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Quantum Andreev effect in two-dimensional HgTe/CdTe quantum well/superconductor systems

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Recently, the topological insulator (TI), a new state of matter, has attracted a lot of theoretical and experimental attention.1–12 The TI has an insulating energy gap in the bulk states, but it has exotic gapless metallic states on its edges or surfaces. The TI is first predicted in two-dimensional (2D) systems, e.g., the graphene and HgTe/CdTe quantum well (QW).2,3 The 2D TI has the gapless helical edge states and systems, e.g., the graphene and HgTe/CdTe quantum well surfaces. The TI is first predicted in two-dimensional (2D) states, but it has exotic gapless metallic states on its edges or surfaces. The TI persists regardless of the system parameters such as the interface, the impurities, the mismatch of the density of states of the conductor and superconductor, and so on. Although the AR has been extensively investigated in various scattering mechanisms such as the contact potential of the interface. Due to the quantum Andreev effect, the conductance exhibits the plateau with the external bias being 4e2/h (2e2/h) for the two-terminal (four-terminal) TI-S hybrid system.

The Andreev reflection (AR) in 2D HgTe/CdTe quantum well-superconductor hybrid systems is studied. A quantized AR with AR coefficient equal to one is predicted, which is due to the multi-Andreev reflection near the interface of the hybrid system. Importantly, this quantized AR is not only universal, i.e., independent of any system parameters and quality of the coupling of the hybrid system, it is also robust against disorder as well. As a result of this quantum Andreev effect, the conductance exhibits a quantized plateau when the external bias is less the superconductor gap.

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is the superconductor gap and $a^\dagger_{\alpha \mathbf{k} \sigma}$ ($a_{\alpha \mathbf{k} \sigma}$) is the creation (annihilation) operators in the superconducting lead with the momentum $\mathbf{k} = (k_x, k_y)$. Here we consider the general $s$-wave superconductor. The coupling Hamiltonian $H_C$ is

$$H_C = \sum_\mathbf{k} \sum_{\alpha \beta} \sum_{\sigma} \sum_{\sigma'} a^\dagger_{\mathbf{k} \alpha \sigma} a_{\mathbf{k} \beta \sigma'} t_{S} \Psi_i + H.C.,$$

where the operator $a_{\alpha \mathbf{k} \sigma} = \sum_\mathbf{k} e^{i \mathbf{k} \cdot \mathbf{r}} a_{\alpha \mathbf{k} \sigma}$ and $t_{S} = (t_{Sa} \, t_{Sb} \, 0 \, 0 \, 0 \, 0 \, -t_{Sa} \, -t_{Sb})$. Here the parameters $t_{Sa}$ and $t_{Sb}$ are the coupling strengths between the superconductor and HgTe/CdTe QW, which depends on the interface’s contact potential and the quality of the coupling in the experiment.

By using the Green’s functions, the charge current $I_{ne}$ and spin current $I_{n}$ from the $n$th terminal of the HgTe/CdTe QW flowing into the device is $I_{ne} = e \left( I_{n\uparrow}^c + I_{n\downarrow}^c \right)$ and $I_{n} = (\hbar/2) (I_{n\uparrow} - I_{n\downarrow})$, where

$$I_{n\sigma} = \frac{1}{\hbar} \int dE \left\{ \sum_m T_{n\sigma m\sigma} (f_{n\sigma} - f_{m\sigma}) + T_{m\sigma n\sigma} (f_{n\sigma} - f_{m\sigma}) \right\}. \tag{2}$$

Here $\sigma = \downarrow$, for $\sigma = \uparrow$, $\downarrow$, $f_{n\uparrow/\downarrow}(E) = f(E \mp eV_n)$, and $f_{\downarrow}(E) = f(E)$, with $f(E)$ being the Fermi distribution function and $V_n$ being the voltage of the terminal $n$. In Eq. (2), $T_{n\sigma m\sigma}(E) = \text{Tr} \left[ \Gamma_{\alpha \mathbf{k} \sigma} G_\sigma n \Gamma_{\beta \mathbf{k} \sigma} G_\sigma m \right]$ and $T_{m\sigma n\sigma}(E) = \text{Tr} \left[ \Gamma_{\alpha \mathbf{k} \sigma} G_\sigma m \Gamma_{\beta \mathbf{k} \sigma} G_\sigma n \right]$ are, respectively, the normal transmission coefficient from the terminal $n$ to the terminal $m$ and to the superconductor terminal, and $T_{n\sigma m\sigma}^A(E) = \text{Tr} \left[ \Gamma_{\alpha \mathbf{k} \sigma} G_\sigma n \Gamma_{\beta \mathbf{k} \sigma} G_\sigma m \right]$ is the AR coefficient with the incident electron from the terminal $n$ and the reflected hole going to the terminal $m$. The linewidth function $\Gamma_{\alpha \mathbf{k} \sigma}(E) = i \left[ \Sigma''_{\alpha \mathbf{k} \sigma} - \left( \Sigma'_{\alpha \mathbf{k} \sigma} \right)^2 \right]$ and the Green’s functions $G^\dagger(E)$ can be calculated from $G^\dagger(E) = \left[ G^\dagger(E) \right]^\dagger = \left[ E - H_{\text{scat}} - \Sigma'_\alpha - \Sigma''_\alpha \right]^{-1}$, where $H_{\text{scat}}$ is the Hamiltonian of the scattering region as shown in Fig. 1 (dotted region). The self-energies functions $\Sigma'_\alpha(E)$ and $\Sigma''_\alpha(E)$ due to terminals of the TI and superconducting lead can be calculated as in Refs. [17–20]. In the following numerical calculations, we choose the parameters from the realistic materials\(^2\):

