{f}_{rs}|$$



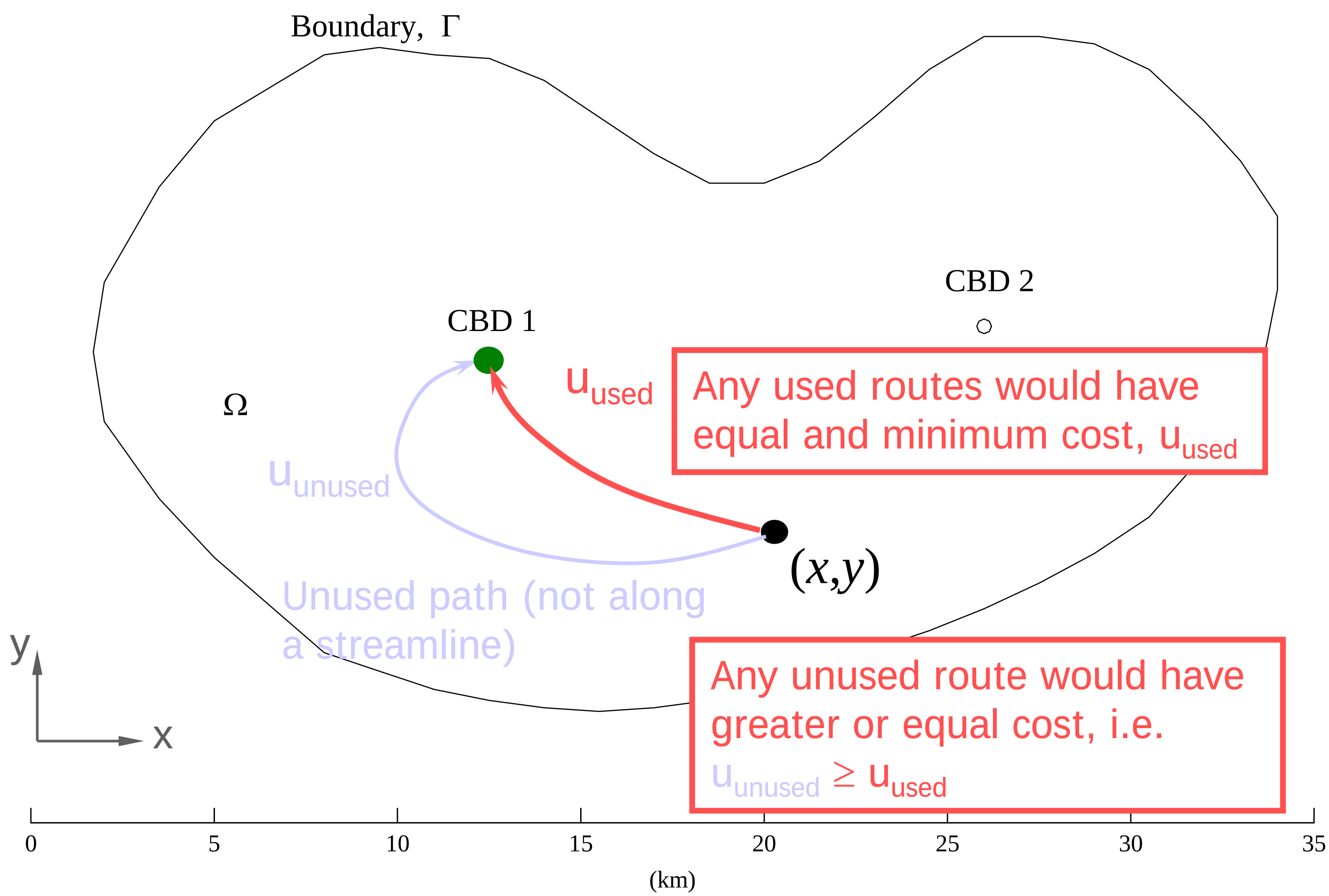
Model formulation and solution algorithm

Path travel cost



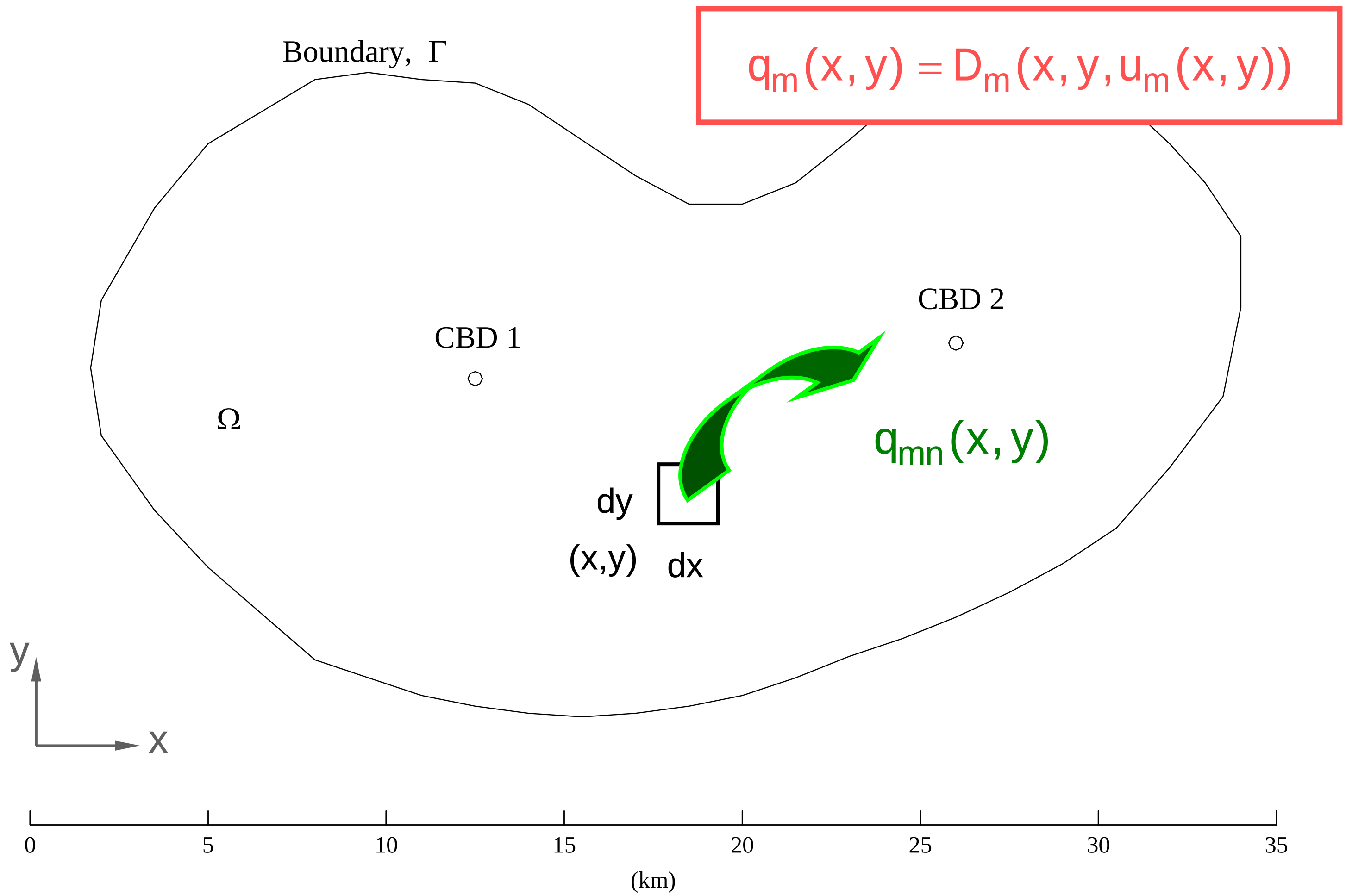
Model formulation and solution algorithm

