

Research Article

Is Classical Energy Equation Adequate for Convective Heat Transfer in Nanofluids?

Jing Fan and Liqiu Wang

Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

Correspondence should be addressed to Liqiu Wang, lqwang@hku.hk

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To address whether the heat transfer in nanofluids still satisfies the classical energy equation, we theoretically examine the macroscale manifestation of the microscale physics in nanofluids. The microscale interaction between nanoparticles and base fluids manifests itself as thermal waves/resonance at the macroscale. The energy equation that governs the heat transfer in nanofluids is of a dual-phase-lagging type instead of the *postulated* and *commonly-used* classical energy equation. The interplays among diffusion, convection, and thermal waves/resonance enrich the heat transfer in nanofluids considerably.

1. Introduction

Choi coined the term “nanofluids” for the fluids with nanoelements (nanoparticles, nanotubes, or nanofibers) suspended in them [1]. Recent experiments on nanofluids have shown, for example, twofold increases in thermal conductivity [2], strong temperature dependence of thermal conductivity [3], substantial increases in convective heat transfer coefficient [4, 5], and threefold increases in critical heat flux (CHF) in boiling heat transfer [2, 3, 6]. State-of-the-art expositions of major advances on the synthesis, characterization, and application of nanofluids are available, for example, in [2, 3, 6–12]. These characteristics make them very attractive for a large number of industries such as transportation, electronics, defense, space, nuclear systems cooling, and biomedicine.

The study of nanofluids is still in its infancy [2, 3, 6–12]. The precise nature and mechanism of the significant improvement of thermal performance are still not known. There is also a lack of agreement between experimental results and between theoretical models. The fact that the enhancement in thermal properties comes from the presence of nanoparticles has directed research efforts nearly exclusively towards thermal transport at nanoscale. The classical conservation equations including the energy equation have

been postulated as the macroscale model of nanofluid convective heat transfer but without adequate justification. Thermal conductivity and convective heat transfer coefficient are a macroscale phenomenological characterization of heat transfer and their measurements are not performed at the nanoscale, but rather at the macroscale. Therefore, interest should focus not only on what happens at the nanoscale but also on how the presence of nanoelements affects the heat transport at macroscale.

We attempt to examine whether the classical energy equation is adequate for describing convective heat transfer in nanofluids at macroscale based on a macroscale heat transfer model in nanofluids, which is rigorously developed by scaling-up the microscale model for the heat transfer in the nanoparticles and in the base fluids. The approach for scaling-up is the volume averaging [13–15] with help of multiscale theorems [15].

2. Macroscale Energy Equation

The microscale model for heat transfer in nanofluids is well known. It consists of the field equation and the constitutive equation. The field equation comes from the conservation laws of mass, momentum, and energy. The commonly-used

constitutive equation includes the Newton law of viscosity and the Fourier law of heat conduction [16].

For transport in nanofluids, the macroscale is a phenomenological scale that is much larger than the microscale and much smaller than the system length scale. Interest in the macroscale rather than the microscale comes from the fact that a prediction at the microscale is complicated due to the complex microscale structure of nanofluids, and also because we are usually more interested in large scales of transport for practical applications. Existence of such a macroscale description equivalent to the microscale behavior requires a good separation of length scales and has been well discussed in [15, 17].

To develop a macroscale model of heat transfer in nanofluids, the method of volume averaging starts with a microscale description. Both conservation and constitutive equations are introduced at the microscale. The resulting microscale field equations are then averaged over a representative elementary volume (REV), the smallest differential volume resulting in statistically meaningful local averaging properties, to obtain the macroscale field equations. In the process of averaging, the *multiscale theorems* [15] are used to convert integrals of gradient, divergence, curl, and partial time derivatives of a function into some combination of gradient, divergence, curl, and partial time derivatives of integrals of the function and integrals over the boundary of the REV.