1. The HgTe/CdTe QW’s parameters are $A = 364.5$ meV nm, $B = -686$ meV nm\(^2\), $C = 0$, and $D = -512$ meV nm\(^2\).
2. The superconductor’s parameters are the gap energy $\Delta = 1$ meV. The lattice constant $a$ is set to 5 nm and the TI-S coupling strengths are taken $t_{Sa} = t_{Sb} = t$.

We first study the two-terminal system. Due to the time-reversal symmetry and the $C_2$ symmetry around the $y$ axis, the AR coefficients have the properties $T_{\uparrow\downarrow, \downarrow\uparrow}^A(E) = T_{\downarrow\uparrow, \uparrow\downarrow}^A(E) \equiv T_A^A(E)$ and $T_{\downarrow\downarrow, \uparrow\uparrow}^A(E) = T_{\uparrow\uparrow, \downarrow\downarrow}^A(E) \equiv T_A^\downarrow(E)$.

**FIG. 1.** (Color online) (a) and (b) are the schematic diagram for the two-terminal and four-terminal devices.
helical edge states, in which the spin-up and spin-down carriers move along the edge of the HgTe/CdTe QW in clockwise and counterclockwise directions, respectively.\textsuperscript{1,3} We consider the case of the spin-up electron coming from terminal-1 when the energy $|E| < \Delta$. As shown in Fig. 1(b), two reflection processes occur at the TI-S interface: (1) It can be Andreev reflected back as a spin-down hole to the same terminal which will contribute to $T_{1,1}^{\uparrow}$. (2) It can also be normal reflected as an electron (spin up) along the TI-S interface, eventually to terminal-3. Note that the normal reflection as an electron back to terminal-1 is prohibited by the time-reversal invariance and the helical edge states being a pair of Kramer states. The reflected electron traveling to terminal-3 has to go along the TI-S interface since the only available transmission channel is the edge state. This results in a reflection again at the TI-S interface where part of the electron is Andreev reflected as the hole back to terminal-1 and the rest is normal reflected as electron toward terminal-3. As this continues, multireflections occur as normal electron traverses along the TI-S interface and eventually the transmission probability $T_{1,1}^{\uparrow}$ becomes zero if the TI-S interface is long enough. Clearly, it is this multi-AR that eventually the transmission probability $T_{1,1}^{\uparrow}$ and $T_{1,1}^{\downarrow}$ has the quantized plateau similar to $T_{1,1}^{\uparrow}$. For $\Delta < E_F$, $T_{1,1}^{\uparrow} = 1$. Furthermore, we have three observations: (i) If the energy $|E| > \Delta$, all AR coefficients decrease as usual (see Fig. 2(d)),\textsuperscript{21} because of the occurrence of the normal tunneling from TI to the superconductor. (ii) When $E_F$ is out of the bulk gap, all AR coefficients have the same behavior: AR coefficient for each transmission channel is in general small and strongly depends on $E_F$ and $E$ (see Figs. 2(c), 2(d), and 2(e)). (iii) For the spin-down incident carrier, it is easy to show that $T_{1,1}^{\downarrow}$ has the quantized plateau similar to $T_{1,1}^{\uparrow}$.

Next, we study how the quantized AR is affected by the system parameters. Figure 3 shows the AR coefficient $T_{1,1}^{\uparrow}$ versus the width $N$ of the HgTe/CdTe QW ribbon and the coupling strength $t$. The results show that the quantization of AR persists for a broad range of the coupling strength $t$. In addition, the wider the width $N$, the broader the quantization plateau is. For $N = 1000$ nm, the AR quantization plateau can sustain when $t$ varies nearly one order of magnitude. Finally, with the increase of the width $N$, the AR coefficient $T_{1,1}^{\uparrow}$ rises monotonously before it reaches quantized value due to the fact that the incident electron has more chance of multi-AR for the longer TI-S interface. These results show the universal feature of the quantum Andreev effect: it is independent of the system parameters.

