Wind Power Investment in Thermal System and Emissions Reduction

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***Abstract*—This paper presents an analytical model for wind power investment. Most generation planning problems are formulated in multiperiod mixed integer programming with cost minimization as objective. We try to resort to finance literature for models able to systematically characterize return and risk. Real option theory is chosen. A primitive function is defined for the fuel cost able to be saved as the revenue of a wind power project. Subsequently the real project is described as a contingent claim on the stochastic fuel prices. Theoretical valuation of the project is thus given by the solution of a partial differential equation derived by Ito lemma. This formulation avoids the ambiguity in analyzing wind power investment based on non-market-based tariffs, but focuses on the welfare to the system as a whole. Finally a hypothetical scenario of carbon emission price is included to demonstrate the incentive it could offer to renewable generation.**

***Index Terms*--Wind power, real option, binomial model, emissions price.**

I. INTRODUCTION

Generation expansion planning (GEP) can be described as an optimization problem concerning when to install, where to install and what type of generation units to be installed over a long horizon. Before restructuring of electricity markets, the planning duty was done by a single entity, the electric utility. Very often the objective of the optimization problem is to minimize costs of various kinds, subject to some reliability criteria of meeting the load demand [1]. Most GEP problems have been modeled as dynamic programming or multiperiod mixed integer programming. The number of objective can be single, to minimize total cost; or multiple, to further minimize emissions [2] or mandate renewable energy [3]. Problem formulations with multi and conflicting objectives do not change the optimization principle, but require multi-criteria approaches or expert knowledge to handle the increased problem complexity.

With restructuring of electricity markets worldwide, generation expansion transits from a planning problem of minimizing costs to individual Gencos' investment decisions of maximizing profits. New analytical tools are needed. With option pricing theory and its application in valuation of firms [4], evaluation of generation assets by financial methods [5] were subsequently developed. More recently, risk-adjusted valuation of generator asset [6] is developed.

It is tempting to analyze wind power investment in the same way as conventional generation investment based on electricity spot price. But usually wind power has its renewable energy support schemes [7]. Three common schemes are feed-in tariff, obligation system and tendering system. Investment analysis based on these schemes are, e.g., capital budgeting on a wind farm based on fixed tariff and average capacity factor [8]; profit of a wind farm based on full obligation and mean-reverting electricity price [9].

Should wind power investment be based on market price of electricity or special tariff for renewable energy? Definite answer may not be available but technically speaking, the effects of wind variability can only be assessed in conjunction with the specifics of the power system where wind farm is connected [10]. The assessment should be an economic dispatch & unit commitment program which contains inputs, system parameters and output results. General and qualitative results of the effects of wind power have been obtained for, e.g., Ireland [11], West Demark [12] and Taiwan [13]. Apparently, quantifying technical impacts of wind power into costs is again difficult. In [14], concise comments are made on various conflicts between wind power and electricity market regulation, in particular, balancing requirement, extent of subsidization and quantification of costs incurred by wind and after all, no clear cut has been arrived to. Nevertheless, [14] points out the market should result in optimal operation of the power system such that the social welfare is maximized. It is not easy as it involves optimal operation in short (economic dispatch) and medium (unit commitment), and optimal planning in long time frames.

The complexity of differentiating wind power related costs leads to the notion that wind power investment should be a centralized planning problem, at least holistically. Back to the question that how should wind power investment be evaluated, this paper stands on a way of thought that to directly base on the fossil fuel cost able to be saved. In this way any technical constraints of the

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system would have been catered theoretically. The approach is also independent of electricity price and hence its mechanism behind. Another important motivation for this approach is to facilitate the use of real option methodology.

In linear programming problems, prices of electricity and fuels are forecasted period by period separately. However, the whole formulation has little address on two critical elements of any investment problem; return and risk. We resort to finance literature for investment planning problem because it accounts price with its drift and volatility, and return (investor's required return, asset expected return, risk-free rate) in a more systematic way. Furthermore, real option can determine the value of managerial flexibility. Right now the usual question is when to invest renewable. The driving forces are environmental and regulatory, but more importantly cost-benefit consideration, primarily due to fossil fuel prices and renewable installation costs. It has been suggested that the value of wind power should depend on how much fuel it can save [12] [14]. This paper explicitly formulates wind power investment as a real option to existing thermal generation. The model will be described step by step in coming sections.

Section II describes how an ideal wind power investment is modeled by differential equation with closed form solution. Section III formulates the analytical investment problem into discrete binomial model; subsequently carbon emission price is incorporated in section IV. Section V contains examples of two base cases. Section VI contains a few discussion points for future research work and section VII is a concluding remark about the methodology offered in this paper.

