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Modeling and Analysis of Supply Chain Risk System under the Influence of Partners' Collaboration

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Abstract

Confining the focus to individual enterprise, traditional risk management literature seems to echo inadequately in the context of collaborative supply chain, partially due to its exclusion of correlation of risks across companies. In this study, we refine the notion of risk in supply chain and propose a model of supply chain risk system (SCRS) that consciously takes into account the correlation among risks resulting from partners' collaboration. Through analytical inference we found that the level of collaboration contributes to the resilience of supply chain. It implies that collaboration can positively affect the topology of SCRS, thus benefiting supply chain operations in terms of risk management. A simulation program has been developed with aim to demonstrate the practical feasibility of the proposed model. Implemented in simulation, two sets of experiments have been conducted for testing the model in actual business scenarios. The experimental results provide supporting evidence to consolidate the analytical findings.

1. Introduction

The first decade of 21st century has witnessed the dramatic growth of supply chain in which a consolidating linkage through a series of processes, from raw material preparation to manufacturing, transportation, storage, and eventually to delivery of services or products to the end consumers, ties up the parties involved with an interest chain so close that supply chain companies are correlated in success and failure in some sense [1]. In such highly interactive business model, the relationship among the parties is rather delicate. Despite conflicts surely existing, every now and then companies in supply chain are inclined to collaborate with each other albeit it is for their own sake, e.g. for financial robustness, strategic advancement or operational excellence [2]. As collaboration goes along, companies may benefit from working with each other. They may seek to expand their collaboration both in breadth and in depth, and hence “partnership”, a much more closed interorganizational relationship, will be established, in which partners are coordinated to behave as a whole and they share not only their profits but also the risks [3]. The impact of partnership is tremendous. The literature has shown a great amount of evidence that it is a critical driving force towards an effective supply chain [4, 5]. While most of studies are focusing on the direct impact of partnership upon the effectiveness and efficiency of the supply chain performance, little attention is paid on the issues of supply chain risks emerging due to such intimate relationship [6]. In fact, under partnership the risk of an individual company is not just the risk of itself and it actually the risk of all the partners. For instance, if the upstream supplier broke down because of a strike occurring in workplace, it would probably jeopardize the operations of the manufacturer in the middle stream and further affect the sales of the retailers in the downstream. In other words, the risk of a company is no longer limited within the boundary of enterprise. Partnership has been expanding the scope of risk management from the enterprise level to the supply chain level. Such a significant change upon risk management under the effect of partnership, we believe, is truly worth to be investigated.

Given the context of collaborative supply chain, risks among partners are linked with each other, which however is seldom considered in traditional risk management research. In attempt to echo the correlation of risks resulting from the effect of partnership and to construct more accurate notion upon supply chain risks, this paper draws from the theories of system science and the literatures of risk management to propose a model of supply chain risk system (SCRS). SCRS organizes supply chain risks in a visual network according to their causal relationships and reflects the partnership effect delicately by the topological distribution of risk nodes and the chain influencing process. Through modeling and simulation, our analysis on SCRS reveals that the partners’ collaboration can significantly contribute to the resilience of supply chain. This offers a novel explanation from the perspective of risk management for why nowadays increasing number of companies is consciously engaged in partnership in supply chain. The rest of this paper is organized as follows: section 2 is devoted to the literature review on supply chain risk management, with intention to identify the key issues as well as the research gap in the existing literature. In section 3, the proposed model of SCRS is presented and the impact of partnership upon supply
chain in terms of topological structure of SCRS is analyzed through analytical inference. The verification experiments and the related results are disclosed in section 4. The main conclusions of the paper are summarized in section 5.

2. Managing risks in supply chain: major issues in the literature

Today, managing risks within the enterprise boundary is far from enough [7]. Companies begin to notice that a number of risks impacting upon them may not exist in the companies but come from their supply chain partners. Therefore, while continuously improving ERM (enterprise risk management) within the companies, they gradually pay more attention upon the risks in supply chain. Supply chain risk management (SCRM), compared to ERM, no doubt is a relatively new discipline. SCRM is more interested in the coordination and collaboration of processes and activities across functions within a network of companies, with an intention to ensure that the supply chain is performing effectively and efficiently as normal [8]. To copy with the uncertainty from the supply side and the demand side, upstream risk management and downstream risk management have emerged, forming two major research areas in SCRM.