User equilibrium



Model formulation and solution algorithm

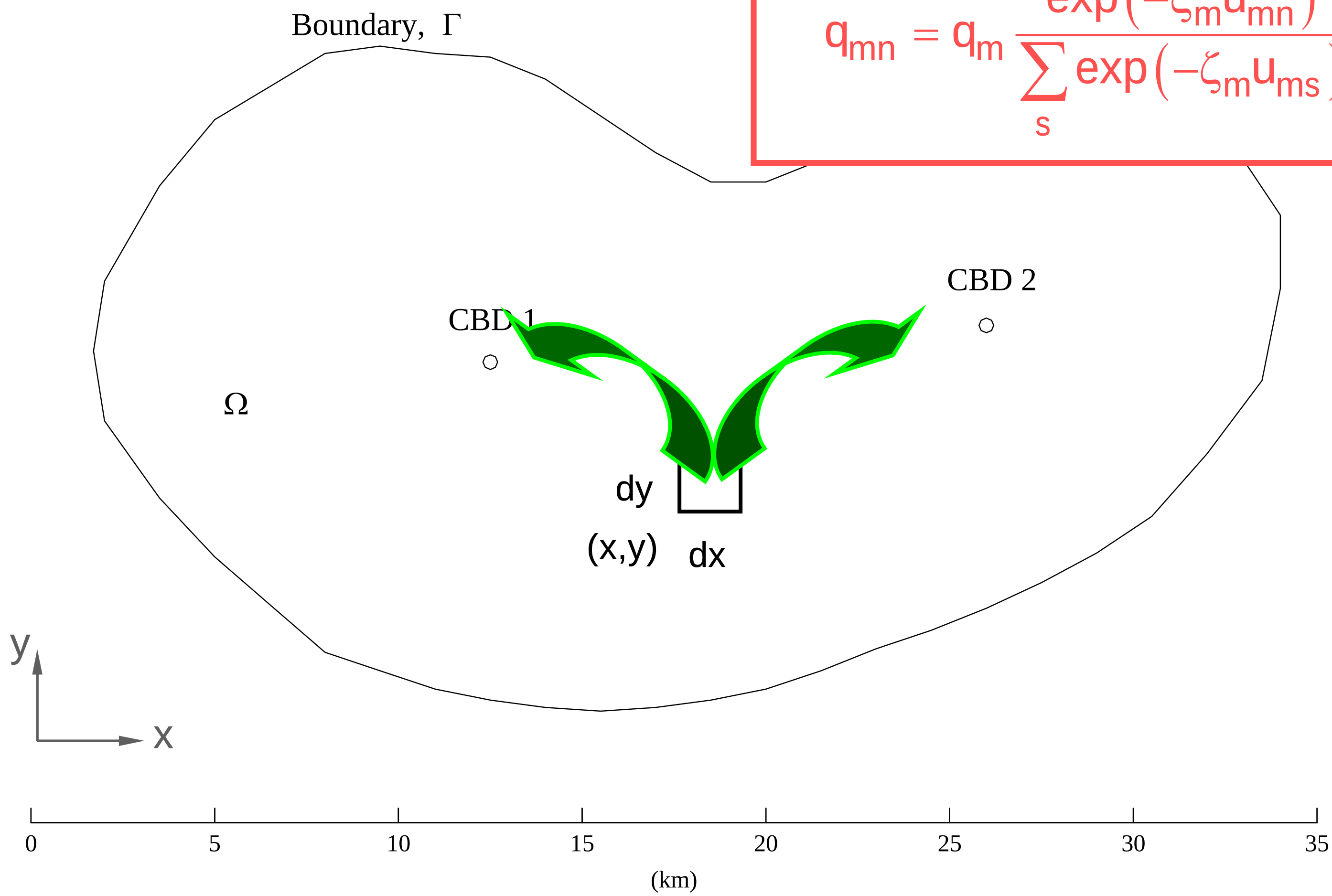
Elastic demand



Model formulation and solution algorithm

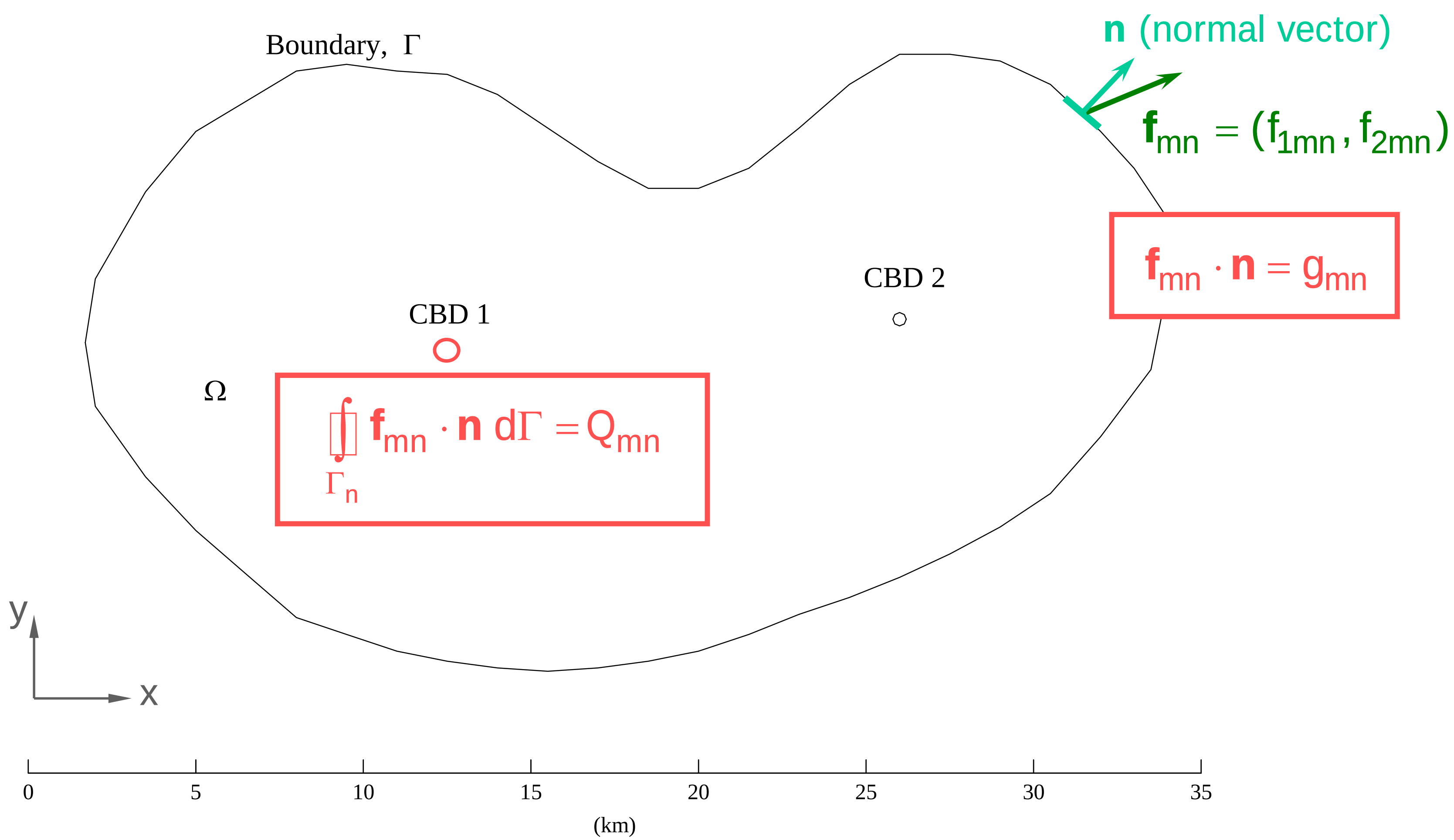
Trip distribution function

$$q_{mn} = q_m \frac{\exp(-\zeta_m u_{mn})}{\sum_s \exp(-\zeta_m u_{ms})}$$



Model formulation and solution algorithm

Boundary conditions



Model formulation and solution algorithm

Mathematical program

$$\underset{\mathbf{f},\mathbf{q},\mathbf{Q}}{\text{Minimize}} \quad z(\mathbf{f},\mathbf{q},\mathbf{Q}) = \sum_m \sum_n \tilde{\theta}_{mn} Q_{mn} + \sum_n \int_0^{\sum_m Q_{mn}} S_n(\xi)d\xi + \iint_\Omega \Bigg\{ \sum_m \sum_n \tilde{a}_m |\mathbf{f}_{mn}|$$

$$+ \frac{1}{2} \sum_m \sum_n \sum_r \sum_s |\mathbf{f}_{mn}||\mathbf{f}_{rs}| + \sum_m \sum_n \frac{1}{\zeta_m b_m} (q_{mn} \ln q_{mn} - q_{mn})$$

$$- \sum_m \frac{1}{b_m} \int_0^{q_m} D_m^{-1}(\xi)d\xi - \sum_m \frac{1}{\zeta_m b_m} (q_m \ln q_m - q_m) \Bigg\} d\Omega$$

subject to

$$\nabla \mathbf{f}_{mn} - q_{mn} = 0, \quad \forall (x,y) \in \Omega, n \in N, m \in M$$

$$q_m - \sum_n q_{mn} = 0, \quad \forall (x,y) \in \Omega, m \in M$$

$$\mathbf{f}_{mn} = 0, \quad \forall (x,y) \in \Gamma, n \in N, m \in M$$

$$\int_{\Gamma_{nc}} \mathbf{f}_{mn} \cdot \mathbf{n} \, d\Gamma - Q_{mn} = 0, \quad \forall n \in N, m \in M$$

Model formulation and solution algorithm

Equivalent set of partial differential equations

$$\nabla \cdot \mathbf{f}_{mn} - q_{mn} = 0, \quad \forall (x,y) \in \Omega, n \in N, m \in M$$ Conservation equation

$$\tilde{c}_m \frac{\mathbf{f}_{mn}}{\left|\mathbf{f}_{mn}\right|} + \nabla \alpha_{mn} = 0, \quad \forall (x,y) \in \Omega, n \in N, m \in M$$ Cost potential function

$$\frac{\ln q_{mn}}{\zeta_m b_m} + \alpha_{mn} + \beta_m = 0, \quad \forall (x,y) \in \Omega, n \in N, m \in M$$ Distribution function

$$\beta_m + \frac{1}{b_m} D_m^{-1}(q_m) + \frac{\ln q_m}{\zeta_m b_m} = 0, \quad \forall (x,y) \in \Omega, m \in M$$ Elastic demand

$$\tilde{C}_{mn} + \pi_{mn} = 0, \quad \forall n \in N, m \in M$$ Boundary conditions

$$q_m - \sum_n q_{mn} = 0, \quad \forall (x,y) \in \Omega, m \in M$$

$$\mathbf{f}_{mn} = 0, \quad \forall (x,y) \in \Gamma, n \in N, m \in M$$

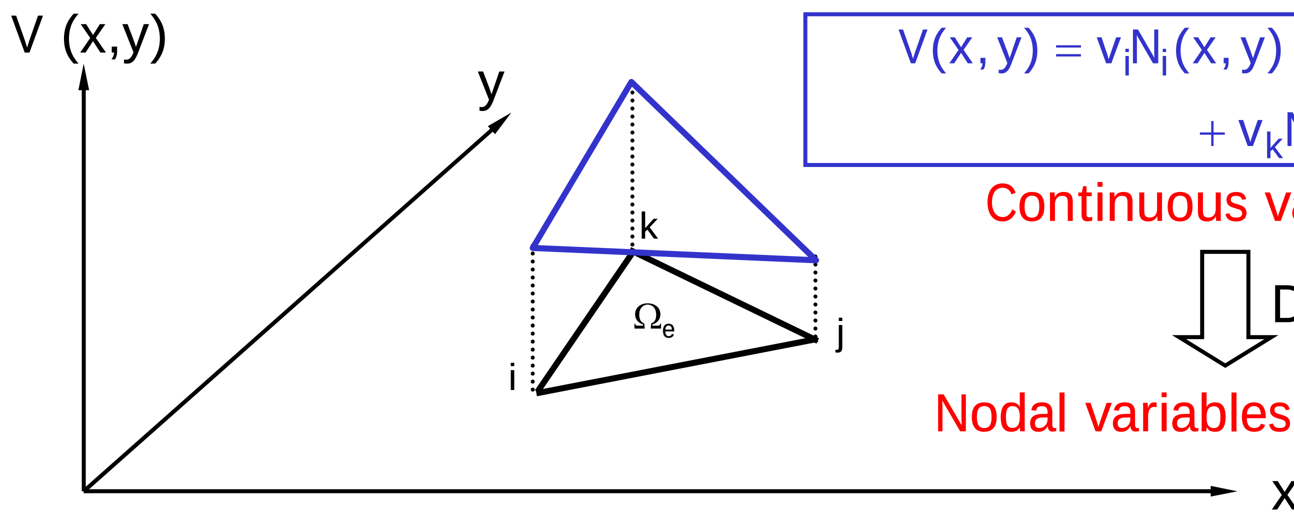
$$\pi_{mn} + \alpha_{mn} = 0, \quad \forall (x,y) \in \Gamma_{nc}, n \in N, m \in M$$

$$\int_{\Gamma_{nc}} \mathbf{f}_{mn} \cdot \mathbf{n} \, d\Gamma - Q_{mn} = 0, \quad \forall n \in N, m \in M$$

Model formulation and solution algorithm

Finite element method

The finite element method (FEM) (Zienkiewicz and Taylor, 1989) is used to approximate the continuous variables in the modeled city

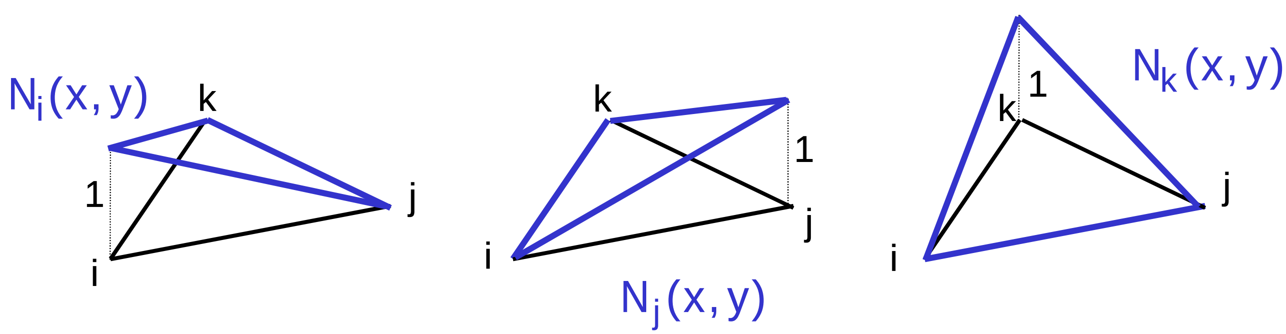


$$V(x,y) = v_i N_i(x,y) + v_j N_j(x,y) + v_k N_k(x,y)$$

Continuous variables

Discretized

Nodal variables: (v_i, v_j, v_k)