II. Problem Description

We start with the stochastic process of a price variable P. The well recognized geometric Brownian motion (GBM) is taken as the stochastic process:

$$d\tilde{P} = \alpha \tilde{P}\, dt + \sigma \tilde{P}\, d\tilde{z} \qquad (1)$$

where α is the drift rate and σ is the volatility rate of the change of logarithm of price, and $d\tilde{z}$ is the standard Wiener process. Denote π(P,t) as the profit (revenue minus cost) to a project per unit time. For any project with π(P,t) contingent on price, contingent claims analysis leads to the following differential equation using Ito lemma:

$$\frac{1}{2}\sigma^2 P^2 \frac{\partial^2 V(P,t)}{\partial P^2} + (r-\delta)P\frac{\partial V(P,t)}{\partial P} - rV(P,t) + \frac{\partial V(P,t)}{\partial t} + \pi(P,t) = 0 \quad (2)$$

where V(P,t) is the valuation of the project as a function P and t, r is the risk-free rate, and δ is equal to μ minus α, i.e. the difference between required return μ on the project and expected return α of price. Without further loss of generality, the quantity of output per period is taken as one. Equation (2) is derived based on two standard assumptions in finance; a riskless portfolio of the real project and the underlying asset can be formed, and no-arbitrage argument [15].

Initially we assume infinite life of the project. Such simplification makes equation (2) independent of time t. The project value one unit of time later looks exactly the project value now, except with a new start state P. Hence the time t in equation (2) can be omitted and (2) becomes

$$\frac{1}{2}\sigma^2 P^2 V''(P) + (r-\delta)PV'(P) - rV(P) + \pi(P) = 0 \qquad (3)$$

Equation (3) is an ordinary differential equation which has closed form solution. Therefore a theoretical valuation V(P) of the project is obtained. Note that V(P) is not equal to net present value (NPV). As we shall see, the solution V(P) to (3) is equivalent to the expected present value of the cash flow stream P. Subsequently, NPV = V(P) – I, where I is the initial investment cost. We try to describe the wind power investment problem using equation (3).

As mentioned in the introduction, the wind power generator (WTG) is viewed as an investment taken by the power system, based on how much economic benefit WTG brings to the system as investing criterion. This idea reconciles with [16]. Our problem formulation also put aside ownership of generation and transmission assets in power market, though it matches with the scenario of a distribution company (Disco) evaluating an investment of distributed generation or wind power [17].

To start with, consider a hypothetical system with selected parameters as shown in Table I.

TABLE I
PARAMETERS OF A SIMPLE HYPOTHETICAL POWER SYSTEM

Symbol	Unit	Description
L	MW	Average annual system load today
T	%	Renewable energy target, a percentage of L
N	-	Target deadline: today is year 0, then N=1 means the deadline is year 1, etc.
Fuel	-	Either coal or gas

That represents a very typical scenario today that Governments stipulate renewable energy targets of annual electricity consumption to be produced by renewable.

The next question is how much wind energy could be generated from the WTG. Assume a wind farm produces more or less the same amount of energy every year. It makes sense since wind speed does not have "growth". The implication of this assumption is, for investment evaluation, it is reasonable to rely on average annual production of wind energy. However, when coming to how much fossil fuel or heat able to be saved, the relationship is not linear because of net heat rate curve and unit commitment status at different times. It is further complicated by the mix of coal-fired plants and gas turbines having different fuel prices. Therefore, to avoid handling this complex issue in course of attempting to fit in the real option framework, we coarsely assume a directly proportionate relationship between the total heat saved and the *average annual* wind energy generated. Then it allows the annual fuel cost saving to be defined as the product of fuel price, described by (1), and the constant annual wind energy output. We discuss a remedial method for dealing with this assumption in the last section.

TABLE II
NOTATIONS FOR WTG AND FUEL

Symbol	Unit	Description
c	-	Capacity factor
C_w	MW	Wind farm installed capacity
S	year	Service life of WTG
I	MUS$	Investment cost
M	MUS$/yr	Maintenance cost
P	US$/MBtu [1]	Fuel price
R(P)	MUS$/yr	Annual fuel cost saving

With the notations for a wind farm given in Table II, we readily write down the average fuel cost saving as

$$R(P) = 8760 \times C_w \times c \times H \times P \quad (4)$$

We assume H a composite unit heat rate at full load. Another simplifying assumption is that the maintenance cost M stays the same as today thereafter. Then the annual profit π is the fuel cost saving less maintenance cost:

$$\pi(P) = R(P) - M \quad (5)$$

When P in (3) is realized as R(P), we state the complete solution of (3) as follows.