Consider heat transfer in nanofluids with the continuous base fluid and the dispersed nanoparticle denoted by c - and d -phases, respectively. Note that the dispersed nanoparticles can be liquid droplets for general nanofluids [10, 12]. As a two-component mixture (base fluid + nanoparticles), its microscale model can be written as [18]

$$\begin{aligned}
& \nabla \cdot \mathbf{v}_c = 0, \quad \text{in the } c\text{-phase,} \\
& \rho_c \frac{\partial \mathbf{v}_c}{\partial t} + \rho_c \mathbf{v}_c \cdot \nabla \mathbf{v}_c \\
& \quad = -\nabla p_c + \rho_c \mathbf{g} + \mu_c \nabla^2 \mathbf{v}_c, \quad \text{in the } c\text{-phase,} \\
& (\rho c_p)_c \frac{\partial T_c}{\partial t} + (\rho c_p)_c \mathbf{v}_c \cdot \nabla T_c \\
& \quad = \nabla \cdot (k_c \nabla T_c), \quad \text{in the } c\text{-phase,} \\
& \nabla \cdot \mathbf{v}_d = 0, \quad \text{in the } d\text{-phase,} \\
& \rho_d \frac{\partial \mathbf{v}_d}{\partial t} + \rho_d \mathbf{v}_d \cdot \nabla \mathbf{v}_d \\
& \quad = -\nabla p_d + \rho_d \mathbf{g} + \mu_d \nabla^2 \mathbf{v}_d, \quad \text{in the } d\text{-phase,} \\
& (\rho c_p)_d \frac{\partial T_d}{\partial t} + (\rho c_p)_d \mathbf{v}_d \cdot \nabla T_d \\
& \quad = \nabla \cdot (k_d \nabla T_d), \quad \text{in the } d\text{-phase,} \\
& \mathbf{v}_c = \mathbf{v}_d, \quad \text{at } A_{cd}, \\
& T_c = T_d, \quad \text{at } A_{cd}, \\
& \mathbf{n}_{dc} \cdot k_c \nabla T_c = \mathbf{n}_{dc} \cdot k_d \nabla T_d, \quad \text{at } A_{cd}.
\end{aligned} \tag{1}$$

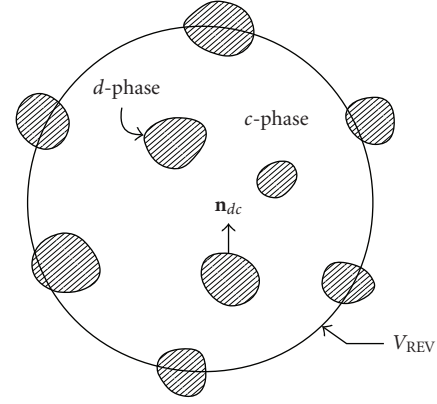


FIGURE 1: Representative elementary volume (REV).

Here \mathbf{v} and T are the velocity and the temperature, respectively. ρ , c_p , k , p , μ , and \mathbf{g} are the density, specific heat, thermal conductivity, pressure, viscosity, and gravitational acceleration, respectively. Subscripts c and d refer to the c - and d -phases, respectively. A_{cd} represents the area of the c - d interface contained in the REV, and \mathbf{n}_{dc} is the outward-directed surface normal from the d -phase toward the c -phase, (Figure 1). To be thorough, we must also specify the initial conditions and the boundary conditions at the entrances and exits of the REV; however, we need not do so for our discussion.

