A New Robust Kalman Filter-Based Subspace Tracking Algorithm in an Impulsive Noise Environment

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***Abstract*—The conventional projection approximation subspace tracking (PAST) algorithm is based on the recursive least-squares algorithm, and its performance will degrade considerably when the subspace rapidly changes and the additive noise is impulsive. This brief proposes a new robust Kalman filter-based subspace tracking algorithm to overcome these two limitations of the PAST algorithm. It is based on a new extension of the adaptive Kalman filter with variable number of measurements (KFVNM) for tracking fast-varying subspace. Furthermore, M-estimation is incorporated into this KFVNM algorithm to combat the adverse effects of impulsive noise. Simulation results show that the robust KFVNM-based subspace tracking algorithm has a better performance than the PAST algorithm for tracking fast-varying subspace and in an impulsive noise environment.**

***Index Terms*—Impulsive noise, Kalman filter, Kalman filter with variable number of measurements (KFVNM), least squares, projection approximation subspace tracking (PAST).**

I. Introduction

Subspace-based methods play a key role in array signal processing and many other system applications. For instance, the conventional direction-of-arrival (DOA) estimation method MUSIC [1] and the more recent work on DOA and mutual coupling coefficient estimation method [2] are both subspace-based methods. Usually, the invariant subspace can be computed from the singular value decomposition (SVD) of the array output or the eigenvalue decomposition (EVD) of the covariance matrix of the array output. However, updating the subspace through EVD or SVD online is computationally expensive. Therefore, subspace tracking methods with low computational complexity have received considerable attention [3]–[7], [10]. An efficient class of algorithms is the projection approximation subspace tracking (PAST) algorithm and its variants [3]–[5]. It employs the recursive least-squares (RLS) algorithm to recursively estimate the signal subspace by minimizing the least-squares (LS) error between the current measurement and a "projection approximation" obtained from previously estimated subspace. However, the conventional PAST method also has two limitations, both of which stem from the nature of the RLS algorithm it uses. First, since the RLS-based PAST method requires the subspace to be slowly varying and the estimate is solely based on the measurements, it may be difficult to track fast changing subspace [6]. Second, the RLS algorithm is vulnerable to impulsive noise, and the performance of the PAST algorithm will be degraded substantially when the measurements are corrupted by impulsive noise, which are frequently encountered in man-made electromagnetic interference and other natural noises [9]. The later problem was studied in [10], where M-estimation [11] in robust statistics is incorporated into PAST to suppress the adverse effect of the impulsive noise. However, the tracking speed is somewhat limited as mentioned above.

In this brief, we propose a new robust Kalman filter-based subspace tracking method in impulsive noise to address the above limitations. We first adopt the Kalman filter, instead of the RLS algorithm, to track the time-varying subspace. The Kalman filter algorithm is an optimal recursive state estimator in the minimum mean-square error sense, and it has a better tracking ability than RLS since it uses a state-space model to describe the subspace dynamics. To further enhance the tracking ability, a new extension of the Kalman filter with variable number of measurements (KFVNM) algorithm in [7] is proposed. A new measure of subspace variations is introduced and incorporated into the KFVNM algorithm to improve its performance. Moreover, a robust method for recursively estimating the covariance matrices of the KFVNM algorithm is proposed to handle their time-varying behaviors. Consequently, when the subspace substantially varies, a small number of measurements will be employed in KFVNM so that the estimation bias introduced by remote and unrelated measurements will be small. On the other hand, if the subspace is slowly varying, a large number of measurements will be used to reduce the estimation variance. Thus, the proposed KFVNM method can provide a better flexibility than the PAST algorithm, particularly in highly dynamic environments. Next, to combat against the impulsive noise, M-estimation in robust statistics [9]–[12] is applied to the KFVNM algorithm. More precisely, a robust statistics-based estimate of the measurement noise variance is calculated and used for detecting whether the incoming measurement vector is contaminated by impulsive noise. The "impulsive-free" noise variance is estimated by assuming that the impulsive noise is a contaminated Gaussian (CG) process as in [9] and [10]. By assigning a small or even zero weight to a potentially impulse-contaminated measurement in the KFVNM algorithm, the adverse effects of impulsive noise can be suppressed. It is worth noting that the particle filter may also be adopted for tracking targets in non-Gaussian noise [13], [14]. However, because of its high complexity, we shall not pursue such direction in this study.