$$V(P) = B_1[R(P)]^{\beta_1} + B_2[R(P)]^{\beta_2} + \frac{R(P)}{\delta} - \frac{M}{r} \quad (6)$$

The first two terms of (6) is the homogenous solution. The remaining part is the particular solution that can be easily verified by substituting it to the ordinary differential equation. To eliminate the effect of speculation of fuel price on the valuation of V(P), B_1 and B_2 should be zero [15]. So V(P) should only take the value of a perpetuity:

$$V(P) = \frac{R(P)}{\delta} - \frac{M}{r} \quad (7)$$

Thus equation (7) constitutes the non-speculative valuation of the wind power investment. In fact the term R(P)/δ or P/δ is just the expected present value of a growing perpetuity discounted at μ since

$$\int_0^\infty E(P_t)e^{-\mu t}dt = \int_0^\infty Pe^{\alpha t}e^{-\mu t}dt = \frac{P}{\mu - \alpha} = \frac{P}{\delta} \quad (8)$$

whereas M/r is the present value of a riskless perpetuity. Both terms in solution (7) substantiate the use of their respective discount rates.

We now turn to examine the investment opportunity of this wind power project. By investment opportunity we mean it is an investment option that can be deployed now or in the future. The option takes into account the benefit of delay or waiting. With the same procedure in deriving (3) a similar differential equation in F(P) denoting the investment option can be obtained [15]:

$$\frac{1}{2}\sigma^2 P^2 F''(P) + (r-\delta)PF'(P) - rF(P) = 0 \quad (9)$$

[1] MBtu = 1 million Btu

Note it is a homogenous one. The solution to (9) is

$$F(P) = A_1[R(P)]^{\beta_1}$$
$$\text{where } \beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}]^2 + \frac{2r}{\sigma^2}} \quad (10)$$

subject to boundary conditions:

$$F(0) = 0 \quad (11)$$
$$F(P^*) = V(P^*) - I \quad (12)$$
$$F'(P^*) = V'(P^*) \quad (13)$$

Equation (12) is so-called the value matching condition. It makes F evaluated at a particular price P* such that F(P*) equals V(P*) – I, i.e. indifferent from the original NPV method. (13) is the smoothing-pasting condition. Using the appropriate form of V(P) as in (7), together with boundary conditions (12) and (13) yield (see Appendix A)

$$A_1 = \frac{(\beta_1 - 1)^{\beta_1 - 1}}{(\frac{M}{r} + I)^{\beta_1 - 1}(\delta\beta_1)^{\beta_1}} \quad (14)$$

$$R(P^*) = \frac{\beta_1\delta(\frac{M}{r} + I)}{\beta_1 - 1} \quad (15)$$

In other words, the investment option is solved with a specific form of V(P). P* is the optimal investment price. Note that infinite service life also implicitly assumes the deadline of investing N is also infinite.

III. BINOMIAL MODEL

In reality, both WTG service life and renewable target date are finite so theoretically equation (2) is preferred. We know that second order partial differential equation can still be solved given appropriate boundary condition. In option pricing theory, it is the Black-Sholes formula. Other numerical methods such as binomial model [18] and Monte Carlo simulation [19] were developed and they both give very satisfactory approximation to the Black-Sholes formula. However, binomial model has the distinct feature able to cater early exercise of an option. Nowadays, the climax is many governments trying to impose deadlines for renewable energy deployment, including wind power. Utilities have to invest wind power no later than the deadline, but still it may not be the most economical to invest in the last minute. The investment decision is very similar to an American option in which the holder (electric utility) exercises the option (WTG investment) early if the payoff now (present value of all future cash flows less WTG cost) is bigger than the discounted future option (investment option). A very brief background of the binomial model and American option, in conjunction with our problem formulation, is given below.

The key idea of binomial model is to discretize the GBM of the underlying price variable into many infinitesimal time step, Δt. For each time step, P can either go up by multiple of u with probability p, or down with multiple of d with probability (1-p). Graphically, it is shown in Figure 1.