Model formulation and solution algorithm

Finite element method

$$\Pi(\Psi(x,y)) = 0$$

A set of partial differential equations

$$\Psi(x,y) = (\mathbf{f}, \mathbf{u}, \lambda)$$

The Galerkin formulation of the weighted residual technique (Cheung et al., 1996; Zienkiewicz and Taylor, 1989)

$$\overline{\Pi}(\overline{\Psi}) = 0$$

A set of nonlinear algebraic equations

$$\overline{\Psi} = (\bar{\mathbf{f}}, \bar{\mathbf{q}}, \bar{\lambda})$$

Model formulation and solution algorithm

Newtonian algorithm

$$\overline{\Pi}(\overline{\Psi}) = \boxed{\overline{\Pi}(\overline{\Psi}^0)} + \boxed{\nabla\overline{\Pi}(\overline{\Psi}^0)}(\overline{\Psi} - \overline{\Psi}^0) + \ldots$$

Residual vector, **R** Jacobian matrix, **J**

- Neglect higher order terms
- Set for a stationary point

$$\overline{\Pi}(\overline{\Psi}) = \mathbf{R} + [\mathbf{J}](\overline{\Psi} - \overline{\Psi}^0) = 0$$

- Set a recursive procedure
- Convergence criterion, $\mathbf{R} \rightarrow 0$

$$\overline{\Psi}^{(k+1)} = \overline{\Psi}^{(k)} - [\mathbf{J}(\overline{\Psi}^{(k)})]^{-1}\mathbf{R}(\overline{\Psi}^{(k)})$$

Some applications

Facility competition problem

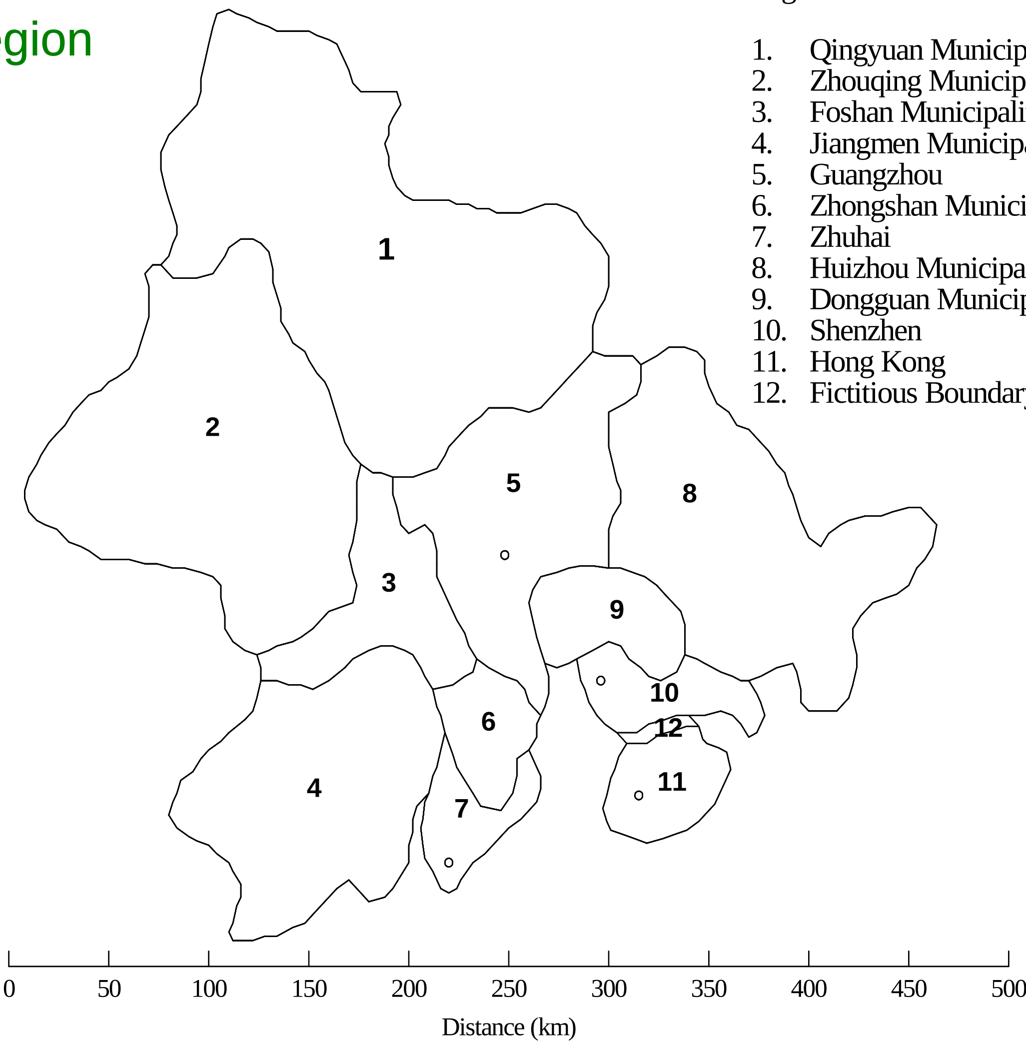
Airports competition within Pearl River Delta Region

- There are four major airports Hong Kong, Guangzhou, Shenzhen and Zhuhai within the PRD region serving both international and domestic flights
- Passengers within this region make their choice of airport based on the geographical location, congestion, and costs
- Other factors such as the cross boundary penalty affect passengers' choice of airport
- A macroscopic model is used to study the route and airport choice of passengers

Some applications

Facility competition problem

The model region



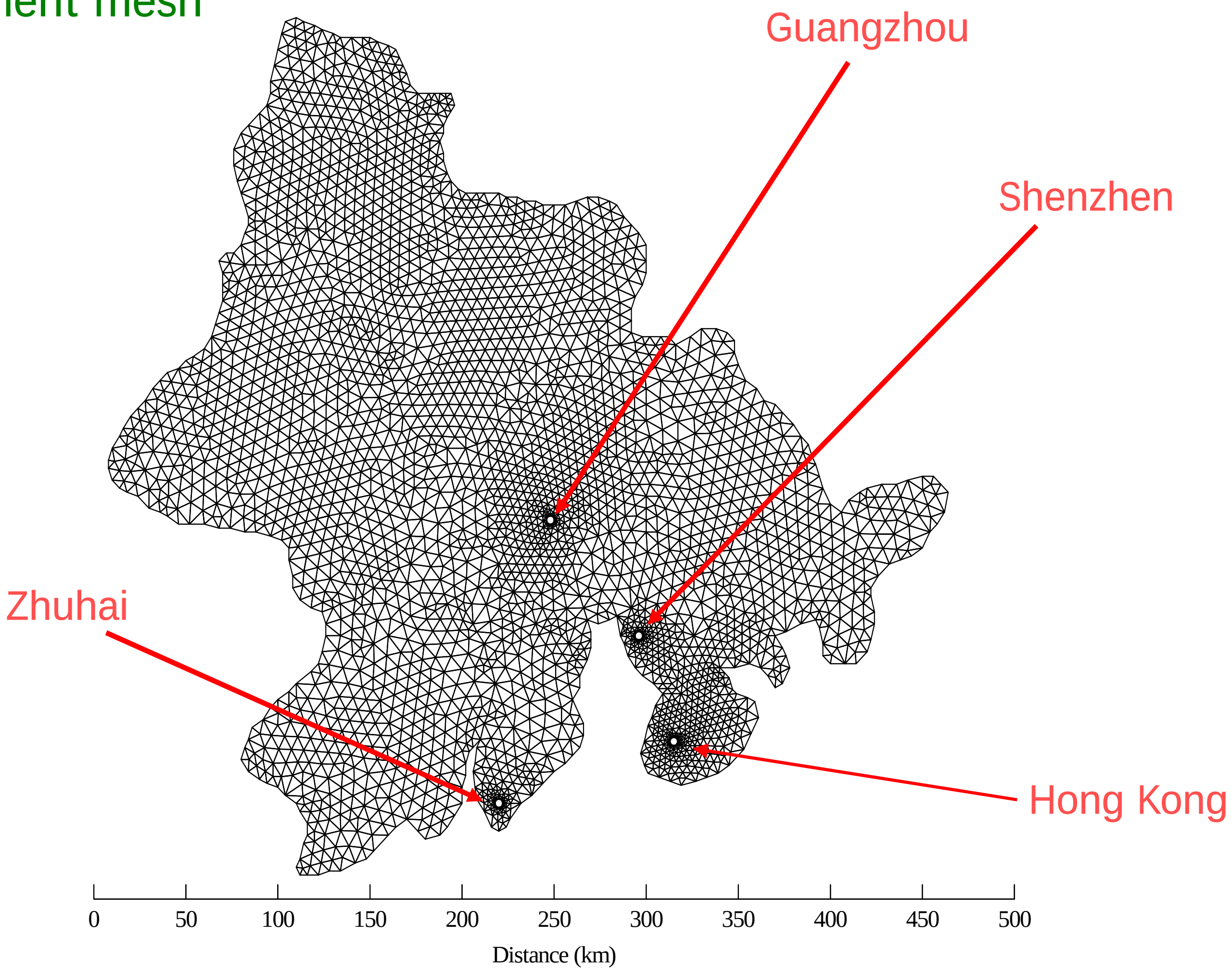
Legend:

1. Qingyuan Municipality
2. Zhouqing Municipality
3. Foshan Municipality
4. Jiangmen Municipality
5. Guangzhou
6. Zhongshan Municipality
7. Zhuhai
8. Huizhou Municipality
9. Dongguan Municipality
10. Shenzhen
11. Hong Kong
12. Fictitious Boundary Layer