Regarding upstream risk management, the main issues include supply network design, supplier relationship management, supplier order allocation and contract management. Helper (1991) reported that supplier relationship in U.S. has experienced a great shift from a common market transaction relationship to a long-term cooperative partnership in 1990s [9]. Dyer and Ouchi (1996) confirmed the similar situation happened in Japan also [10]. Spekman and Davis (2004) pointed out that interdependency resulting from partnership is the main source of risks in the supply side, but it is not unmanageable [11]. Zsidins (2003) provided some useful suggestions to mitigate the upstream risks, including: 1) establishing buffer stock and enhancing inventory management; 2) avoiding single sourcing and using alternative sources of supply; 3) utilizing supply contracts to manage the fluctuation in sourcing price; and 4) adopting quality initiatives to implement comprehensive quality management [12].

Normally in supply chain, the supply capacity is fixed in most cases whereas the demand is changing all the time. Given that, companies are required to apply various downstream risk management strategies to lower the probability of mismatch between the static constant supply and the dynamic uncertain demand. The literature is abundant in this area. Carr and Lovejoy (2000) developed a mathematical model to identify an optimal portfolio of demand distributions [13], and Van Mieghem and Dada (2001) from an economic view made an in-depth analysis on how to through dynamic pricing make an appropriate response to the unstable demand [14]. Regarding the phenomena that companies set different price at different time, the literature explained this as a strategy of risk mitigation. Pricing higher in peak seasons is actually to shift demand to off seasons rather than to grasp high profit margin from the customers [15]. It is found that the pricing mechanism is not only able to shift the demand across the time but also to shift the demand across the products. Chod and Rudi (2005) showed us a vivid case in which a higher profit can be obtained through adopting differential pricing strategy to entice customers to shift the demand from one product to another [16].

In the context of supply chain, physical experimentations, suffering from technical and cost related limitations, are difficult to implement in risk research [17]. Simulation is considered to be an effective method to model and analyze risk cases of large scale. There are a number of good studies of SCRM using simulation in the literature. E.g. Swaminathan et al (1998). It adopted agent-based simulation to model supply chain dynamics and provided a rapid decision supporting tool for risk management [18]. Deleris and Erhun (2005) presented in their paper a Monte Carlo simulation with which different levels of risks in supply chain can be analyzed sophisticatedly [19]. Jain and Leong (2005) viewed supply chain as a complex system and demonstrated the advantages of constructing supply chain simulations to “stress test” the system and investigate its performance under risk situations [20].

After a brief literature review above, we are aware that the previous literature more or less has propensity to focus SCRM upon discrete sectors, mainly confined in either upstream or downstream risk management. Most of research is concentrating upon solely one area, and there are few studies that provide an integrated view of the whole supply chain from risk management perspective. Furthermore, the correlation among the risks of the partners is few discussed in the literature. Such gap provides a good research opportunity for our study. One of the goals of this research is to bridge this gap through investigating supply chain risks as a system and reflecting the correlation effect among the risks in supply chain. Considering the scale as well as the constraints of this study, simulation is chosen as the research methodology, for its fitness to SCRM studies suggested by the literature.
3. Conceptual development and model building

3.1. Modeling individual supply chain risk

What is a risk? How to conceptualize it? Those are the fundamental problems every study of risk has to confront. Although countless answers subjected to various perceptions are conferred by the literature, their essence is never out of the below four interpretations, including viewing risk as hazard [21], as probability [22], as consequence [23], and as potential adversity or threat [24].

No matter hazard, probability or potential threat, they emphasized that risk should be uncertain, which will happen some time and will not happen other time. It is believed that there exists a law of probability governing its occurrence [25]. Once a risk occurs, it will result in some negative consequence, such as hazard or adversity. Most of the times, the reasons behind causing the risk are attracting people’s attention, and people are unstoppably trying every means to seek for the origin of the risk. But it at the end always turns out to be a tantalizing conundrum when traced back to the root of the reasons. The origin seems like a black box unknown or too complicated to be known for people, thus scholars give it a name “uncertainty”. Although there used to be a period in which academia was confused by “does uncertain produce risk or risk begets uncertain?”, nowadays scholars have reached a consensus that it is uncertainty that leads to risk [22].

The discussion above uncovers an important fact that risk is not a simple object, in other words, it is a system made up of components [26]. Kaplan (1997) proposed that risk can be expressed by a triplet with the combination of scenario, probability and consequence[27]. Inheriting the similar idea, we believe that risk at least comprises three essential parts: 1) the drivers that generate the risk; 2) an event with probability that signifies the occurrence of risk; and 3) the consequence brought by the risk. By taking into account the three essential components of risk and incorporating the conception of correlation among risks, we propose a model named Risk with Casual Relationship (RCR) to depict an individual risk. It is visualized in Fig.1.