Applying the volume averaging and multiscale theorems to scale-up the microscale model yields a macroscale model [12, 19], where the energy equation reads

$$\begin{aligned}
& \varepsilon_c (\rho c_p)_c \frac{\partial \langle T_c \rangle^c}{\partial t} + \varepsilon_c (\rho c_p)_c \langle \mathbf{v}_c \rangle^c \cdot \nabla \langle T_c \rangle^c - \mathbf{u}_{cc} \cdot \nabla \langle T_c \rangle^c \\
& \quad - \mathbf{u}_{cd} \cdot \nabla \langle T_d \rangle^d \\
& \quad = \nabla \cdot (\mathbf{K}_{cc} \cdot \nabla \langle T_c \rangle^c + \mathbf{K}_{cd} \cdot \nabla \langle T_d \rangle^d) \\
& \quad - a_v h (\langle T_c \rangle^c - \langle T_d \rangle^d), \\
& \varepsilon_d (\rho c_p)_d \frac{\partial \langle T_d \rangle^d}{\partial t} + \varepsilon_d (\rho c_p)_d \langle \mathbf{v}_d \rangle^d \cdot \nabla \langle T_d \rangle^d - \mathbf{u}_{dc} \cdot \nabla \langle T_c \rangle^c \\
& \quad - \mathbf{u}_{dd} \cdot \nabla \langle T_d \rangle^d \\
& \quad = \nabla \cdot (\mathbf{K}_{dc} \cdot \nabla \langle T_c \rangle^c + \mathbf{K}_{dd} \cdot \nabla \langle T_d \rangle^d) \\
& \quad - a_v h (\langle T_d \rangle^d - \langle T_c \rangle^c).
\end{aligned} \tag{2}$$

Here ε_i is the volume fraction of the i -phase (the index i can take c or d), with V_i and V_{REV} as the volume of the i -phase in REV, and the volume of the REV respectively:

$$\varepsilon_i = \frac{V_i}{V_{\text{REV}}}. \tag{3}$$

The *intrinsic average* is defined by

$$\langle \Psi_i \rangle^i = \frac{1}{V_i} \int_{V_i} \Psi_i dV. \quad (4)$$

\mathbf{K}_{cc} , \mathbf{K}_{cd} , \mathbf{K}_{dc} , \mathbf{K}_{dd} , \mathbf{u}_{cc} , \mathbf{u}_{cd} , \mathbf{u}_{dc} , \mathbf{u}_{dd} , and $a_v h$ represent the effects of microscale physics on the macroscale heat transfer. The readers are referred to [12, 19, 20] for their governing equations and numerical computations.

3. Results and Discussion

Rewrite (2) in their operator form:

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} \begin{bmatrix} \langle T_c \rangle^c \\ \langle T_d \rangle^d \end{bmatrix} = 0. \quad (5)$$

where $\mathcal{A} = \gamma_c(\partial/\partial t) + \gamma_c \langle \mathbf{v}_c \rangle^c \cdot \nabla - \mathbf{u}_{cc} \cdot \nabla - \nabla \cdot (\mathbf{K}_{cc} \cdot \nabla) + a_v h$, $\mathcal{B} = -\mathbf{u}_{cd} \cdot \nabla - \nabla \cdot (\mathbf{K}_{cd} \cdot \nabla) - a_v h$, $\mathcal{C} = -\mathbf{u}_{dc} \cdot \nabla - \nabla \cdot (\mathbf{K}_{dc} \cdot \nabla) - a_v h$, and $\mathcal{D} = \gamma_d(\partial/\partial t) + \gamma_d \langle \mathbf{v}_d \rangle^d \cdot \nabla - \mathbf{u}_{dd} \cdot \nabla - \nabla \cdot (\mathbf{K}_{dd} \cdot \nabla)$. We then obtain an uncoupled form by evaluating the operator determinant such that

$$\begin{aligned} & \left[\left(\gamma_c \frac{\partial}{\partial t} + \gamma_c \langle \mathbf{v}_c \rangle^c \cdot \nabla - \mathbf{u}_{cc} \cdot \nabla - \nabla \cdot (\mathbf{K}_{cc} \cdot \nabla) + a_v h \right) \right. \\ & \times \left(\gamma_d \frac{\partial}{\partial t} + \gamma_d \langle \mathbf{v}_d \rangle^d \cdot \nabla - \mathbf{u}_{dd} \cdot \nabla - \nabla \cdot (\mathbf{K}_{dd} \cdot \nabla) + a_v h \right) \\ & - (\mathbf{u}_{cd} \cdot \nabla + \nabla \cdot (\mathbf{K}_{cd} \cdot \nabla) + a_v h) \\ & \left. \times (\mathbf{u}_{dc} \cdot \nabla + \nabla \cdot (\mathbf{K}_{dc} \cdot \nabla) + a_v h) \right] \langle T_i \rangle^i = 0, \quad (6) \end{aligned}$$