Some applications

Facility competition problem

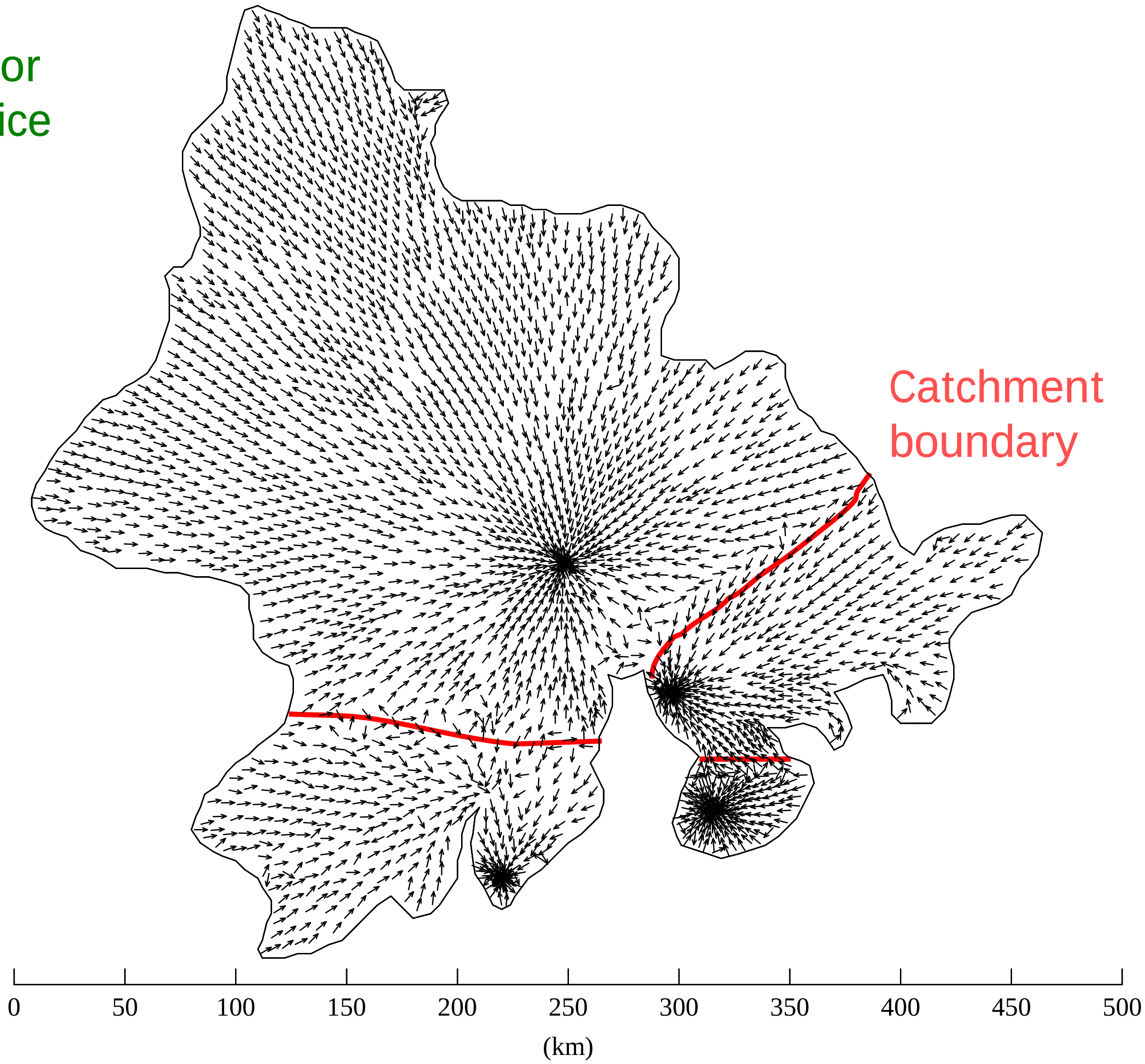
Finite element mesh



Some applications

Facility competition problem

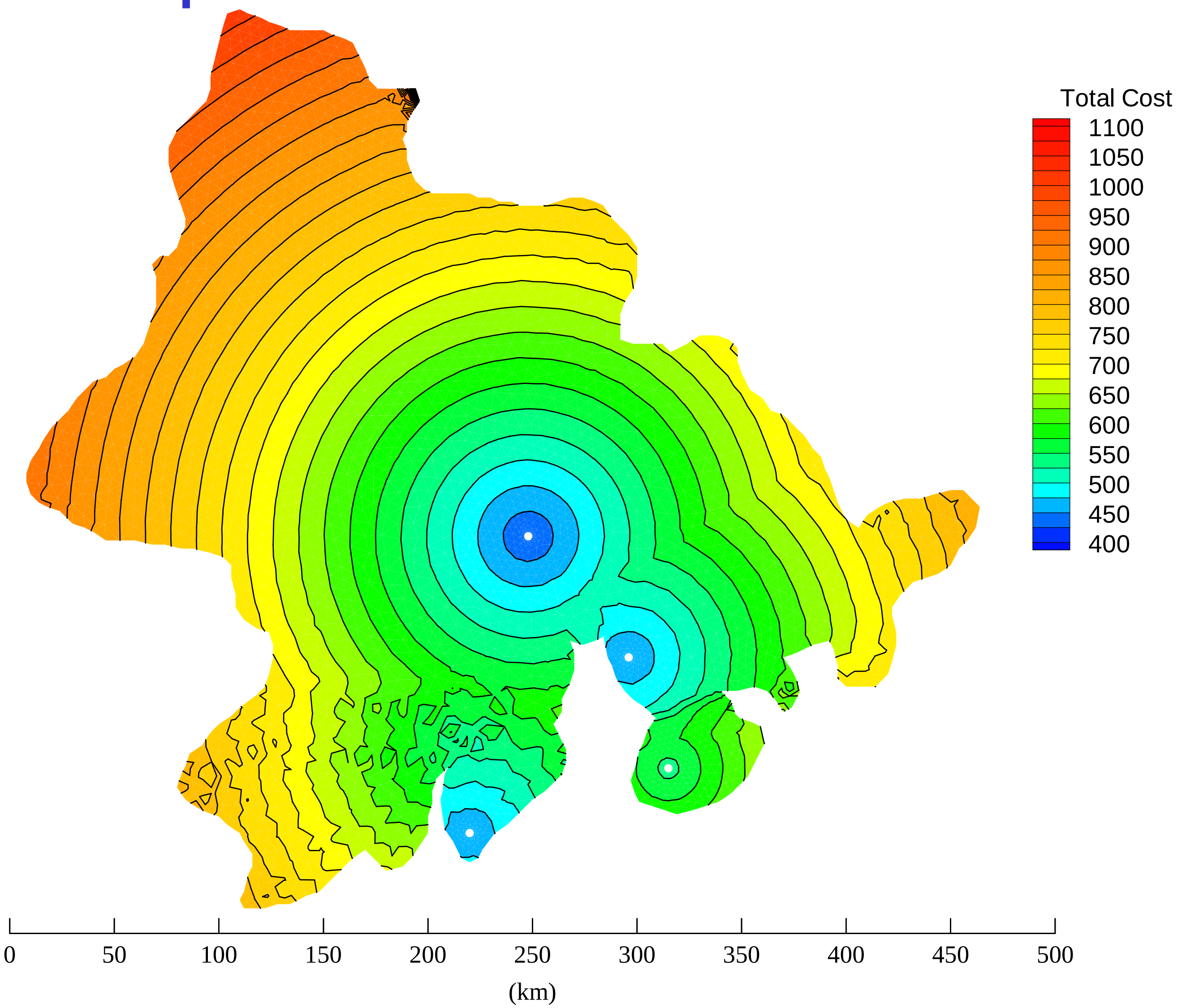
Flow pattern for domestic service



Some applications

Facility competition problem

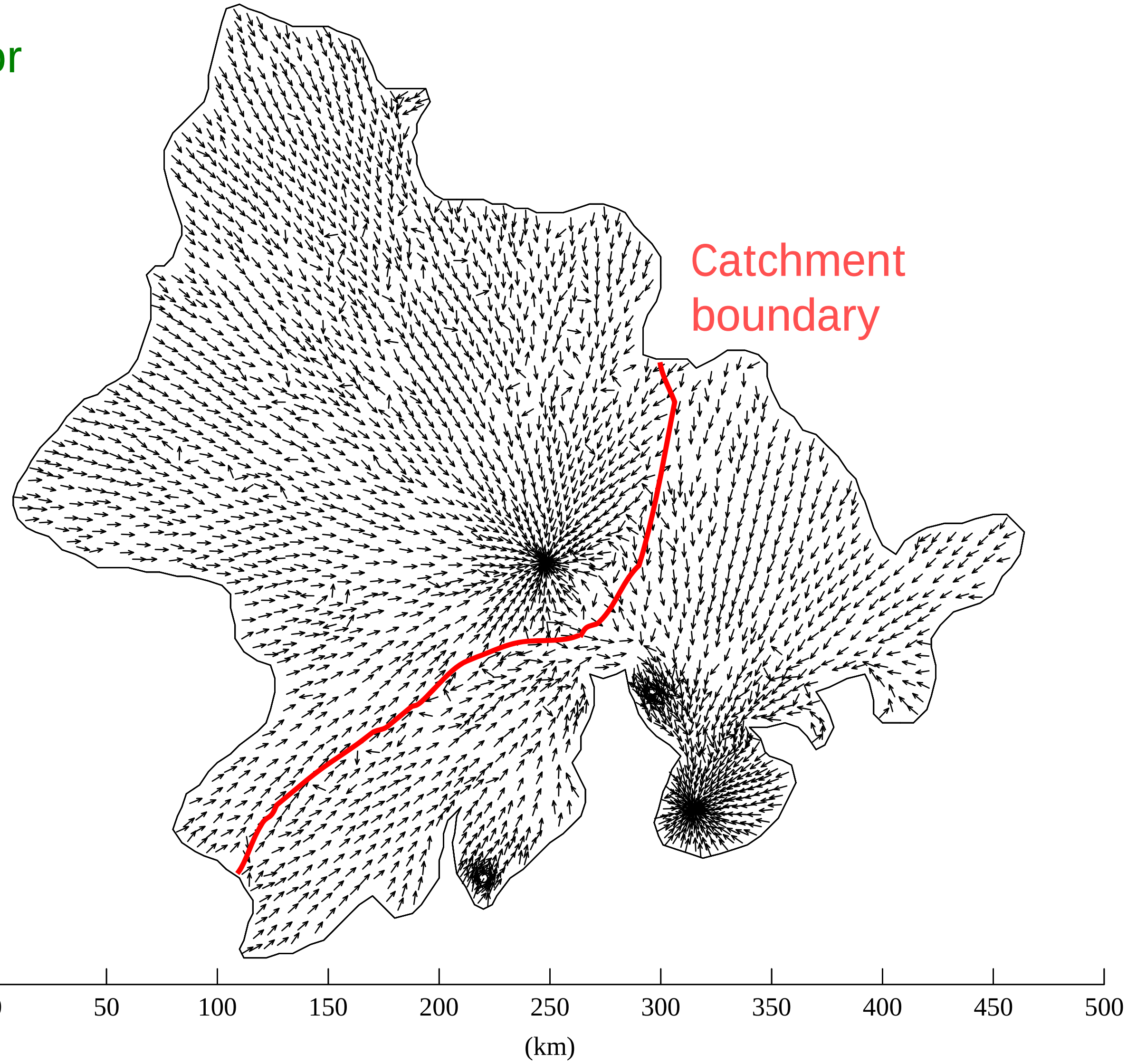
Cost potential
for domestic
service



Some applications

Facility competition problem

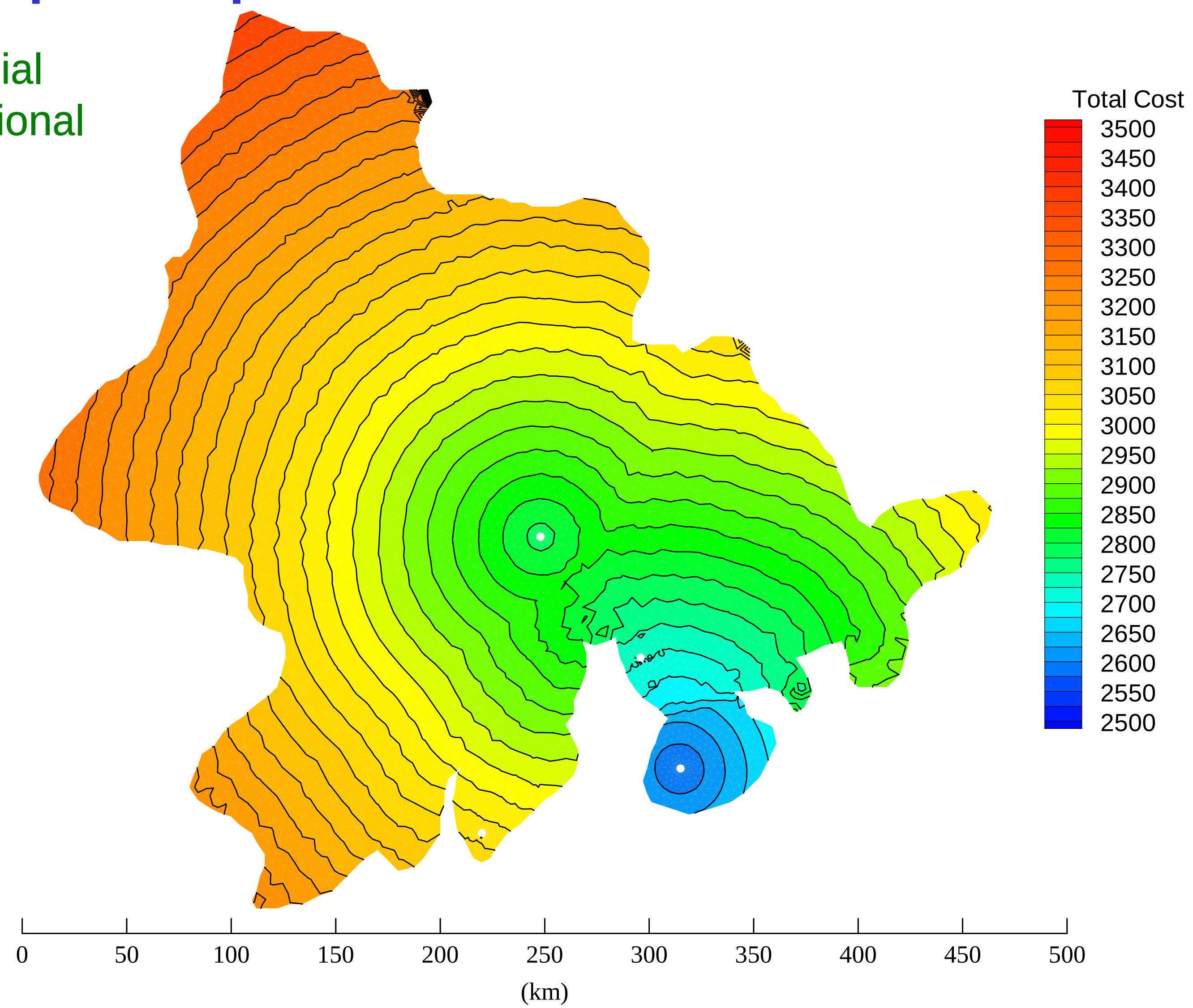
Flow pattern for international service



Some applications

Facility competition problem

Cost potential
for international
service



Some applications

Facility competition problem

Market shares among airports in the region

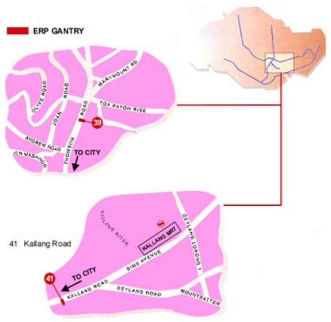
	Domestic Base Case	
Airport	**Demand (passengers/ year)**	**Estimated market share (%)**
Hong Kong	5,618,463	22.7
Shenzhen	5,467,913	22.1
Zhuhai	2,661,705	10.7
Guangzhou	10,993,485	44.4
Cross Boundary flow from Hong Kong	1,773,651 passengers/year	