![Fig.1 A model of Risk with Casual Relationship (RCR)](image)

The model RCR provides an understanding of risk with causal relationship. It portrays the process of causation rather than giving a static definition of an individual risk. Keeping in line with traditional interpretations of risk, RCR involves “drivers”, “event” and “consequences” in the conception. It conceptualizes risk as an event that occurs in probability, which is denoted as a circle in the middle (See Fig.1). However, an isolated circle itself cannot be called risk. It is qualified as a true risk only and only if it happens with reasons (those refer to drivers) and consequences (the consequences should be negative). Thereby, to identify a risk, we cannot make judgment sheer based on whether it is an event with probability only. Like a customer visiting a supermarket, nobody will treat it as a risk despite that it happens stochastically. A true risk is able to cause negative consequences. Moreover, the occurrence of such event requires drivers which are the antecedents that cause the risk. Aware of the correlation among the risks, we infer that the antecedents should not include just the factors that lead to the risk event, and it is also possible that some risks which happened antecedently would trigger a ripple effect and drive other risk event to happen. Given that, drivers we propose can be factors or risks:

\[\text{Driver (D)} = \text{Factor (F)} \mid \text{Risk (R)}\]

Factors in RCR mainly refer to the causes uncontrollable or immeasurable. They can be perceived abstractly, and it is difficult to confirm their occurrence with a specific indicator. The major difference between a factor and a commencing risk lies in that a commencing risk happens in probability while a factor is similar to uncertainty that is nearly impossible to quantified, and even though it can be quantified to some extent, it nearly cannot be changed. A typical example for “factor” is political climate. Political climate or political situation is uncontrollable to supply chain companies, and an unstable political situation is able to trigger a number of risks, e.g. a strike risk in the manufacturing site, which may cause serious disruption in supply chain. Nevertheless, companies can do very little to political situation, and in most cases they have to accept the factor and adapt to the situation.

Consequences in RCR are referred to the negative result of a risk. They can be a direct harmful impact quantified financially or another consequent risks being triggered which may cause other bigger loss in the later time:

\[\text{Consequence (C)} = \text{Impact (I)} \mid \text{Risk (R)}\]

Impacts in RCR are referred to the ultimate and specific loss resulting from the risk, which can be measured quantitatively.
In a short summary, RCR defines risk as a stochastic event with causes and effects. The occurrence of such event requires drivers which can be some factors or commencing risks. When the drivers are active (or say the conditions to trigger the risk event are satisfied), it does not mean the risk will happen definitely but just implies that the probability of risk to occur is extremely high. Some risks may cause economical damage in a direct manner while others may result in tremendous harms indirectly through triggering other risks in a consequence.

3.2. Modeling supply chain risk system

When we link up the risks according to their causal relationships, those will constitute a great network with intensive connections among the items in it. Fig 2 illustrates a visualized picture of this network for us.

Excluding the factor nodes as well as the impact nodes, the region inside the dash line composes a risk causality network (RCN). RCN is constructed on the basis of a number of individual risks under the model of RCR. To secure the causality in between, we suggest that the risks involved be at the same level. Up to specific application, RCN can be aggregated to a more strategic level or be drilled down to a more operational level. In fact, its organization is not arbitrary and there are 3 rules, we are aware of, governing the construction of RCN:

1. The initial risk node in RCN must be triggered solely by factor(s);
2. The RCN should end with the risk nodes that result in negative impacts only.
3. The causality among risks implies the time sequence, and thereby the antecedent risk must happen before the triggered risk. The property of time sequence determines that the RCN must be an acyclic graph.

RCR and RCN are two fundamental concepts that assist us in modeling risks in supply chain. Though conceived in a general level that ignored the application context, they have been conceptualized with the key features of supply chain risks. The correlation of risks in supply chain is reflected fully in the casual relationship both in RCR and in RCN. RCR provides an appropriate template for supply chain risks modeling, and supply chain risks can easily be specified following the conception of RCR. TABLE 1 shows a list of supply chain risks in the process of sourcing. It demonstrates a well fitness of applying RCR in the context of supply chain. Similarly, the supply chain risk system (SCRS) can be engendered by a specific application of RCN in supply chain (Please see Fig. 3).