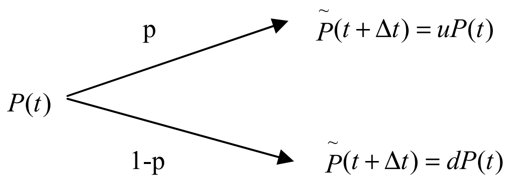

Fig. 1. Sample period of a binomial model

Our aim is to determine parameters u, d and p such that the binomial model can approximate the GBM satisfactorily. Consider one time step Δt:

$$\frac{\tilde{P}(t+\Delta t)}{P(t)}=e^{\tilde{w}} \qquad \tilde{w}=\begin{cases} a & \text{with probability p} \\ -a & \text{with probability 1-p}\end{cases} \tag{16}$$

Let $u=e^{a}$ and d=1/u, by matching the mean and variance of the binomial defined above with the GBM's, we obtain

$$u=e^{\sigma\sqrt{\Delta t}},\ d=e^{-\sigma\sqrt{\Delta t}} \text{ and } p=\frac{\exp[(\alpha-\frac{1}{2}\sigma^2)\Delta t]-d}{u-d} \tag{17}$$

and with the condition $\Delta t\le(\frac{\sigma}{\alpha-0.5\sigma^2})^2$.

Once the binomial model for the variable is constructed, the payoff function of the investment opportunity can be defined. Suppose the target deadline N equals 10 years. In binomial model, it is usual to denote the current time state as zero, here year 0. It has the number of periods equal to N. The last time state would then be year 10, where the terminal payoff is first calculated. Then the investment option F at present is determined recursively backward from the last time state in dynamic programming manner. Figure 2 shows the extract of the last period of the binomial lattice for illustrative purpose.

$$R(P)_{10}=u^{10}R(P)_0$$

$$F_{10,10}=(1-e^{-25\delta})\frac{u^{10}R(P)_0}{\delta}-(1-e^{-25r})\frac{M}{r}-I$$

$$R(P)_9=u^9R(P)_0$$

$$F_{9,9}=\max[Invest,Wait]$$

$$R(P)_{10}=u^9dR(P)_0$$

$$F_{10,9}=(1-e^{-25\delta})\frac{u^9dR(P)_0}{\delta}-(1-e^{-25r})\frac{M}{r}-I$$

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$$R(P)_{10}=d^{10}R(P)_0$$

$$F_{10,0}=(1-e^{-25\delta})\frac{d^{10}R(P)_0}{\delta}-(1-e^{-25r})\frac{M}{r}-I$$

Where

$$\begin{aligned} Invest&=(1-e^{-25\delta})\frac{u^9R(P)_9}{\delta}-(1-e^{-25r})\frac{M}{r}-I \\ Wait&=e^{-r}(p_\pi F_{10,10}+(1-p_\pi)F_{10,9}) \\ Risk-neutral\ probability&=p_\pi=\frac{e^{r-\delta-0.5\sigma^2}-d}{u-d}\end{aligned} \tag{18}$$

Fig. 2. The last period of the binomial lattice

The values of F's should be read after the price lattice. Suppose further the service life S of WTG is 25 years and the construction lead time is assumed zero for simplicity. At year 10, the project must be invested and the payoff is a growing annuity of revenue less the annuity of maintenance cost and construction cost of the WTG [20]. One period earlier, the decision is either to invest immediately or wait till the end. The payoff of waiting is the two option value F's in year 10 discounted at risk-neutral probabilities. The delta δ is embedded in the risk-neutral probabilities [20]. Repeat the process to obtain F at present time, which is the value of investment option F of wind power we are looking for. The results of NPV and F of two cases based on either gas or coal cost savings are given in Section V.

As a recap, we have applied the closed form solution of ordinary differential equation as the valuation of wind power project. Then finite service life S and target deadline N are imposed to give realistic results. Two base cases of displacing either purely natural gas or coal consumption are constructed and their discrete NPV and F are calculated. In practice, for a system using two fuels, there are many reasons determining how the coal-fired plants and gas turbines are dispatched. After a wind farm is built, one could expect some proportions of coal and gas consumption are reduced. Following the previous problem formulation that fuel cost saving is the revenue to the WTG, a logical extension is to build a model that can cater two stochastic fuel prices simultaneously. Binomial model with two variables is the right choice and the formulation would be addressed in another piece of work.

IV. Effects of Carbon Emission Price

The motivation to consider carbon emission price is twofold. It encourages emissions reduction. It also signifies renewable generation without subsidy is somewhat economically unfeasible. In the next section of this paper, it would be shown that wind power as a substitute to gas consumption is not completely unviable. Naturally we would ask how much carbon price could provoke the transition. Knowing that coal is more carbon-intensive and polluting than natural gas, the society should be more interested in using less coal. The analysis suggested in this section is based on the notion that any emissions saved are credited to the WTG investment. It is applicable to both gas and coal consumption saving.