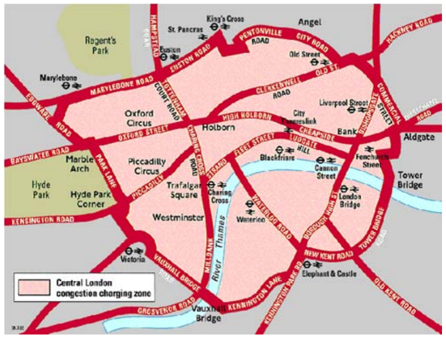
	International Base Case	
Airport	**Demand (passengers/ year)**	**Estimated market share (%)**
Hong Kong	26,971,302	97.6
Shenzhen	0	0
Zhuhai	0	0
Guangzhou	671,665	2.4
Cross Boundary flow from Mainland	609,170 passengers/year	

Some applications

Cordon-based congestion-charging problem

- Common charging methods: point-based, time-based, distance-based, and area-based
- Cordon-based charging schemes have been implemented in Singapore, Oslo, Trondheim, Bergen, and London
- Easy implementation, socially acceptable
- Location of cordons?
- Charging levels?
- Social benefits: first or second best?
- Exhaustive evaluations by discrete network optimization approach

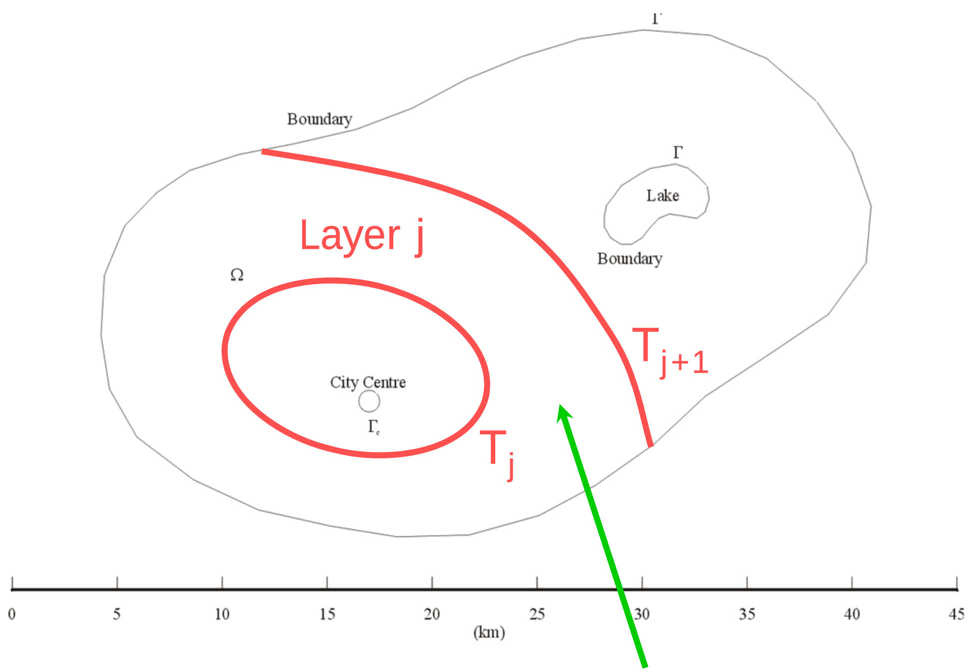




Some applications

Cordon-based congestion-charging problem

Cordon-based toll for each cordon layer



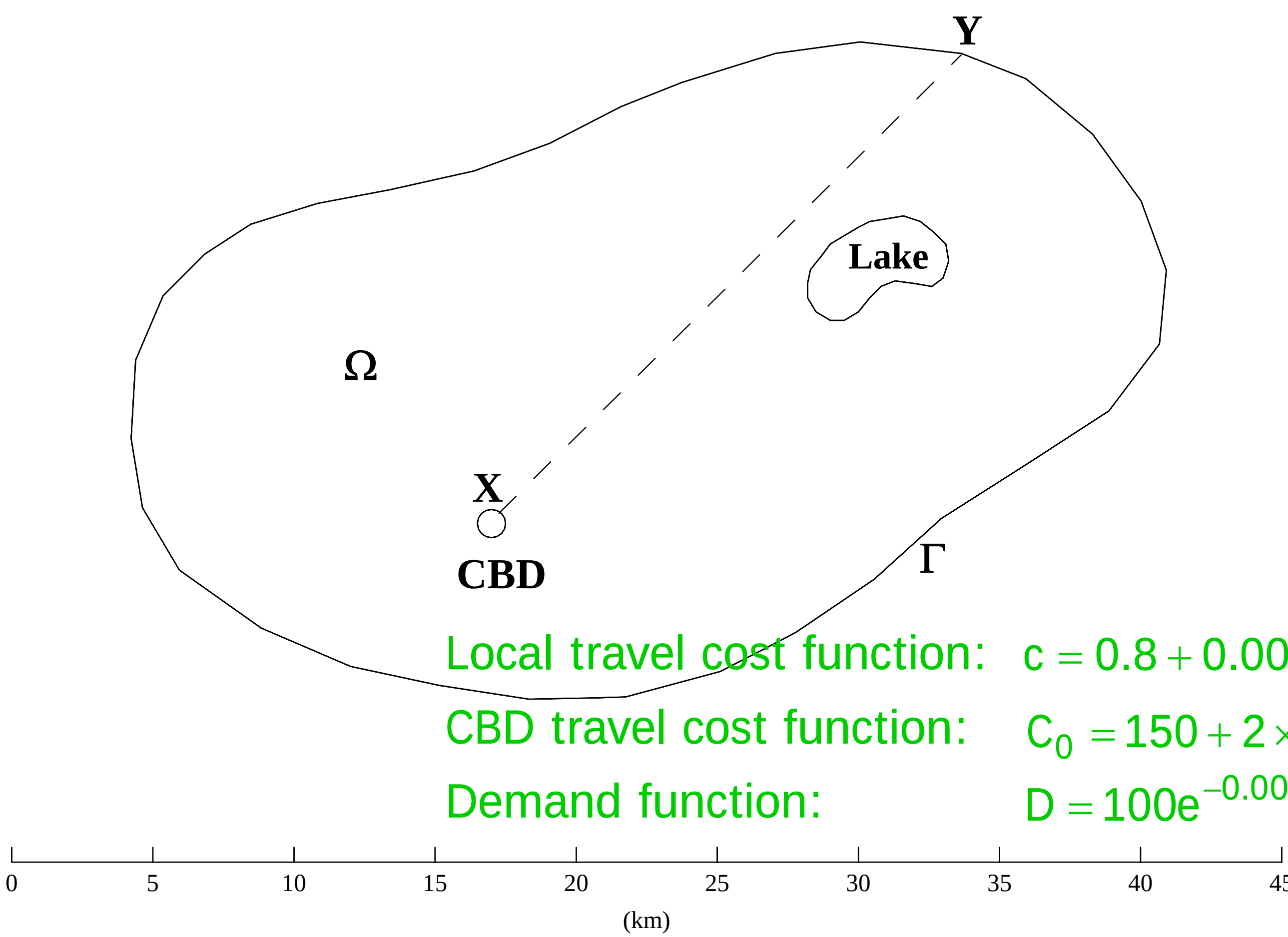
Flat toll charged for entering Layer j, $\tau_j = T_{j+1} - T_j$