<table>
<thead>
<tr>
<th>Risk Name</th>
<th>Drivers</th>
<th>Event Indicator</th>
<th>Consequences</th>
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<tbody>
<tr>
<td>Supply disruption risk</td>
<td>- Unstable political climate(F) - Payment delay (R)</td>
<td>Supplier terminates the supply</td>
<td>- Damaging the trust in partnership (I)</td>
</tr>
<tr>
<td>[28, 29]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Supplier commitment risk</td>
<td>- Competitor seducing supplier to jump out of the contract (F) - Supply disruption risk (R) - Payment delay (R)</td>
<td>Supplier terminates the supply contract</td>
<td>- Manufacturing stoppage risk (R) - Material inventory shortage risk (R)</td>
</tr>
<tr>
<td>[28, 29]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long lead time risk</td>
<td>- Supplier transportation risk (R) - Offshore procurement risk (R)</td>
<td>The lead time runs out of the safety range</td>
<td>- Material inventory shortage risk (R) - Manufacturing stoppage risk (R)</td>
</tr>
<tr>
<td>[12, 30]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material inventory shortage risk</td>
<td>- Long lead time risk (R)</td>
<td>The material inventory level is lower than the safety stock level</td>
<td>Manufacturing stoppage risk (R)</td>
</tr>
<tr>
<td>[12, 30]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Payment delay</td>
<td>- Cash in short (F) - Problems with the accounting system (F)</td>
<td>Late pay to the supplier</td>
<td>- Supply disruption risk(R) - Damaging the trust in partnership (I)</td>
</tr>
<tr>
<td>[30, 31]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Offshore procurement risk</td>
<td>- Low cost of procurement (F)</td>
<td>Purchase the raw material from offshore suppliers</td>
<td>- Long lead time risk (R)</td>
</tr>
<tr>
<td>[31]</td>
<td></td>
<td></td>
<td></td>
</tr>
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Note: F: factor; R: risk; I: impact

TABLE 1: A List of Supply Chain Risks in the Process of Sourcing

Fig.2 Risk Causality Network (RCN)

Fig.3 Supply Chain Risk System (SCRS)
3.3. Measuring the supply chain operation. In contrary to robustness, negative impact of risk will cause very little harm to the robust supply chain is able to cope with it so that the supply chain operation is not affected. Facing risk, a supply chain must have the ability to resist the impact of risk and retain the same stable situation as it had before the occurrence of risk [33]. Robustness and resilience are two key concepts in SCRM. To large extent, they indicate the performance of risk management in supply chain. Robustness of supply chain is referred to ability of supply chain to adapt to the new change brought by risks very quickly and then operate in a stable manner again. In this study, our interest is in measuring the resilience of supply chain using the model of SCRS.

In the model of SCRS, risk node has two states: one is “healthy”, indicating that the risk is inactive and not triggered; the other is “infected”, indicating that the risk is active and has been triggered by some factors or other risks. At each time point t, the healthy risk node is possible to be infected, which is subjected to its probability to be triggered. If its antecedent risk nodes are not infected, it will be triggered by its marginal probability; but if some of its antecedent risk nodes have been infected, it will be triggered by its conditional probability which will be much higher than its marginal probability. In case that some risk nodes have been infected already, the supply chain companies concerned will take actions, trying their best to recover the risk nodes back to the healthy state. The attempt may succeed or fail, subjected to the recovery rate which measures how likely a risk node can be restored from a chaos back to normality. Generally, the higher the recovery rate is, the more quickly the risk node is able to return to health. Given that, the recovery rate in SCRS can be a good indicator to measure the resilience of supply chain.

In fact, the improvement of the recovery rate comes at cost. It requires supply chain companies to devote significant investment in risk analysis and the development of mitigation approach. However, in reality few companies are able to afford such huge investment in SCRM whereas they know that it is truly crucial for them to maintain an acceptable level of the recovery rate otherwise the business may suffer huge loss. In SCRS, we are aware that there must exist a minimum threshold for the recovery rate. If the recovery rate fails to catch up with the threshold value, there will be a persistent chaos in the supply chain in which the healthy risk nodes will be triggered by conditional probability rather than marginal probability. Under this circumstance, the chain effect of risk infection will continue and never die out because the recovery rate is too low to suppress the spawning chaos. If this were the case, it would be a catastrophe to the whole supply chain.