Is the quantized AR robust against the disorder? To answer this question, we consider the on-site Anderson disorder in a region near the TI-S interface [see the light gray (red) region in Fig. 1(b)]. Because of the disorder, an extra on-site term $\Psi_i \bar{\Psi}_i \Psi_i$ is added on each site $i$ in the disorder region, where $\Psi_i$ is the $4 \times 4$ diagonal matrix with diagonal elements ($w_i, w_i, -w_i, -w_i$). $w_i$ is assumed uniformly distributed in the range $[-W/2, W/2]$ with the disorder strength $W$. Figure 4 shows the AR coefficients $T_{1,1}^{\uparrow}(t)$, and its fluctuation versus $E_F$ and $E$. The results show that the quantized AR plateau in $T_{1,1}^{\uparrow}$ is very robust: the AR plateau can persist and its fluctuation $\text{rms}(T_{1,1}^{\uparrow})$ is zero for the disorder strength $W$ up to 100 meV because of the helical edge states being very robust. Upon further increasing of $W$ from 100 meV, $T_{1,1}^{\uparrow}$ starts to decrease and the fluctuation becomes nonzero because at large disorders the system reaches the diffusive regime and the helical edge states are destroyed. Hence, as long as the edge state is survived, disorder has no effect on the quantum Andreev effect.

Let us investigate the conductance $G_{nc}$ ($G_{nc} \equiv d I_{nc}/d V$) and spin conductance $G_{ns}$ ($G_{ns} \equiv d I_{ns}/d V$). We set the biases of the HgTe/CdTe QW terminals, $V_1 = V_2 = V_3 \equiv V$, and the superconductor-terminal bias $V_4 = 0$. Figure 5(a) shows the conductance $G_{nc}$ and spin conductance $G_{ns}$ versus the bias $V$ for the four-terminal system. When $E_F$ is inside the bulk gap, $G_{nc}$ (in the unit $e^2/h$) is exactly equal to $G_{ns}$ (in the unit $e^2/4\pi$) since only the spin-up electron traverses from the terminal-1 to the TI-S interface with the spin-down hole Andreev reflected back. In particular, when $V < \Delta/e$, a (spin) conductance plateau emerges with the plateau value $2e^2/h$ ($e^2/2\pi$) because of the quantized AR with $T_{1,1}^{\uparrow} = 1$. Here we emphasize that like the quantized AR this conductance plateau is also universal (i.e., it is independent of the system parameters and the quality of TI-S coupling) and robust against the disorder. Since the TI phase in HgTe/CdTe QW has been realized experimentally,\textsuperscript{7,8} this predicted quantum Andreev effect with quantized conductance plateau should
Other parameters in (a)–(d) are the same as Fig. 2. (b) is the conductance $G_e$ vs. the bias $V$ for different $E_F$ [with the legend being the same in (b)] for the four-terminal device. (c) and (d) are the conductance $G_{sr}$ vs. the bias $V$ for the two-terminal device. (c) and (d) are $T_{11}^\uparrow\downarrow$ vs. $E_F$ (c) and conductance $G_{1e}$ vs. bias $V$ (d) for the four-terminal device with the positive $M = 2$ meV. The other parameters in (a)–(d) are the same as Fig. 2.

not be difficult to observe using the present technology. On the other hand, when $E_F$ is out of the bulk gap, both $G_{1e}$ and $G_{sr}$ are not equal and they are sensitive to the system parameters. Finally, we comment on following two points: (i) If HgTe/CdTe QW is in the normal state (i.e., $M > 0$), no plateau emerges for both the AR coefficient $T_{11}^\uparrow\downarrow$ and conductance $G_{nr}$ (see Figs. 5(c) and 5(d)), similar to the ordinary conductor-superconductor hybrid system. (ii) For the two-terminal device with the HgTe/CdTe QW in TI regime, the conductance $G_e$ also exhibits the quantum Andreev effect with quantized AR conductance $4e^2/h$ at $V < \Delta/e$ (see Fig. 5(b)). However, the spin conductance $G_{sr}$ is exactly zero due to the fact that the left and right edges of the HgTe/CdTe QW ribbon carry the same current but opposite spin current.

In summary, we predict a quantum Andreev effect in the 2D TI-S hybrid system, in which the AR coefficient is quantized with the value one. Importantly, the quantized AR plateau is independent of various system parameters and the quality of TI-S coupling, and it is also robust against the disorders. Due to the excellent properties of the quantum Andreev effect and the 2D TI having experimentally been realized in HgTe/CdTe quantum well, so it should be easy to observe it using the present technology. In addition, due to the quantized AR, the conductance and spin conductance versus the bias also exhibit the quantized plateau when the bias is within the superconductor gap.

Recently, we noticed that the perfect AR with the AR coefficient $T_{11}^\uparrow\downarrow = 1$ is also addressed by Adroguer et al. using the one-dimensional model. Here we consider the two-dimensional TI-S device, the full band structures of the TI and superconductor are involved, and in particular we point out that the AR plateau is universal and robust in contrast to the conclusion of Ref. 22.

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20In fact, from the continuous effective Hamiltonian $H_{11}$ and using the Nambu representation, this problem can also be solved. For example, one can obtain the transmission and AR coefficients by solving the scattering wave function in the real space with the matching boundary conditions, as done in Ref. 22 and Q.-F. Sun and X. C. Xie, Phys. Rev. B 71, 155321 (2005). No matter which methods are applied, the results are the same, because the present system can exactly be solved.