Charging levels for the multi-layer cordons: $\{\tau_0, \tau_1, \tau_2, \tau_3, \ldots\}$

Some applications

Cordon-based congestion-charging problem

The example city



Local travel cost function: $c = 0.8 + 0.0002|\mathbf{f}|^{1.2}$

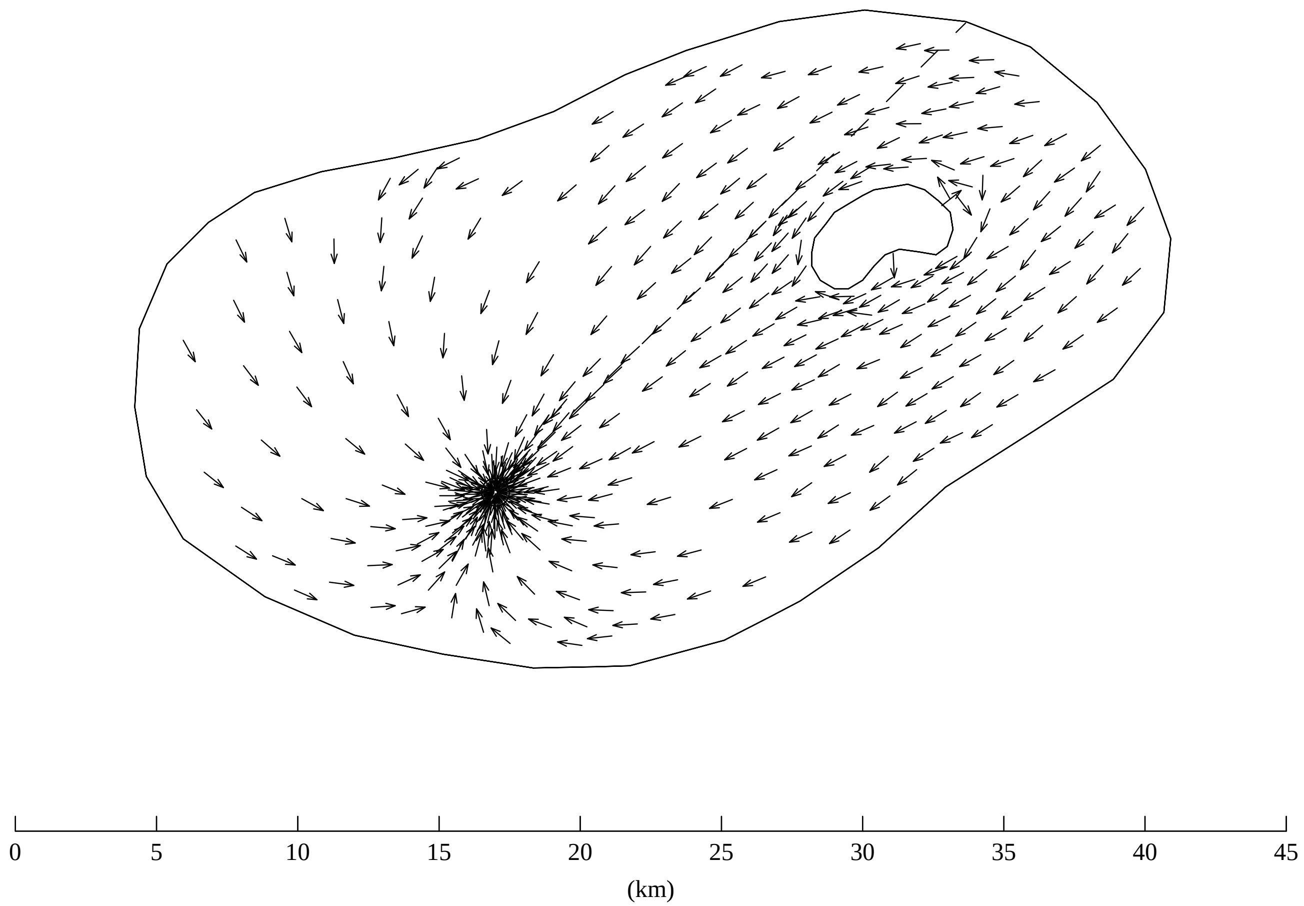
CBD travel cost function: $C_0 = 150 + 2 \times 10^{-17} Q^4$