Below, we would like to use an analytical model based on SCRS to analyze the recovery rate and try to deduce the threshold value through mathematical inference.

3.3. Measuring resilience of supply chain risk system

Vulnerability is an opposite concept against responsiveness and it is used to describe a supply chain in lack of robustness and resilience with respect to various risks that imposes negative impact upon supply chain [32, 33]. Robustness and resilience are two key concepts in SCRM. To large extent, they indicate the performance of risk management in supply chain. Robustness of supply chain is referred to the ability of supply chain to resist the impact of risk and retain the same stable situation as it had before the occurrence of risk [33]. Facing risk, a robust supply chain is able to cope with it so that the negative impact of risk will cause very little harm to the supply chain operation. In contrary to robustness, resilience is referred to ability of supply chain to return to a new stable or desirable situation after the impact of risk [33].

In SCRS, risk nodes are connected according to their casual relationship in a network structure. The initial nodes are normally triggered by some unpredictable environmental factors. For example, severe fluctuation in raw material price may cause flawed co-forecasting risk in supply, and it may trigger inventory risk if the price is so high that the manufacturer is not able to afford the high price and thus have to shrink the import quota of inventory. The shortage in inventory may cause some bigger problem if it cannot be fixed appropriately, one such like manufacturing stoppage that can result in appalling disruption of the whole supply chain.

Actually the state of each supply chain risk node at a time is determined by two factors: its marginal probability and its correlation with other risk nodes. Given that, the state of SCRS can be presented in a form of a set with the time property:

\[ RS_t = \{ p_i, corr_{ij} \} \] ……………………………(1)

Where \( RS_t \) denotes the state of SCRS at time t, \( p_i \) is marginal probability of a risk node i, and \( corr_{ij} \) is the causal correlation between risk node i and j.

\( p_i \) measures how likely the individual risk would happen, and \( corr_{ij} \) not only reflects the relationship between nodes but also determines the topological structure of SCRS. Both \( p_i \) and \( corr_{ij} \) can be obtained from the historical data stored in enterprise data warehouse. But if the data is unavailable in the database, it can be estimated or suggested by the experts in the field of SCRM. \( RS_t \) would vary at different time t in terms of the layout structure or the risk likelihoods in SCRS. It can be influenced by lots of factors, such as economic climate, industrial practices or partnership (See Fig.3.). In this study, we would like to investigate how partnership impacts on the topological structure of SCRS.
which risk node $i$ is triggered at time $t$, as a result of infection by its antecedent infected nodes at time $(t-1)$. Suppose that $j=1,2,…,k$ are the antecedent nodes of node $i$. $\Delta$ is the average recovery rate of the supply chain. $\Gamma$ is the triggered rate matrix of $n \times n$, defined as below:

$$
\Gamma = \begin{bmatrix}
\gamma_{11} & \gamma_{12} & \ldots & \gamma_{1n} \\
\gamma_{21} & \gamma_{22} & \ldots & \gamma_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\gamma_{n1} & \gamma_{n2} & \ldots & \gamma_{nn}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_n
\end{bmatrix} =
\begin{bmatrix}
p_{11} & p_{12} & \ldots & p_{1n} \\
p_{21} & p_{22} & \ldots & p_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
p_{n1} & p_{n2} & \ldots & p_{nn}
\end{bmatrix}
\begin{bmatrix}
p_1 \\
p_2 \\
\vdots \\
p_n
\end{bmatrix}
$$

Where $p_i$ is the marginal probability of risk node $i$, and $p_{ij}$ is the conditional probability given that node $j$ is infected. $p_{ij}$ is positively associated with $\gamma_{ij}$. It is said that given other condition unchanged, in case that the correlation between node $i$ and $j$ is stronger, if node $j$ is infected, then the probability of node $i$ to be triggered will be higher. $\gamma_{ij}$ is the triggered rate of risk node $i$ given that node $j$ is infected, therefore

$$
\gamma_{ij} = \begin{cases} 
p_i & i \neq j \\
p_j & i = j
\end{cases}
$$

If risk node $i$ is not infected at time $(t-1)$, then the probability of infection at time $t$ is the sum of probabilities of being triggered by the $k$ antecedent nodes. By definition, the probability that node $i$ is not infected at $(t-1)$ is $[1 - p_i(t-1)]$, and then the probability of being triggered by antecedent nodes is