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II. System Model and PAST Algorithm

We consider the subspace tracking problem in an antenna array with N sensors impinged by r narrow-band incoherent signals. The array output observed at the tth snapshot consists of the N sensor outputs, which can be written as

$$\boldsymbol{x}(t) = \sum_{k=1}^{r} \boldsymbol{a}(\omega_k)s_k(t) + \boldsymbol{n}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{n}(t) \tag{1}$$

where $\boldsymbol{a}(\omega_k)$ is the steering vector corresponding to the kth DOA for the given array geometry. For a uniform linear array (ULA), $\boldsymbol{a}(\omega_k) = [1, e^{j\omega_k}, \ldots, e^{j(N-1)\omega_k}]^T$ and $\omega_k = 2\pi\lambda^{-1}d \sin\theta_k$, with λ, d, and θ_k denoting the carrier wavelength, intersensor spacing, and kth DOA, respectively. $\boldsymbol{A} = [\boldsymbol{a}(\omega_1), \boldsymbol{a}(\omega_2), \ldots, \boldsymbol{a}(\omega_r)]$ denotes the steering matrix, $\boldsymbol{s}(t)$ is the vector of signal waveforms, and $\boldsymbol{n}(i)$ is the noise vector, which is commonly considered to be an additive white Gaussian noise (AWGN) vector with zero mean and covariance matrix $\sigma^2\boldsymbol{I}$, where $\boldsymbol{I}$ is an identity matrix. Conventionally, the signal subspace $\boldsymbol{U}_s$ is obtained from the EVD of the covariance matrix of the array output, i.e.,

$$\boldsymbol{R}_{xx} = E\left[\boldsymbol{x}(t)\boldsymbol{x}^H(t)\right] = \boldsymbol{A}\boldsymbol{R}_{ss}\boldsymbol{A}^H + \sigma^2\boldsymbol{I} \tag{2}$$

where $\boldsymbol{R}_{ss} = E[\boldsymbol{s}(t)\boldsymbol{s}^H(t)]$ is the signal autocovariance matrix.

For online implementation, the PAST algorithm estimates the subspace by minimizing the following cost function of $\boldsymbol{W}(t)$:

$$J\left(\boldsymbol{W}(t)\right) = \sum_{i=1}^{t} \beta^{t-i} \left\|\boldsymbol{x}(i) - \boldsymbol{W}(t)\boldsymbol{y}(i)\right\|_2^2 \tag{3}$$

where $\boldsymbol{y}(i) = \boldsymbol{W}^H(i-1)\boldsymbol{x}(i)$ is the projection approximation used, and $0 < \beta \leq 1$ is the forgetting factor. It has been shown in [3] that (3) is minimized when the column of $\boldsymbol{W}(t)$ is equal to the signal subspace. However, as mentioned before, the performance of the PAST algorithm is considerably degraded in fast-varying subspace. Moreover, because of the LS cost function adopted in (3), the estimate will be significantly deteriorated by impulsive noise. Hence, a robust KFVNM algorithm is proposed in the following sections to overcome these problems.

III. Kalman Filter-Based Subspace Tracking With Variable Number of Measurements

A. KFVNM

We first introduce the proposed KFVNM algorithm. This algorithm is derived from the following state-space model:

$$\boldsymbol{z}(t) = \boldsymbol{F}(t)\boldsymbol{z}(t-1) + \boldsymbol{w}(t) \tag{4}$$

$$\boldsymbol{x}(t) = \boldsymbol{H}(t)\boldsymbol{z}(t) + \boldsymbol{\delta}(t) \tag{5}$$

where $\boldsymbol{z}(t)$ and $\boldsymbol{x}(t)$ are the state vector and the observation vector, respectively. $\boldsymbol{F}(t)$ and $\boldsymbol{H}(t)$ are the state transition matrix and the observation matrix, respectively, and $\boldsymbol{w}(t)$ and $\boldsymbol{\delta}(t)$ are zero-mean Gaussian noise with covariance matrix $\boldsymbol{Q}_w(t)$ and $\boldsymbol{R}_\delta(t)$, respectively. It is well known that the standard Kalman filter can be utilized to update the state $\boldsymbol{z}(t)$ recursively based on the state-space model [15]. Moreover, it has been proved that the Kalman filter is equivalent to a linear regression problem since (4) and (5) can be stacked together as [16]

$$\begin{bmatrix} \boldsymbol{I} \\ \boldsymbol{H}(t) \end{bmatrix} \boldsymbol{z}(t) = \begin{bmatrix} \boldsymbol{F}(t)\hat{\boldsymbol{z}}(t-1) \\ \boldsymbol{x}(t) \end{bmatrix} + \begin{bmatrix} \boldsymbol{F}(t)\left[\boldsymbol{z}(t-1) - \hat{\boldsymbol{z}}(t-1)\right] + \boldsymbol{w}(t) \\ -\boldsymbol{\delta}(t) \end{bmatrix}. \tag{6}$$

We notice that only one single measurement $\boldsymbol{x}(t)$ is used for the state update at each iteration. The number of measurements used in the Kalman filter can be carefully selected to achieve a better bias–variance tradeoff.