where the index i can take c or d . $\gamma_c = \varepsilon_c(\rho c)_c$ and $\gamma_d = \varepsilon_d(\rho c)_d$ are the c -phase and d -phase effective thermal capacities, respectively. Its explicit form reads, after dividing by $a_v h(\gamma_c + \gamma_d)$,

$$\begin{aligned} & \frac{\partial \langle T_i \rangle^i}{\partial t} + \frac{\gamma_c \gamma_d}{a_v h(\gamma_c + \gamma_d)} \frac{\partial^2 \langle T_i \rangle^i}{\partial t^2} + \frac{1}{(\gamma_c + \gamma_d)} \\ & \times \left(\gamma_c \langle \mathbf{v}_c \rangle^c + \gamma_d \langle \mathbf{v}_d \rangle^d \right) \cdot \nabla \langle T_i \rangle^i \\ & = \frac{1}{\gamma_c + \gamma_d} \left[\nabla \cdot (\mathbf{K}_{cc} \cdot \nabla) + \nabla \cdot (\mathbf{K}_{cd} \cdot \nabla) + \nabla \cdot (\mathbf{K}_{dc} \cdot \nabla) \right. \\ & \quad \left. + \nabla \cdot (\mathbf{K}_{dd} \cdot \nabla) \right] \langle T_i \rangle^i \\ & + \frac{1}{a_v h(\gamma_c + \gamma_d)} \\ & \times \left\{ \gamma_c \frac{\partial}{\partial t} [\nabla \cdot (\mathbf{K}_{dd} \cdot \nabla)] + \gamma_d \frac{\partial}{\partial t} [\nabla \cdot (\mathbf{K}_{cc} \cdot \nabla)] \right. \\ & \quad \left. - \gamma_c \gamma_d \frac{\partial}{\partial t} \left[\left(\langle \mathbf{v}_c \rangle^c + \langle \mathbf{v}_d \rangle^d \right) \cdot \nabla \right] \right\} \langle T_i \rangle^i \end{aligned}$$

$$\begin{aligned} & + \frac{1}{a_v h(\gamma_c + \gamma_d)} \\ & \times \left\{ \left[\nabla \cdot (\mathbf{K}_{cd} \cdot \nabla) \right] \left[\nabla \cdot (\mathbf{K}_{dc} \cdot \nabla) \right] \right. \\ & \quad \left. - \left[\nabla \cdot (\mathbf{K}_{cc} \cdot \nabla) \right] \left[\nabla \cdot (\mathbf{K}_{dd} \cdot \nabla) \right] \right\} \langle T_i \rangle^i \\ & + \left[\gamma_c \frac{\partial}{\partial t} (\mathbf{u}_{dd} \cdot \nabla \langle T_i \rangle^i) + \gamma_d \frac{\partial}{\partial t} (\mathbf{u}_{cc} \cdot \nabla \langle T_i \rangle^i) \right] \\ & - \left\{ (\mathbf{u}_{cc} \cdot \nabla) [\nabla \cdot (\mathbf{K}_{dd} \cdot \nabla)] \right. \\ & \quad + (\mathbf{u}_{dd} \cdot \nabla) [\nabla \cdot (\mathbf{K}_{cc} \cdot \nabla)] \\ & \quad - (\mathbf{u}_{cd} \cdot \nabla) [\nabla \cdot (\mathbf{K}_{dc} \cdot \nabla)] \\ & \quad - (\mathbf{u}_{dc} \cdot \nabla) [\nabla \cdot (\mathbf{K}_{cd} \cdot \nabla)] \\ & \quad - \gamma_c (\langle \mathbf{v}_c \rangle^c \cdot \nabla) [\mathbf{u}_{dd} \cdot \nabla + \nabla \cdot (\mathbf{K}_{dd} \cdot \nabla)] \\ & \quad \left. - \gamma_d (\langle \mathbf{v}_d \rangle^d \cdot \nabla) [\mathbf{u}_{cc} \cdot \nabla + \nabla \cdot (\mathbf{K}_{cc} \cdot \nabla)] \right\} \langle T_i \rangle^i \\ & - \left[(\mathbf{u}_{cc} \cdot \nabla) (\mathbf{u}_{dd} \cdot \nabla) - (\mathbf{u}_{cd} \cdot \nabla) (\mathbf{u}_{dc} \cdot \nabla) \right. \\ & \quad \left. + \gamma_c \gamma_d (\langle \mathbf{v}_c \rangle^c \cdot \nabla) (\langle \mathbf{v}_d \rangle^d \cdot \nabla) \right] \langle T_i \rangle^i. \quad (7) \end{aligned}$$