$$
\sum_j \gamma_{ij} p_j(t-1)
$$

We put it into matrix form, we can get

$$
P(t) = [1 - p_i(t-1)] \Gamma P(t-1) + (1 - \Delta) P(t-1)
$$

Supposing supply chain is in a chaos state $[0(1)$, which does not fit nicely into the matrix formulation. Given that, we would like to assume $p_i(t-1)$ is rather small and such that

$$
1 - p_i(t-1) \approx 1
$$

Put it into matrix form, we can get

$$
P(t) = ([1 - p_i(t-1)] \Gamma + (1 - \Delta) I) P(t-1)
$$

Formula (2) shows in an elegant form of dynamical system with Markov chain property. The state equation is ideal except for the term $[1 - p_i(t-1)]$, which does not fit nicely into the matrix formulation. Given that, assuming $p_i(t-1) \approx 0$ may not lead to great accuracy loss in $P(t)$, and formula (3) will be a good approximation in calculating $P(t)$.

Let $[\Gamma + (1 - \Delta) I]$ be $S$, called the system matrix, we can obtain the first order difference equation:

$$
P(t) = S \cdot P(t-1)
$$

When $P(0)$ is given

$$
P(t) = S^t P(0)
$$

We define $\rho(S)$ as the largest nontrivial eigenvalue of system matrix $S$ and $\rho(\Gamma)$ as the largest nontrivial eigenvalue of triggered rate matrix $\Gamma$. $\rho(S)$ and $\rho(\Gamma)$ can be shown to have the following relationship:

$$
\rho(S) = \rho(\Gamma) + (1 - \Delta)
$$

Formula (5) and (6) convey a message that SCRS is a convergent system if $\rho(S) < 1$, or equivalently, if $\Delta$ is at least larger than $\rho(\Gamma)$. Supposing supply chain is in a chaos state $P(0)$ at time $t=0$, with time passes by, if the recovery rate $\Delta > \rho(\Gamma)$, $P(t)$ will converge and return to a stable or normal state that each risk node in SCRS is

\[\footnote{Due to the space limit, the proof is omitted. The readers can contact the authors for the detail of the proof if they are interested in it.}\]
triggered by its marginal probability. The convergent speed reflects the resilience of supply chain, which is subjected to the recovery rate $\Delta$ given $\rho(\Gamma)$ unchanged. The SCRS can quickly return to normality if the recovery rate $\Delta$ is very high. But if $\Delta < \rho(\Gamma)$, the chaos will persist in the supply chain, in which most of risk nodes in SCRS are triggered by its conditional probability. Hence, risk will happen by much higher chance, and accidental events will continuously break out anywhere in the supply chain, probably able to result in long-term disruption in operation and tremendous loss in finance. To prevent such disaster, $\Delta$ should at least maintain at the level of $\rho(\Gamma)$, and $\Delta' = \rho(\Gamma)$ is the minimum threshold for the recovery rate.

3.4. Supply chain resilience and partnership

The threshold value of the recovery rate $\Delta^*$ is equal to the largest nontrivial eigenvalue of triggered rate matrix $\Gamma$. It is associated with the topological structure of SCRS. The topological structure is concerned with the problems such as how the risk nodes are distributed and in which patterns they are organized. In terms of the orderliness and regularity, there exist two typical types of structures: random graph and scale-free network. Random graph is referred to the system in which its components are organized randomly, and there is no pattern able to be recognized regarding its topological structure [34]. Scale-free network is referred to the system in which small portion of its components are with high degree of connections with other components but most of its components are with low degree of connections with other components, and this regulation holds regardless the scale of the system [26, 34]. The rationale behind is that each time when a new node enter into the network, it will first choose the node with highest degree of connections to connect with, in other words, the node with higher degree of connections gets higher chance to be connected with, and the node with lower degree of connections gets lower chance to be connected with. Therefore, it will result in such scale-free system.

With dramatic growth of partnership in supply chain, companies are inclined to closely collaborate with each other. They enjoy being engaged in joint activities in which companies cooperate together to accomplish certain operations. Such joint activities may be associated with lots of other operations, thus capable to influence the supply chain in a wide extent. If problems happen in those activities, it may harm the supply chain greatly. In SCRS, the risk hidden in joint activities is named as collaboration risks, and the collaboration risks normally are the risk nodes with high degree of connections. Therefore, when partnership goes deep, partners will collaborate more frequently and they may choose co-working in form of joint activities rather than working individually. Hence, that will result in more collaboration risks in supply chain. Such change reflected in SCRS in terms of topological structure is a process of transformation from random graph to scale-free network (See Fig.4).