We consider some fixed levels of carbon dioxide price only. Denote C (€/tonne) as the carbon price, CC (lbs/MBtu) as the carbon dioxide content of the fuel, R(C) (MUS$) as the carbon saving. Then

$$R(C)=\frac{xC}{2204.6226}\times CC\times E\times H \tag{19}$$

where x is the exchange rate, 1 tonne = 2204.6226 lbs, E is the electrical energy concerned and H is again the composite unit heat rate at full load. As before, we first describe the infinite model. Suppose the emissions policy is already implemented and carbon price is constant forever, then the effect of R(C) is simply a perpetuity denoted by V*

$$V^* = \frac{R(C)}{r} \quad (20)$$

It can be treated as an additional term to the solution V(P) from equation (7), in which it becomes

$$V(P) = \frac{R(P)}{\delta} - \frac{M - R(C)}{r} \quad (21)$$

With effectively the same form of V(P), closed form solution of investment option F can again be obtained.

V. Examples

We presents closed form solutions for both gas and coal scenario. Each base case is further differentiated by onshore or offshore WTG. The typical parameters of a system and wind farms are given in Table III.

TABLE III
PARAMETERS OF THE SYSTEM AND WTG

Symbol	Value	Unit	Description
L	5000	MW	Average annual system load
T	3	%	Consumption-wise
c	0.3	-	Capacity factor onshore
	0.4	-	Capacity factor offshore
C_w	500	MW	Defined as LxT/c
S	25	year	Service life of WTG
N	10	year	Renewable target deadline
I	700	M US$	1.4M per MW onshore [21]
	1050		2.1M per MW offshore [21]
M	15	M US$/yr	30k per MW onshore [21]
	30		60k per MW offshore [21]

The calculations of the average fuel cost savings are presented in Table IV (A & B). NPV and investment option F (with μ designed at 11%) of infinite case are first calculated analytically using spreadsheet as in Table V (A & B), with underlined values as output results. Then their discrete approximations are generated using Matlab and recorded in Table VI (A & B).

TABLE IV A
CALCULATION OF GAS COST SAVING

Symbol	Parameter	Value		Unit	Description
		Onshore	Offshore		
H	Unit heat rate	7000	7000	Btu / kWh	Average efficiency of gas turbine [21]
P	Current fuel price	6.96	6.96	US$ / k ft^3	Natural gas for US electric power users in Dec 2008 [22]
		6.96	6.96	US$ / Mbtu	1 cubic feet of gas = 1000 Btu
E x H	Heat saved	9.20	12.26	TBtu	8760C_wcH
R(P)	Fuel cost saving	64.0	85.4	M US$	R(P) = EHP

TABLE V A
NPV AND F OF GAS CASE

Parameters	Value		Unit	Description
	Onshore	Offshore		
r	4.0%	4.0%	p.a.	30 yrs Treasury bond yield as proxy
fuel price volatility σ	30.7%	30.7%	p.a.	Derived from 1976-2008 monthly US natural gas wellhead logarithmic price [22]
fuel price drift α	5.6%	5.6%	p.a.	
μ	11.0%	11.0%	p.a.	required return
δ	5.4%	5.4%	p.a.	δ=μ-α
R(P)	64	85.4	M US$	Current annual fuel cost saving
M	15	30	M US$	Annual maintenance
I	700	1050	M US$	Initial investment
V(P)	810	831	M US$	R(P)/δ - M/r
NPV	110	-219	M US$	V(P) - I
F(P)	379	424	M US$	$A1[R(P)]^{\beta 1}$
β1	1.78	1.78		
A1	0.24	0.16		
R(P*)	132.93	222.58	M US$	Optimal R(P) to invest
V(P*)	2086.69	3371.89	M US$	equals F(P*) at P*

TABLE VI A
DISCRETE APPROXIMATION OF NPV AND F FOR GAS CASE

S (yr)	N (yr)	NPV (M US$)	F(P) (M US$)	NPV (M US$)	F(P) (M US$)
		Onshore		Offshore	
infinity	infinity	110	379	-219	424
100	100	110	368	-212	413
25	100	-59	242	-352	272
25	10	-59	199	-352	215

TABLE IV B
CALCULATION OF COAL COST SAVING

Symbol	Parameter	Value		Unit	Description
		Onshore	Offshore		
H	Unit heat rate	10000	10000	Btu / kWh	Average efficiency of coal-fired steam turbine [21]
P	Current fuel price	36.06	36.06	US$ / short ton	Average price for coal for US electric power users in 2007 [22][2]
		1.80677	1.80677	US$ / Mbtu	1 metric ton = 1.1023 short ton, 1 ton of (bituminous) coal = 22 Mbtu
E x H	Heat saved	13.14	17.52	TBtu	8760C_wcH
R(P)	Fuel cost saving	23.7	31.7	M US$	R(P) = EHP