Demand function: $D = 100e^{-0.0015u}$

Some applications

Cordon-based congestion-charging problem

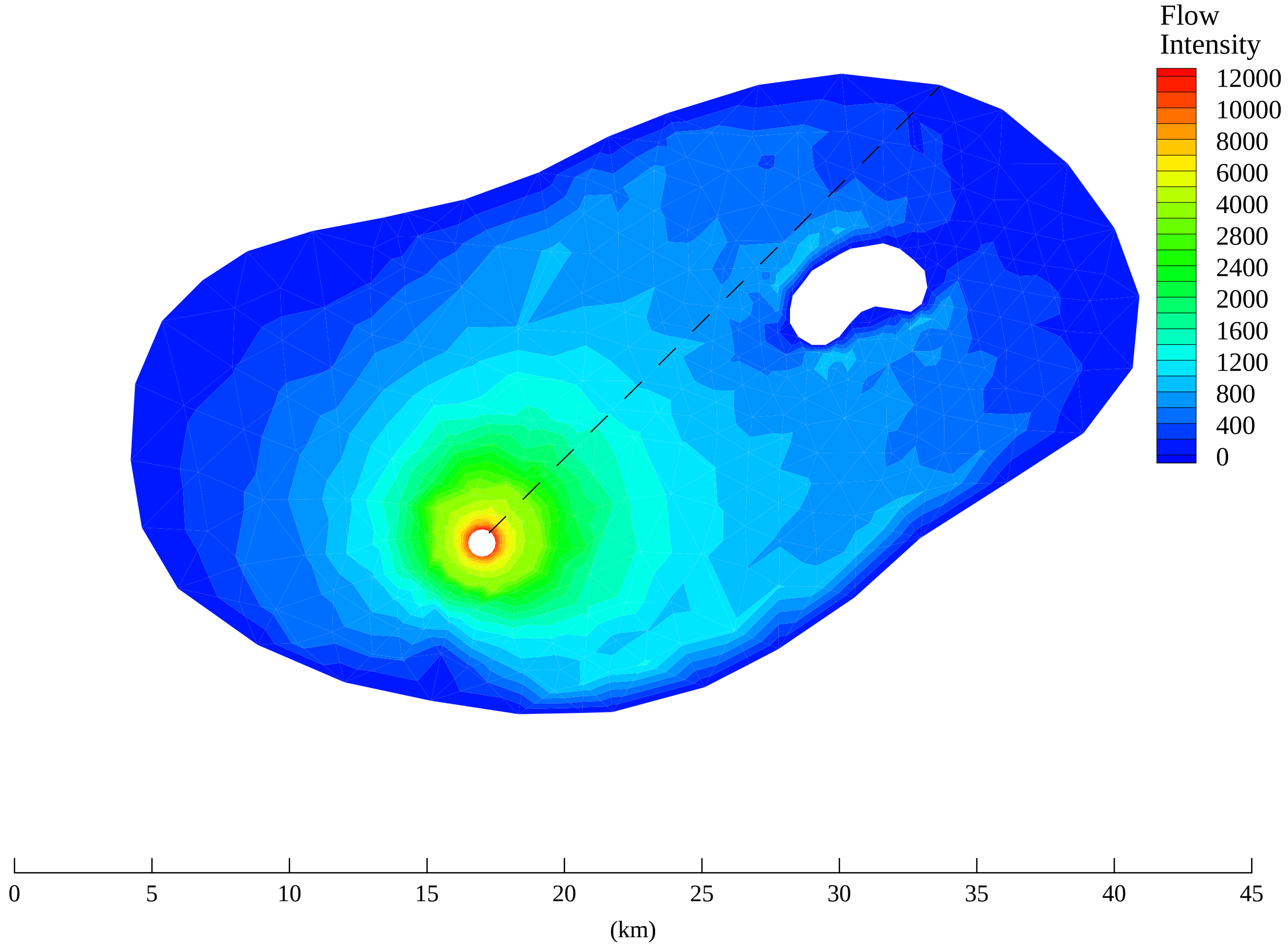
Traffic flow pattern



Some applications

Cordon-based congestion-charging problem

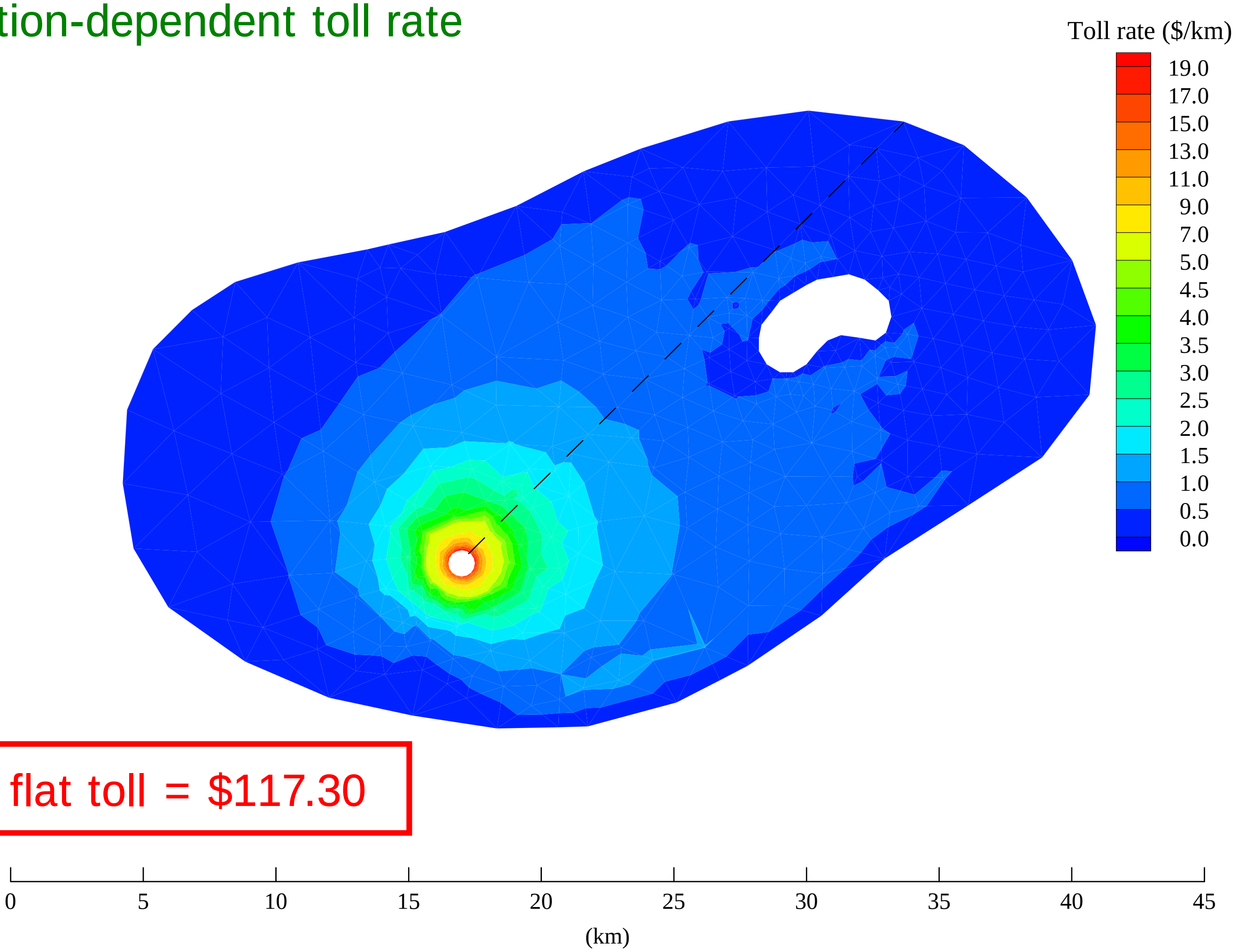
Flow intensity



Some applications

Cordon-based congestion-charging problem

Location-dependent toll rate

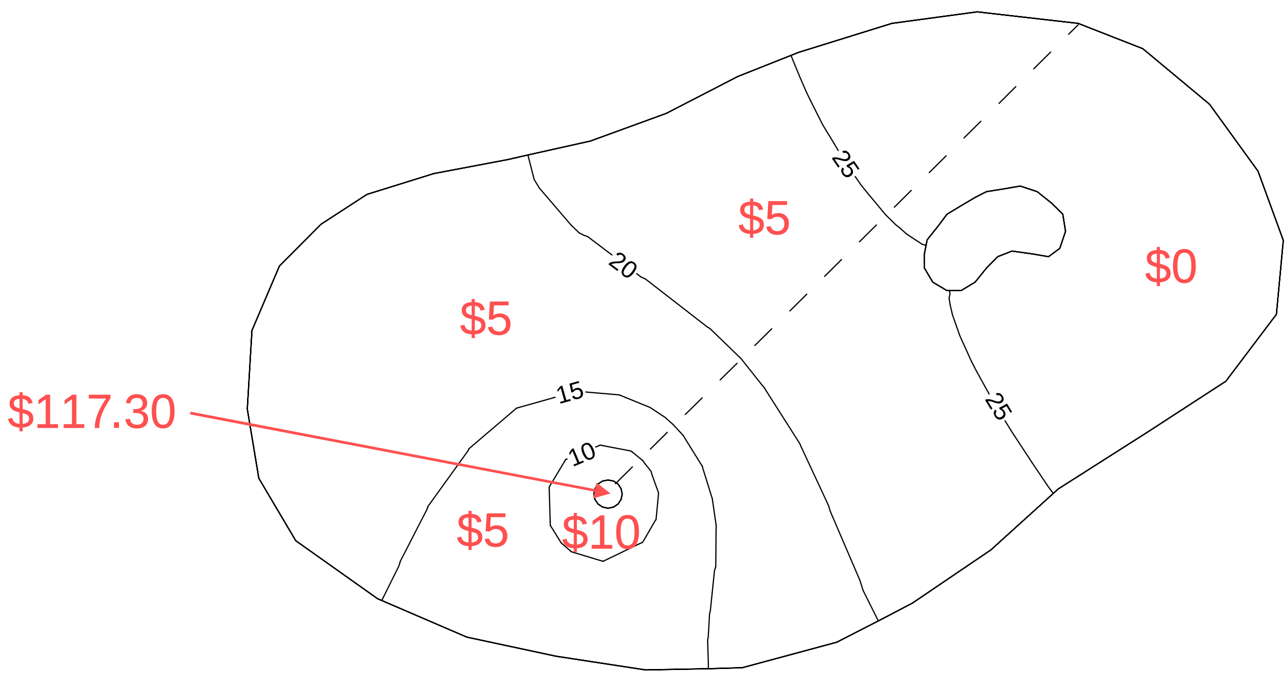


CBD flat toll = $117.30

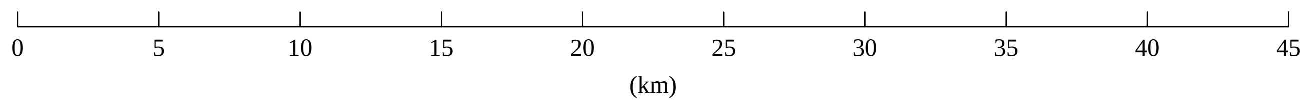
Some applications

Cordon-based congestion-charging problem

Distance-based toll paid by a user



For cordons were determined at $10, $15, $20 and $25



Some applications

Housing problem

- It is well recognized that land use and transportation systems interact strongly with each other.
- For example, if the accessibility of a place increases, residents and investors will be attracted to this place. Thus, the land use will be changed.
- Also, if a large residential estate is built, the accessibility and demand of travel will be altered.
- Arnott (1995) and Boyce and Matsson (1999) studied the interaction of the land use and transportation systems by incorporating the concept of “rent”
- This study aims to develop a continuum user equilibrium model for the land use and transportation problem.

Some applications

Housing problem

Optimum housing allocation using bi-level programming approach

For a given budget, find an **optimum housing allocation pattern** that maximizes the total utility of the system users

Upper-level model

Optimized housing provision pattern, subject to
- Maximum housing development
- Budgetary constraint
- Meeting housing needs
- Non-negativity constraint

Sensitivity information of the lower level variables at optimum point

Lower-level model

For a given housing provision pattern, find the **distributions of different classes of system users** such that all users travel in an user optimal manner

Some applications

Housing problem

Numerical example

- Total demand:

  Class 1 commuters: 60,000 units (more sensitive to rent)

  Class 2 commuters: 80,000 units (more sensitive to travel cost)

- Cost impedance functions:

  Class 1 commuters: $c_1 = 0.50v(x,y) + 0.0004v(x,y)\left(|\mathbf{f}_1| + |\mathbf{f}_2|\right)^{1.2}$

  Class 2 commuters: $c_2 = 0.75v(x,y) + 0.0006v(x,y)\left(|\mathbf{f}_1| + |\mathbf{f}_2|\right)^{1.2}$

  where $v(x,y) = 1.10 - 0.005\sqrt{(x-14)^2 + (y-20)^2}$

- Housing rent functions:

  Class 1 commuters: $h_1 = 80\left(1 + 10q/(350 - q)\right)$

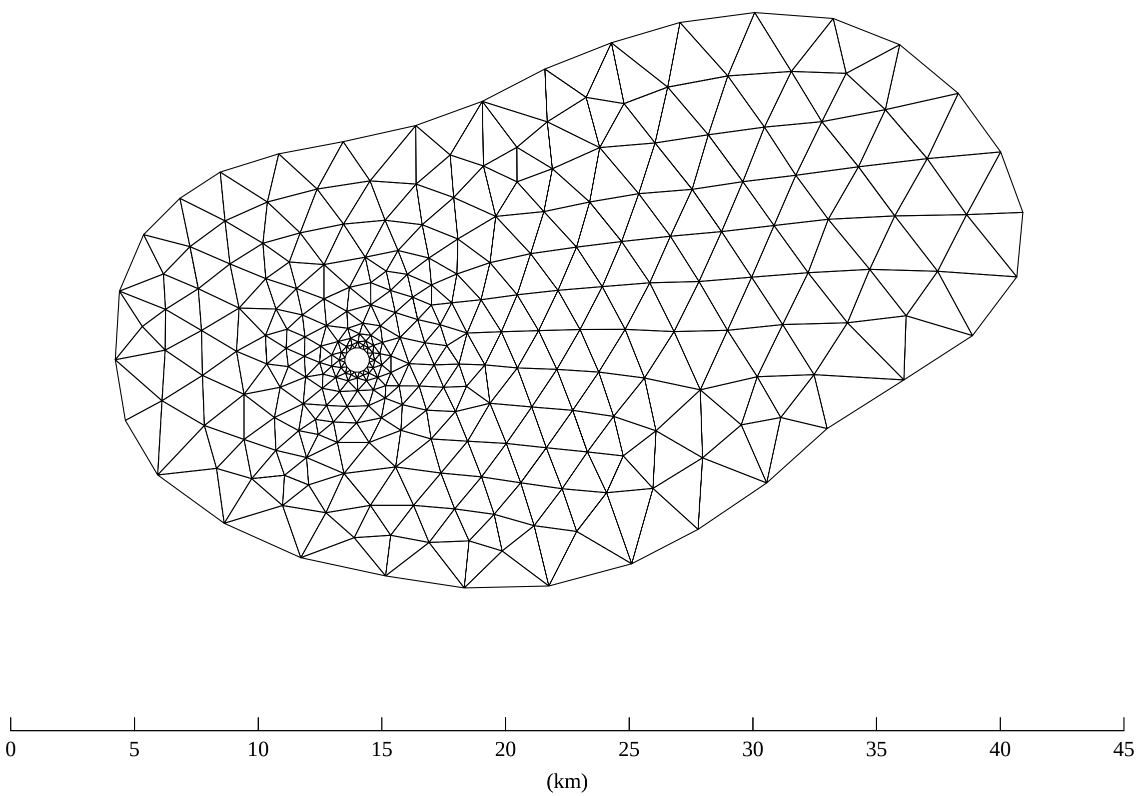
  Class 2 commuters: $h_2 = 80\left(1 + q/(350 - q)\right)$