Fig 4 Effect of Partnership on Topological Structure of SCRS

A fact has been revealed by system science that scale-free network is a more organized system compared to random graph [34]. Given that, theoretically speaking, $\rho(\Gamma)$ of a scale-free network should be less than the one of random graph. Thus, $\Delta'$ for scale-free network is less than the one for random graph. In other words, partnership makes the SCRS evolve towards a more organized and regular system, and then lowers the minimum threshold $\Delta'$ required for the recovery rate. Moreover, referred to formula (6) $\rho(S) = \rho(\Gamma) + (1 - \Delta)$, when $\rho(\Gamma)$ decreases, $\rho(S)$ will drop from 1 to 0 given that $\Delta > \rho(\Gamma)$. It implies that the decrease of $\rho(\Gamma)$ is able to speed up SCRS from chaos to normality. Thereby $\rho(\Gamma)$ is negatively associated with the resilience of supply chain. The effect of partnership upon SCRS reflects in reducing $\rho(\Gamma)$ and making the system inclined towards a more organized structure. Hence, partners’ collaboration contributes to the resilience of supply chain. Through partnership and collaboration, supply chain can recover from chaos or disruption more quickly.

4. Verification experiments

In the previous section, two conclusions have been drawn theoretically:
1. There exists a minimum threshold value $\Delta'$ that prevents the SCRS from persistent chaos, and $\Delta' = \rho(\Gamma)$
2. Partnership makes the SCRS more inclined to
evolve towards scare-free network, a more organized and regular system compared to random graph, and such impact benefits the resilience of supply chain by reducing \( \rho(\Gamma) \) and lower \( \Delta' \).

These two findings are mainly relied on analytical inference in theory, and thus lack empirical evidence to manifest their correctness. Given that, in this paper, simulation is employed as the verification method to test the conclusions mentioned above. As a matter of fact, simulation is the only feasible choice under the rigorous research requirement of this study. First, it is too difficult to collect empirical data in the industry. Despite that risk management has been studied for more than 15 years, few companies have the records of risk data in their database [35]. Moreover, the research target is on supply chain which involves multiple companies. Obtaining consistent historical data across companies is extremely difficult. Furthermore, the key research scenario is fixed on a supply chain from chaos back to normality. It is only simulation that can construct such a specific scenario that the research interest is able to be satisfied.

4.1. Design of experiments

A simulation program has been developed to implement the attributes and behaviors of the model of SCRS. Fig.5 shows a snapshot of this system.

With the help of this simulation program, researchers can construct the scenarios they are really interested and conduct the experiments following their research plan. In this paper, two set of experiments are designed to test the conclusions drawn analytically using the model of SCRS. The first set of experiments is named threshold tests, which are intended to verify that \( \Delta' = \rho(\Gamma) \) through simulation. The experiments involve 4 key steps: Step1 is to construct a SCRS with reasonable setting. In this step, a SCRS is built by applying a real scenario in business. The risk nodes and their causal relationships are suggested by 4 experts in the field of SCRM. The marginal probability and conditional probability, due to the need for large data, are generated using the computer algorithm. Suggested by the literature, a small probability should be below 0.3 [26]. Given that, for marginal probability, it is generated uniformly within the interval of \((0, 0.2)\), and conditional probability is generated within the interval of \(\text{marginal probability} \times 0.3\) since conditional probability must be larger than the marginal one. Under this network setting, we can obtain the value of \( \rho(\Gamma) \) by calculating the largest eigenvalue of \( \Gamma \) matrix. According to Frobenis-Perron theorem, it is assured that a positive value of \( \rho(\Gamma) \) can be obtained because for a positive square matrix there must be at least one positive real eigenvalue. Step2 is to record the normal state of SCRS. Through simulation, we are allowed to imitate the process in which the SCRS is operating in a normal state. During the process, the program would record down the number of the triggered nodes at each time \( t \) and store the data into a database. For each time \( t \), it is called a run. For each experiment, the process would be simulated for 2000 runs in order to capture a comprehensive data of normal state of SCRS. Step3 is to simulate the process of SCRS from chaos to normality and obtain \( \Delta' \) in this recovery process. In this step the first task is to simulate the chaos state of SCRS. The chaos state is referred to a situation in which the risk network is overwhelming by risk events and nearly all risk nodes are triggered by conditional probability. Then in a chaos state, by assigning different recovery rates, we would find out the minimum one that is able to recover the SCRS from chaos to normality. To judge whether the SCRS has already returned to normality, we compare the current state with the normal state that has been obtained in step 2. By using two-sample T test, if the P-values of T statistic and F statistic both exceed 0.05, we can confirm that the SCRS has returned from chaos to normality. Step4 is to examine the correlation between \( \Delta' \) and \( \rho(\Gamma) \). In this step we would like to check the relationship between \( \Delta' \) and \( \rho(\Gamma) \), and see if they are strongly correlated as predicted in the analytical model.