[2] The figure for 2008 was not yet published at time this paper was written

TABLE V B
NPV AND F OF COAL CASE

Parameters	Value		Unit	Description
	Onshore	Offshore		
r	4.0%	4.0%	p.a.	30 yrs Treasury bond yield as proxy
fuel price volatility σ	10.2%	10.2%	p.a.	Derived from 1949-2006 yearly US bituminous coal logarithmic price [22]
fuel price drift α	4.0%	4.0%	p.a.	
μ	11.0%	11.0%	p.a.	required return
δ	7.0%	7.0%	p.a.	δ=μ-α
R(P)	23.7	31.7	M US$	Current annual fuel cost saving
M	15	30	M US$	Annual maintenance
I	700	1050	M US$	Initial investment
V(P)	-36	-297	M US$	R(P)/δ - M/r
NPV	-736	-1347	M US$	V(P) - I
F(P)	0	0	M US$	$A1[R(P)]^{\beta 1}$
β1	7.76	7.76		
A1	0.00	0.00		
R(P*)	86.38	144.64	M US$	Optimal R(P) to invest
V(P*)	859.07	1316.35	M US$	equals F(P*) at P*

TABLE VI B
DISCRETE APPROXIMATION OF NPV AND F FOR COAL CASE

S (yr)	N (yr)	NPV (M US$)	F(P) (M US$)	NPV (M US$)	F(P) (M US$)
		Onshore		Offshore	
infinity	infinity	-736	0	-1347	0
100	100	-730	0	-1333	0
25	100	-657	0	-1150	0
25	10	-657	0	-1150	0

It can be seen from Table VI B that if coal-fired units are displaced by wind power, both onshore and offshore, NPV's are very negative. Negative NPV means the project should not be invested *immediately* (before considering F) given the specified required return. But F's are zeros too, meaning the investment options are still not worthwhile even the option of delay/waiting is considered. Indeed we observe that NPVs are less negative when service life is shortened. It is because the total discounted profit V(P) itself is negative, let alone initial cost. The longer the project is run, the more to lose. Therefore such project should not be operated at the very beginning.

NPV results for the gas case are mostly negative as well, except for onshore WTG with hypothetically long service life, see Table VI A. Only after option of delay/waiting is considered, the investment looks feasible. A positive F means we should not invest immediately, but still is worth to do so sometime later. As an intrinsic feature of option pricing, value of option arises as a result of the underlying asset volatility. If the investment is committed immediately, a premium, which is the difference between F and NPV for P below P*, will be lost. The difference can be viewed as a waiting premium in this context. Here the volatility of gas price is high enough to give a meaningful size of waiting premium. For optimality, the wind power investment should not be deployed until the natural gas price rises to the threshold. Using the result from Table V A, the threshold translates into R(P*)=133 for onshore case, at which the linear NPV and perpetual investment option F coincides as in Figure 3. It is theoretically better to wait as long as price is less than the threshold, beyond it waiting is indifferent from investing immediately (for finite horizon, further waiting would be even worse).

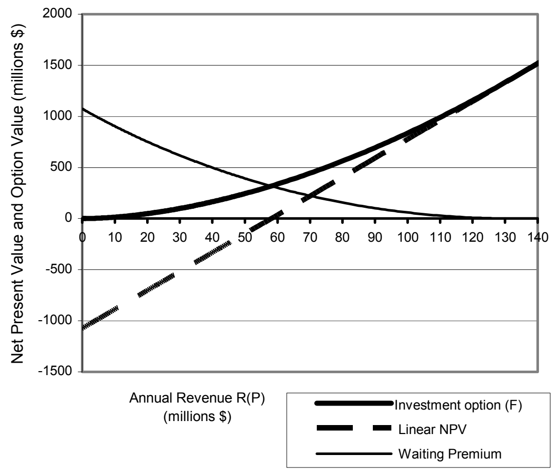

Fig. 3. Perpetual option to invest and linear NPV

Finally we choose the base case of coal cost saving and onshore WTG for the analysis of carbon price to build on. We jump directly to the binomial model to generate NPV and F, again with service life S=25 of onshore WTG and renewable target deadline N=10. Results are shown in Table VII. It is shown that carbon price has to be greater than 24 Euro/tonne for breakeven. Another observation is that NPV and F converge very quickly, meaning that the carbon price at level around €24 just sends the R(P) very close to its optimal threshold R(P*). The waiting premium in this case is therefore not significant.