When the system is isotropic and the physical properties of the two phases are constant, it reduces to

$$\begin{aligned} & \frac{\partial \langle T_i \rangle^i}{\partial t} + \tau_q \frac{\partial^2 \langle T_i \rangle^i}{\partial t^2} + \langle \mathbf{v}_{cd} \rangle^{cd} \cdot \nabla \langle T_i \rangle^i \\ & = \alpha \Delta \langle T_i \rangle^i + \alpha \tau_T \frac{\partial}{\partial t} (\Delta \langle T_i \rangle^i) \\ & \quad + \frac{\alpha}{k} \left[F(\mathbf{r}, t) + \tau_q \frac{\partial F(\mathbf{r}, t)}{\partial t} \right], \quad (8) \end{aligned}$$

where

$$\begin{aligned} & \langle \mathbf{v}_{cd} \rangle^{cd} = \frac{1}{(\gamma_c + \gamma_d)} \left(\gamma_c \langle \mathbf{v}_c \rangle^c + \gamma_d \langle \mathbf{v}_d \rangle^d \right), \\ & \tau_q = \frac{\gamma_c \gamma_d}{a_v h(\gamma_c + \gamma_d)}, \\ & \tau_T = \frac{\gamma_c k_{dd} + \gamma_d k_{cc}}{a_v h(k_{cc} + k_{cd} + k_{dc} + k_{dd})}, \\ & k = k_{cc} + k_{cd} + k_{dc} + k_{dd}, \\ & \alpha = \frac{k_{cc} + k_{cd} + k_{dc} + k_{dd}}{\gamma_c + \gamma_d}, \quad (9) \end{aligned}$$