The second set of experiments is named topological experiments. They are through simulation to test whether \( \Delta' \) of a scale-free network is actually
lower than $\Delta'$ of a random graph system. The most difficult part in these experiments is to generate the SCRS with topological structure of scale-free or random graph. For scale-free network, Barabasi-Albert (BA) approach is chosen as the network generating algorithm while for random graph, Erdos-Renyi (ER) approach is used to construct the network. To make these two types of network comparable, the simulation program by default generates the same amount of risk nodes with constant marginal probability of 0.1 and constant conditional probability of 0.15, and the same amount of links within the SCRS also.

4.2. Experimental results and discussion

The thresholds tests consist of 4 business scenarios pervasively existing in supply chain and each scenario includes 20 experiments. The final experimental results are summarized in Fig.6:

![Regression Statistics in Threshold Tests](image)

The statics of regression analysis manifest that there is a strong correlation between $\Delta'$ and $\rho(\Gamma)$. The simulation results confirm the inference derived from the analytical model that $\Delta'$ is equal to $\rho(\Gamma)$. Through extensive experiments, it is able to reach a conclusion that $\Delta'$ the threshold value of recovery rate is highly correlated with $\rho(\Gamma)$ and their correlation is approaching nearly 1, and this provides strong supporting evidence to bolster the argument that $\Delta' = \rho(\Gamma)$, which has been shown theoretically.

The topological experiments in all contain 10 experiments. They are contrasting $\Delta'$ of SCRS in scale-free and random graph. TABLE 2 briefly shows the summary of the experimental results.

<table>
<thead>
<tr>
<th>Type</th>
<th>Exp 1</th>
<th>Exp 2</th>
<th>Exp 3</th>
<th>Exp 4</th>
<th>Exp 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Random</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=10</td>
<td>0.2517</td>
<td>0.359</td>
<td>0.3147</td>
<td>0.3272</td>
<td>0.2906</td>
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<tr>
<td>L=10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Random</td>
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<td></td>
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<td>0.1068</td>
<td>0.1042</td>
<td>0.1063</td>
<td>0.1087</td>
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<tr>
<td>L=30</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Scale-free</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N=10</td>
<td>0.2516</td>
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<td>0.4112</td>
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<td>0.3413</td>
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<tr>
<td>L=10</td>
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<td></td>
</tr>
<tr>
<td>Scale-free</td>
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</tr>
<tr>
<td>N=30</td>
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<td>0.1121</td>
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<tr>
<td>L=30</td>
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<td></td>
</tr>
</tbody>
</table>

Note: N: number of risk nodes; L: number of links

TABLE 2 uncovers a fact that $\Delta'$ of a scale-free network is always smaller than the one of a random graph. It indicates that the threshold of the recovery rate reduces when the SCRS evolves towards a more organized and regular network. The effect of partnership will make supply chain more inclined to transform from random graph to scale-free. Such change in topological structure can significantly contribute to the resilience of supply chain. It can reduce the minimum threshold required to prevent chaos from persisting in supply chain, and it also can speed up the recovery process for a risk node from chaos to normality. In a word, partnership does make positive influence on the management of supply chain risks.

5. Conclusions

Supply chain risk management is a principle component in the body of supply chain management research. Previous studies mostly focusing on specific type of risk paid little attention upon the internal correlation existing among the risks in supply chain. This study re-conceptualizes supply chain risks into a general form and proposes a model through which risks can be integrated into a supply chain risk system (SCRS) capable to reflect not only the mutual correlation but also the causal relationship among them. Through modeling and simulation, it is revealed that the partnership can significantly contribute to the resilience of supply chain and benefit the risk management in supply chain. This offers a novel view from the perspective of risk management to explain why more and more supply chain companies are consciously engaged in partnership nowadays. It is hoped that this study would inspire practitioners and help them to form a new perception regarding the relationship between partnership and supply chain risk management.
6. References