TABLE VII
NPV AND F OF ONSHORE WTG AND COAL WITH EMISSION POLICY

Carbon price C (Euro€/tonne)	Carbon content[3] CC (lbs/MBtu)	Carbon saving[4] R(C) (MUS$/yr)	NPV (MUS$)	F(P) (MUS$)
0	205	0	-657	0
10	205	17	-388	0
20	205	34	-120	1
24	205	41	-9	10
24.5	205	42	6	15
25	205	43	22	22
30	205	51	148	148

[3] For average Bituminous coal in US; figure of natural gas is 117 [22]
[4] Take 1 Euro = 1.4 US$

VI. Discussion

On the capacitor factor c. As raised in the introduction, how much wind power can be utilized depends on wind speed as well as the system itself. In this work, only average annual wind energy production is considered. Monte Carlo simulation can be employed based on the probability distribution of wind speed, which may be more persuasive in the analysis.

On the unit heat rate H. Since there are times gas turbines and coal-fired plants operating at part-load at lower efficiency, if we stick to only values of unit heat rate at full load, potentially we underestimate the heat able to be saved by WTG. Furthermore, the forced outage rates of thermal generators shall be considered. It leads to the consideration of probabilistic production cost model to estimate the difference in system total fuel cost before and after wind power is added. The whole discussion section is planned as future work.

VII. Conclusion

This section is primarily a remark of the methodology used in this paper instead of stating particular conclusion from the model results. The methodology is typical in finance literature and is about valuation of future profits. Future profits are all valuated in present value (PV) such that when this PV minus any initial cost of the investment, we obtain the net present value (NPV). When NPV is positive, the investment gains, vice versa. Since future profits are uncertain or volatile, we can only describe the profit stream by an expected growth rate and use one discount rate at a time; the volatility of the profit is not catered in this simple setting. Later economists developed a way to incorporate volatility if one has an option to do or not do something. That is the famous Black-Sholes option pricing theory. It was initially derived on stock price and gives the stock option value based on (1) and (2). Because the option is sort of a claim of all future inflows, the concept had been extended to real projects in such a way that option value is analogous to the PV of the project's profit stream and exercise price is analogous to the initial cost. Then equation (2) is used to describe the valuation of the investment and the valuation is a floating value independent of the initial cost. Furthermore, the idea of investment opportunity, so defined by equation (9) and its boundary conditions, captures a value of delaying investment in addition to the conventional NPV for any projects. The investment opportunity has been formulated with the decision to invest or not [15] and is regarded as a more comprehensive indicator of the prospects. Using differential equation to model a real project can be collectively described as real option [20]. A successful application in generator asset valuation is [5].

In this paper, the wind power investment problem is similarly separated into two components; the initial cost and valuation of the present value of profit stream. Meanwhile, there are two parts in the calculation need discrete approximation. The first one is the binomial model used to discretize the continuous stochastic process (1). Subsequently the decision to invest or not is allowed every period before the renewable target deadline and is determined by the investment payoff. The procedure of evaluating payoff is in fact an American option and is partly illustrated in figure 2. The second part with discretization is the growing annuity of the wind farm's future profits. It is made discrete because in reality the service life of wind turbine is necessarily finite. The profit of the wind investment project is defined as fossil fuel cost saving less maintenance cost of wind turbine, therefore the stochastic process is made on fuel price. To conclude, American option is the essential part of the discrete model governing the decision to invest or not.

VIII. Appendix

A. Solving ordinary second order homogenous differential equation with boundary conditions

Restate (10) as the solution to a second order homogenous differential equation (9),

$$F(P) = A_1[R(P)]^{\beta_1}$$
$$where\ \beta_1 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} + \sqrt{[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}]^2 + \frac{2r}{\sigma^2}} \tag{A.1}$$

and the boundary conditions (12) and (13):

$$F(P^*) = V(P^*) - I \tag{A.2}$$
$$F'(P^*) = V'(P^*) \tag{A.3}$$

Based on the functional form of V(P) as in (7),

$$V(P) = \frac{R(P)}{\delta} - \frac{M}{r} \tag{A.4}$$

we can rewrite (A.2) and (A.3) respectively as

$$A_1[R(P^*)]^{\beta_1} = \frac{R(P^*)}{\delta} - \frac{M}{r} - I \tag{A.5}$$

$$\beta_1 A_1[R(P^*)]^{\beta_1 - 1} = \frac{1}{\delta} \tag{A.6}$$

Solving the coefficient A_1 and the optimal threshold R(P*) from the above two equations yields (14) and (15).

IX. References

[1] X. Wang and J. R. McDonald, *Modern Power System Planning,* London: McGraw-Hill, 1994.