$$\begin{aligned}
& F(\mathbf{r}, t) + \tau_q \frac{\partial F(\mathbf{r}, t)}{\partial t} \\
&= \frac{1}{a_v h} \left\{ (k_{cd} k_{dc} - k_{cc} k_{dd}) \Delta^2 \langle T_i \rangle^i \right. \\
&\quad + \left[\gamma_c \frac{\partial}{\partial t} (\mathbf{u}_{dd} \cdot \nabla \langle T_i \rangle^i) + \gamma_d \frac{\partial}{\partial t} (\mathbf{u}_{cc} \cdot \nabla \langle T_i \rangle^i) \right. \\
&\quad \left. - \gamma_c \gamma_d \frac{\partial}{\partial t} (\langle \mathbf{v}_c \rangle^c + \langle \mathbf{v}_d \rangle^d) \cdot \nabla \langle T_i \rangle^i \right] \\
&\quad - \left\{ k_{dd} \Delta [(\mathbf{u}_{cc} + \gamma_c \langle \mathbf{v}_c \rangle^c) \cdot \nabla] \right. \\
&\quad \left. + k_{cc} \Delta [(\mathbf{u}_{dd} + \gamma_d \langle \mathbf{v}_d \rangle^d) \cdot \nabla] \right. \\
&\quad \left. - k_{dc} \Delta (\mathbf{u}_{cd} \cdot \nabla) - k_{cd} \Delta (\mathbf{u}_{dc} \cdot \nabla) \right\} \langle T_i \rangle^i \\
&\quad - \left\{ [(\mathbf{u}_{cc} + \gamma_c \langle \mathbf{v}_c \rangle^c) \cdot \nabla] (\mathbf{u}_{dd} \cdot \nabla) \right. \\
&\quad \left. - [(\mathbf{u}_{dd} + \gamma_d \langle \mathbf{v}_d \rangle^d) \cdot \nabla] (\mathbf{u}_{cc} \cdot \nabla) \right. \\
&\quad \left. - [(\mathbf{u}_{cd} + \gamma_c \langle \mathbf{v}_c \rangle^c) \cdot \nabla] \right. \\
&\quad \left. \times [(\mathbf{u}_{dc} + \gamma_d \langle \mathbf{v}_d \rangle^d) \cdot \nabla] \right\} \langle T_i \rangle^i \Big\}. \tag{10}
\end{aligned}$$

This is a dual-phase-lagging heat-conduction equation with τ_q and τ_T as the phase lags of the heat flux and the temperature gradient, respectively [15, 21, 22]. Here, $F(\mathbf{r}, t)$ is the volumetric heat source. k , ρc , α , and $\langle \mathbf{v}_{cd} \rangle^{cd}$ are the effective thermal conductivity, capacity, diffusivity, and velocity of nanofluids, respectively. Therefore, the presence of nanoparticles shifts the classical energy equation for the heat transfer in the base fluid into the dual-phase-lagging energy equation in nanofluids at the macroscale. This is significant because all results regarding dual-phase-lagging heat transfer can thus be applied to study heat transfer in nanofluids [21, 22]. For the case of heat conduction in nanofluids, the inclusion of the solid-fluid interface heat transfer also leads to dual-phase-lagging heat-conduction equation, which resolves the conflict between experimental data of nanofluid thermal conductivity and classical theories of effective thermal conductivity of suspensions [23].

It is interesting to note that there are nontraditional convective terms $-\mathbf{u}_{cc} \cdot \nabla \langle T_c \rangle^c - \mathbf{u}_{cd} \cdot \nabla \langle T_d \rangle^d$ and $-\mathbf{u}_{dc} \cdot \nabla \langle T_c \rangle^c - \mathbf{u}_{dd} \cdot \nabla \langle T_d \rangle^d$ in (2). In (8), however, such terms disappear because of the constraint from the mass conservation. Therefore, the microscale physics does not manifest itself as the macroscale convection. The velocity-like terms appear only in the source term in (8).

The presence of nanoparticles gives rise to variations of thermal capacity, conductivity, and diffusivity, which are given by, in terms of ratios over those of the base fluid,

$$\begin{aligned}
\frac{\rho c}{(\rho c)_c} &= (1 - \varepsilon_d) + \varepsilon_d \frac{(\rho c)_d}{(\rho c)_c}, \\
\frac{k}{k_c} &= \frac{k_{cc} + k_{cd} + k_{dc} + k_{dd}}{k_c}, \\
\frac{\alpha}{\alpha_c} &= \frac{k}{k_c} \frac{(\rho c)_c}{\rho c}. \tag{11}
\end{aligned}$$

Therefore, $\rho c/(\rho c)_c$ depends *only* on the volume fraction of nanoparticles and the nanoparticle-fluid capacity ratio. However, both k/k_c and α/α_c are affected by the geometry, property and dynamic process of nanoparticle-fluid interfaces.