Some applications

Housing problem

Numerical example

- The finite element mesh adopted

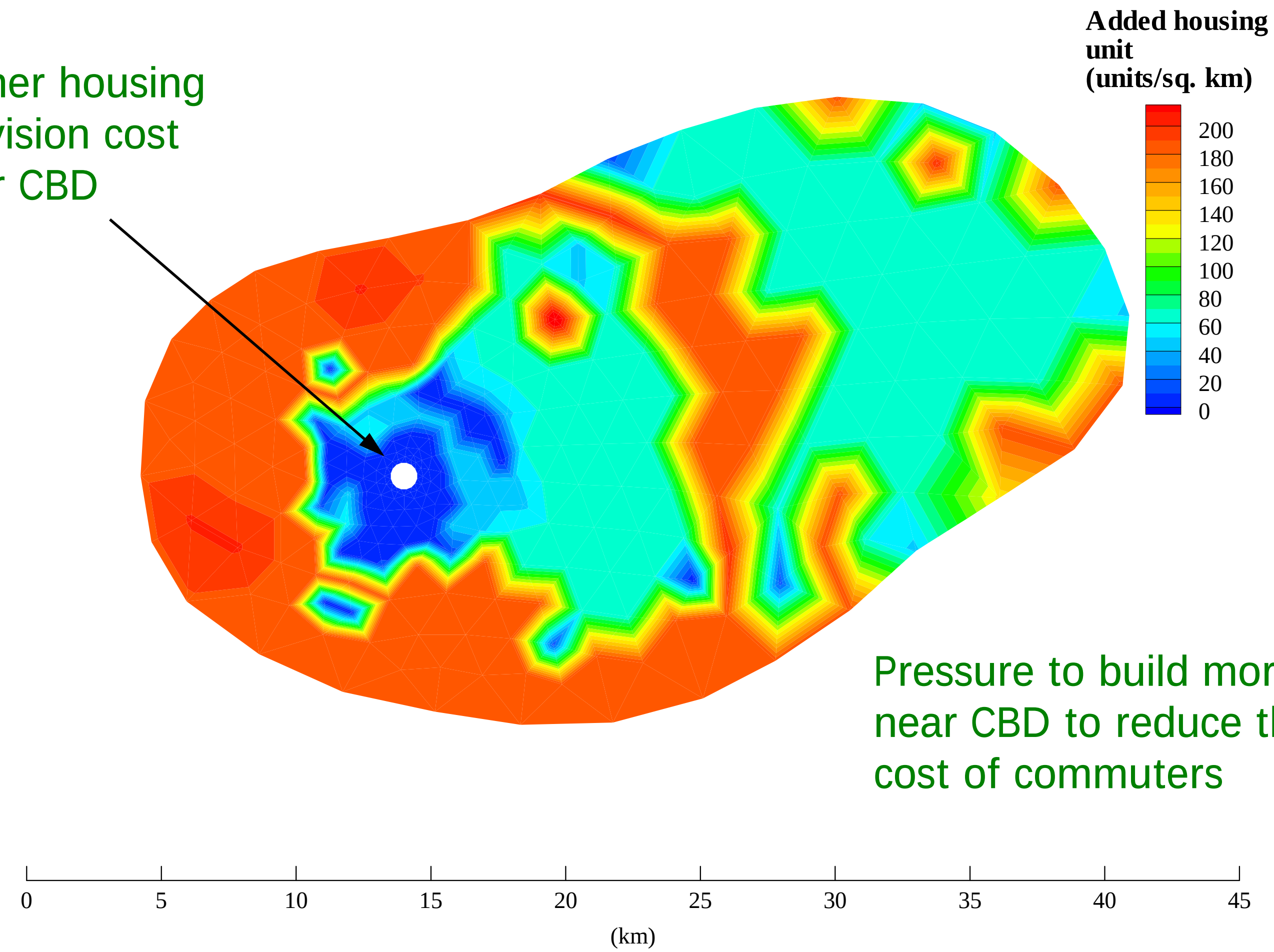


Some applications

Housing problem

Distribution of the additional housing units

Higher housing provision cost near CBD

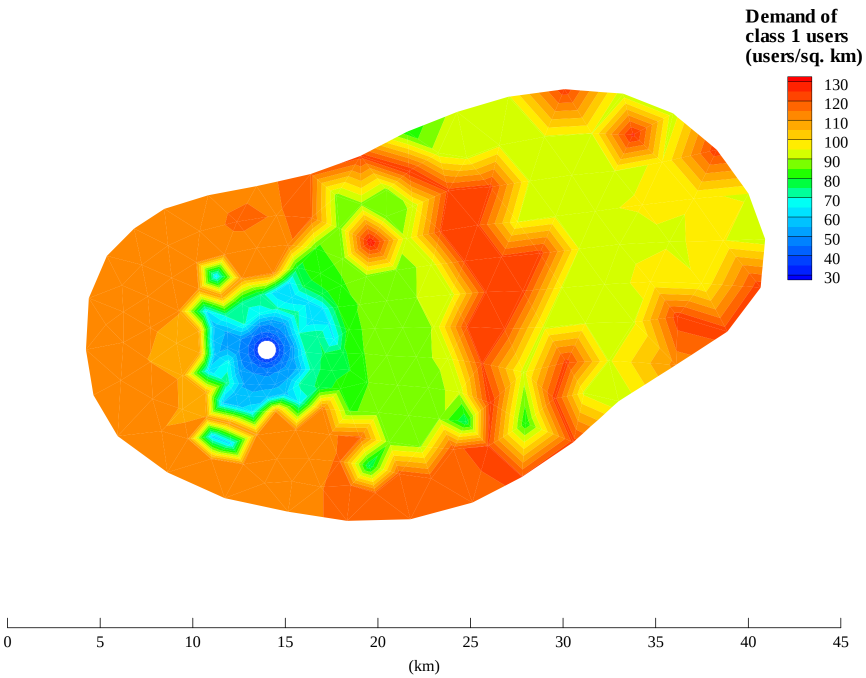


Pressure to build more housing near CBD to reduce the travel cost of commuters

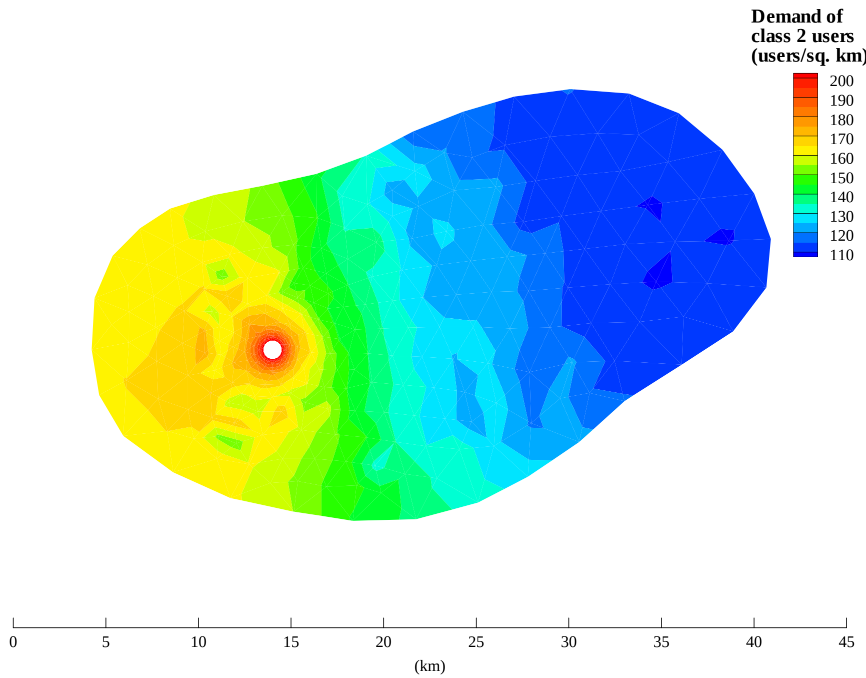
Some applications

Housing problem

Demand contour for class 1 users



Demand contour for class 2 users

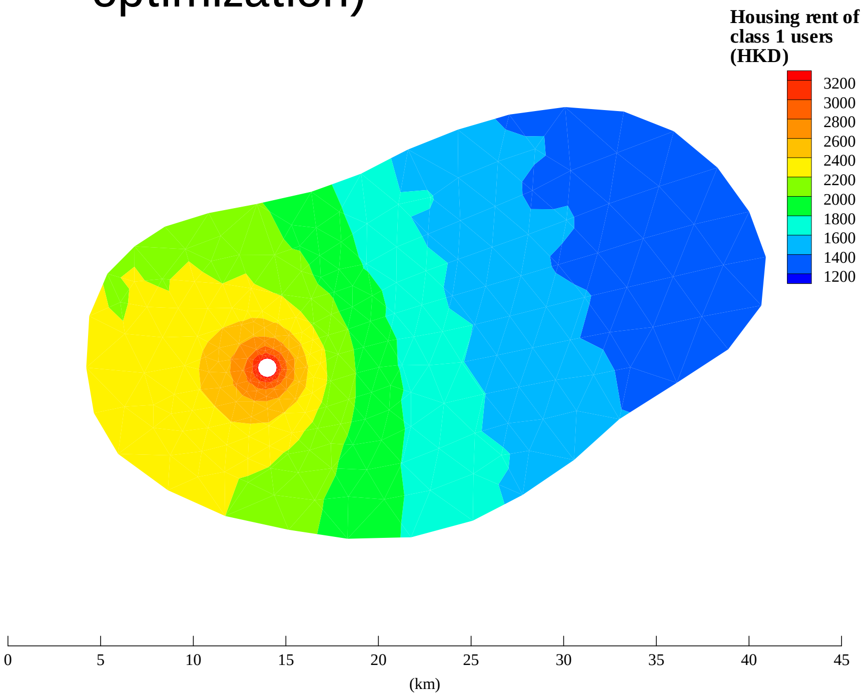


Class 1 commuters are more seriously affected as they are more sensitive to rent

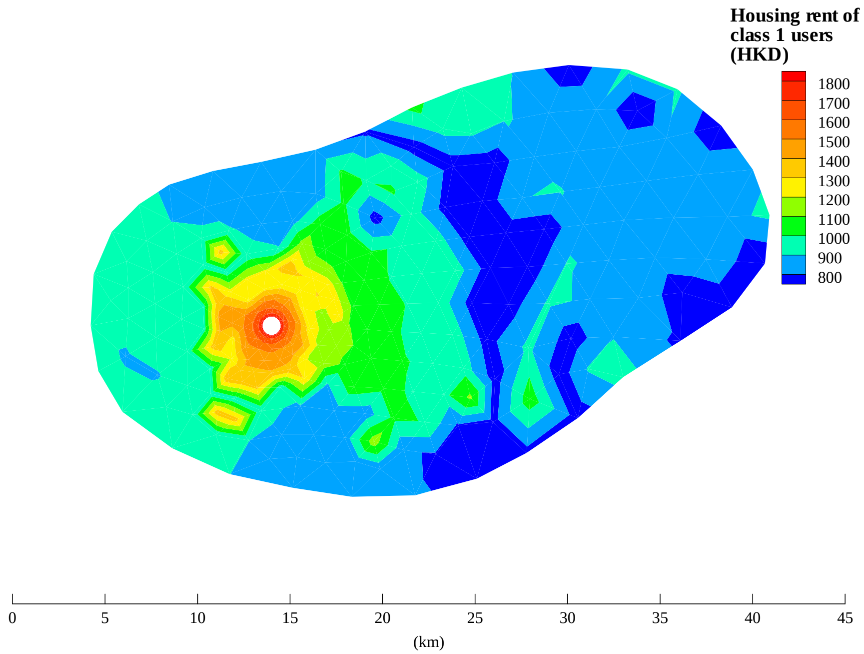
Some applications

Housing problem

Housing rent of class 1 user (before upper-level optimization)



Housing rent of class 1 user (after upper-level optimization)



Some applications

Housing problem

Conclusions and Further Works

- The continuum modeling approach to traffic equilibrium problems in an urban city has been briefly described
- The model formation and finite element solution algorithm for a typical continuum model have been discussed
- Some potential applications of this continuum approach, such as facility competition, cordon-based congestion-pricing, and housing problems, have been given
- Directions of future research
  - Extension to discrete/continuous model, in which the major freeway are modeled by discrete links, and surface streets by continuum
  - Extension to dynamic problems
  - Evaluation of environmental impacts, such as greenhouse gas and air pollutions