[2] Jose L. Ceciliano Meza, Mehmet Bayram Yildirim, and Abu S. M. Masud, "A Model for the Multiperiod Multiobjective Power Generation Expansion Problem," *IEEE Trans. Power System*, vol. 22, No. 2, pp. 871-878, May 2007.

[3] Ronan Doherty, Hugh Outhred, Mark O'Malley, "Establishing the Role that Wind Generation May Have in Future Generation Portfolios," *IEEE Trans. Power System*, Vol. 21, No. 3, pp. 1415-1422, Aug. 2006.

[4] Robert L. McDonald and Daniel R. Siegel, "Investment and the Valuation of Firms When There is an Option to Shut Down," *International Economic Review*, Vol. 26, No. 2 (Jun., 1985), pp. 331-349.

[5] Shijie Deng, Blake Johnson, and Aram Sogomonian, "Spark Spread Options and the Valuation of Electricity Generation Assets," *in Proc. the 32nd Hawaii International Conference on System Sciences,* 5-8 Jan. 1999.

[6] Felix F. Wu, Yang He and Yunhe Hou, "Risk-adjusted Valuation of Generation Asset," to be appeared in *IEEE Trans. Power System*

[7] M. Mendonca, *Feed-in Tariffs, Accelerating the Development of Renewable Energy,* London: Earthscan, 2007.

[8] Fotios E. Karagiannis, "Wind Energy Investments in Greece – an Economical Approach," *in Proc. The 10th Mediterranean Electrotechnical Conference,* MEleCon 2000, Vol.3, pp. 1141-1144.

[9] Hui Zhou, Yunhe Hou, Yaowu Wu, Haiqiong Yi, Chengxiong Mao and Gonggui Cheng, "Analytical Assessment of Wind Power Generation Asset in Restructured Electricity Industry," *in Proc. the Universities Power Engineering Conference***,** UPEC, 2007, pp. 1086-1092.

[10] Bart C. Ummels, Madeleine Gibescu, Engbert Pelgrum, Wil L. Kling, and Arno J. Brand, "Impacts of Wind Power on Thermal Generation Unit Commitment and Dispatch," *IEEE Trans. Energy Conversion*, vol. 22, No. 1, pp. 44-51, March 2007.

[11] Electricity Supply Board National Grid (ESBNG), "Impact of Wind Power Generation in Ireland on the Operation of Conventional Plant and the Economic Implications," ESBNG, Ireland, 2004a.

[12] Hannele Holttinen and Jens Pedersen, "The Effect of Large Scale Wind Power on a Thermal System Operation," in *Proc. the 4th International Workshop on Large Scale Integration of Wind Power and Transmission Networks for Offshore Wind Farms*, pp. E1-E7, 20-22 Oct. 2003.

[13] C. L. Chen, T. Y. Lee, and R. M. Jan, "Optimal Wind-thermal Coordination Dispatch in Isolated Power Systems with Large Scale Integration of Wind Capacity," *Energy Conversion and Management*, 47 (2006) 3456-3472.

[14] Brendan Fox, Damian Flynn, Leslie Bryans, Nick Jenkins, David Miborrow, Mark O'Malley, Richard Watson, and Olimpo Anaya-Lara, *Wind Power Integration, Connection and System Operation Aspects*, London: IET Power and Energy Series, 2007.

[15] Avinash K. Dixit and Robert S. Pindyck, *Investment Under Uncertainty*, New Jersey: Princeton, 1994.

[16] Hugo A. Gil and Geza Joos, "Models for Quantifying the Economic Benefits of Distributed Generation," *IEEE Trans. Power System*, vol. 23, No. 2, pp. 327-335, May 2008.

[17] Walid El-Khattam, Kankar Bhattacharya, Yasser Hegazy and M. M. A. Salama, "Optimal Investment Planning for Distributed Generation in a Competitively Electricity Market," *IEEE Trans. Power System*, vol. 19, No. 3, pp. 1674-1684, Aug 2004.

[18] John C. Cox, Stephen A. Ross and Mark Rubinstein, "Option Pricing: A Simplified Approach," *Journal of Financial Economics,* 7, 229-263, Sept. 1979.

[19] Phelim P. Boyle, "Options: A Monte Carlo approach," *The Journal of Financial Economics*, Vol. 4, No. 3, pp. 323-338, May 1977.

[20] Richard L. Shockley, *An Applied Course in Real Options Valuation*, Mason: Thomson, 2007.

[21] Allen J. Wood and Bruce F. Wollenberg, *Power Generation Operation and Control*, New York: Wiley, 1996.

[22] [Online]. Available: www.eia.doe.gov

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