Consider

$$\frac{\tau_T}{\tau_q} = 1 + \frac{\gamma_c^2 k_{dd} + \gamma_d^2 k_{cc} - \gamma_c \gamma_d (k_{cd} + k_{dc})}{\gamma_c \gamma_d (k_{cc} + k_{cd} + k_{dc} + k_{dd})}. \tag{12}$$

It can be larger, equal, or smaller than 1 depending on the sign of $\gamma_c^2 k_{dd} + \gamma_d^2 k_{cc} - \gamma_c \gamma_d (k_{cd} + k_{dc})$. Therefore, by the condition for the existence of thermal waves that requires $\tau_T/\tau_q < 1$ [22, 24], we may have thermal waves in nanofluid heat transfer when

$$\begin{aligned}
& \gamma_c^2 k_{dd} + \gamma_d^2 k_{cc} - \gamma_c \gamma_d (k_{cd} + k_{dc}) \\
&= \left(\gamma_c \sqrt{k_{dd}} - \gamma_d \sqrt{k_{cc}} \right)^2 + \gamma_c \gamma_d \left(2\sqrt{k_{cc} k_{dd}} - k_{cd} - k_{dc} \right) < 0. \tag{13}
\end{aligned}$$

A necessary (but not sufficient) condition for (13) is $k_{cd} + k_{dc} > 2\sqrt{k_{cc} k_{dd}}$. Note also that for heat transfer in nanofluids the microscale physics yields a time-dependent source term $F(\mathbf{r}, t)$ in the dual-phase-lagging energy equation ((8) and (10)). Therefore, the resonance can also occur. These thermal waves and possibly resonance are believed to be the driving force for the enhancement of heat transfer. When $k_{cd} + k_{dc} = 0$ so that τ_T/τ_q is always larger than 1, thermal waves and resonance would not appear. The sum $k_{cd} + k_{dc}$ is thus responsible for thermal waves and resonance in nanofluid heat transfer. It is also interesting to note that although each τ_q and τ_T is $a_v h$ -dependent, the ratio τ_T/τ_q is not. Therefore the evaluation of τ_T/τ_q will be much simpler than τ_q or τ_T .

Therefore, the molecular physics and the microscale physics (interactions between nanoparticles and base fluids at the microscale in particular) manifest themselves as heat diffusion and thermal waves/resonance at the macroscale, respectively. Their overall macroscopic manifestation shifts the classical energy equation for the heat transfer in the base fluid into the dual-phase-lagging energy equation in nanofluids. When $\tau_T/\tau_q < 1$, thermal waves dominate and (8) is of a hyperbolic type [22]. When $\tau_T/\tau_q \geq 1$, however, heat diffusion dominates and (8) is parabolic [22]. Depending on factors like material properties of nanoparticles and base fluids, nanoparticles' geometrical structure and their distribution in the base fluids, and interfacial properties and dynamic processes on particle-fluid interfaces, the heat diffusion, thermal waves/resonance, and convection may either enhance or counteract each other. Consequently, the heat transfer in nanofluids is endowed with much richer features than that in the base fluid.

4. Concluding Remarks

In an attempt to determine how the presence of nanoparticles affects the heat transfer at the macroscale and isolates the mechanism responsible for the reported variation of thermal properties, a macroscale energy equation is developed and

examined analytically for nanofluid heat transfer. The model is obtained by scaling-up the microscale model for the heat transfer in the nanoparticles and in the base fluids. The approach for scaling-up is the volume averaging with help of multiscale theorems. The result shows that the presence of nanoparticles leads to a dual-phase-lagging energy equation in nanofluids at the macroscale. Therefore, the molecular physics and the microscale physics manifest themselves as heat diffusion and thermal waves at the macroscale, respectively. Depending on factors like material properties of nanoparticles and base fluids, nanoparticles' geometrical structure and their distribution in the base fluids, and interfacial properties and dynamic processes on particle-fluid interfaces, the heat diffusion, convection, and thermal waves may either enhance or counteract each other, which will enrich heat-transfer performance significantly.

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