If $L(t)$ measurements, i.e., $\widetilde{\boldsymbol{x}}(t) = [\boldsymbol{x}^T(t-L(t)+1), \ldots, \boldsymbol{x}^T(t-1), \boldsymbol{x}^T(t)]^T$, are used for the state update at time t, the linear state-space model will be extended to

$$\begin{bmatrix} \boldsymbol{I} \\ \tilde{\boldsymbol{H}}(t) \end{bmatrix} \boldsymbol{z}(t) = \begin{bmatrix} \boldsymbol{F}(t)\hat{\boldsymbol{z}}(t-1) \\ \tilde{\boldsymbol{x}}(t) \end{bmatrix} + \boldsymbol{\Delta}(t) \tag{7}$$

where $\tilde{\boldsymbol{H}}(t) = [\boldsymbol{H}^T(t-L(t)+1), \ldots, \boldsymbol{H}^T(t-1), \boldsymbol{H}^T(t)]^T$,

$$\boldsymbol{\Delta}(t) = \begin{bmatrix} \boldsymbol{F}(t)\left[\boldsymbol{z}(t-1) - \hat{\boldsymbol{z}}(t-1)\right] + \boldsymbol{w}(t) \\ -\tilde{\boldsymbol{\delta}}(t) \end{bmatrix} \tag{8}$$

with $\widetilde{\boldsymbol{\delta}}(t) = [\boldsymbol{\delta}^T(t-L(t)+1), \ldots, \boldsymbol{\delta}^T(t-1), \boldsymbol{\delta}^T(t)]^T$ and covariance

$$E\left[\boldsymbol{\Delta}(t)\boldsymbol{\Delta}^T(t)\right] = \begin{bmatrix} \boldsymbol{P}(t/t-1) & \boldsymbol{0} \\ \boldsymbol{0} & \bar{\boldsymbol{R}}_\delta(t) \end{bmatrix} \tag{9}$$

where $\boldsymbol{P}(t/t-1) = \boldsymbol{F}(t)\boldsymbol{P}(t-1/t-1)\boldsymbol{F}^T(t) + \boldsymbol{Q}_w(t)$ is the *a priori* estimate covariance with $\boldsymbol{P}(t-1/t-1)$ being *a posteriori* estimate covariance in the standard Kalman filter recursions and $\bar{\boldsymbol{R}}_\delta(t) = \mathrm{diag}\{\boldsymbol{R}_\delta(t-L(t)+1), \ldots, \boldsymbol{R}_\delta(t-1), \boldsymbol{R}_\delta(t)\}$. Let $\boldsymbol{S}(t)$ be the Cholesky decomposition of (9), then multiplying both sides of (7) by $\boldsymbol{S}^{-1}(t)$ will lead to a linear regression as follows:

$$\bar{\boldsymbol{X}}(t) = \bar{\boldsymbol{H}}(t)\boldsymbol{z}(t) + \bar{\boldsymbol{n}}(t) \tag{10}$$

where $\bar{\boldsymbol{X}}(t) = \boldsymbol{S}^{-1}(t)[(\boldsymbol{F}(t)\hat{\boldsymbol{z}}(t-1))^T, \tilde{\boldsymbol{x}}^T(t)]^T$, $\bar{\boldsymbol{H}}(t) = \boldsymbol{S}^{-1}(t)[\boldsymbol{I}, \tilde{\boldsymbol{H}}^T(t)]^T$, and $\bar{\boldsymbol{n}}(t) = -\boldsymbol{S}^{-1}(t)\boldsymbol{\Delta}(t)$, with $E[\bar{\boldsymbol{n}}(t)\bar{\boldsymbol{n}}^T(t)] = \boldsymbol{I}$. The LS solution of (7) is given by

$$\begin{aligned} \hat{\boldsymbol{z}}(t) &= \arg\min_{z} \left\|\bar{\boldsymbol{X}}(t) - \bar{\boldsymbol{H}}(t)\boldsymbol{z}(t)\right\|_2^2 \\ &= \left(\bar{\boldsymbol{H}}^T(t)\bar{\boldsymbol{H}}(t)\right)^{-1} \bar{\boldsymbol{H}}^T(t)\bar{\boldsymbol{X}}(t) \end{aligned} \tag{11}$$

with covariance $\boldsymbol{P}(t) = (\bar{\boldsymbol{H}}^T(t)\bar{\boldsymbol{H}}(t))^{-1}$. Hence, $\hat{\boldsymbol{z}}(t)$ and $\boldsymbol{P}(t)$ will be adopted for updating $\hat{\boldsymbol{z}}(t/t)$ and $\boldsymbol{P}(t/t)$ instead of those in the standard Kalman filter. We know that the variance of the estimator in (11) will be decreased when more measurements are included. However, if the system state $\boldsymbol{z}(t)$ rapidly varies, more measurements, corresponding to different underlying states, will result in a large estimation bias. Thus, it is desirable to select an appropriate number of measurements $L(t)$ to achieve a proper tradeoff between bias and variance. As shown in [7], $L(t)$ can be selected based on the approximation of subspace variations. We first define state variation as

$$\hat{\boldsymbol{e}}(t) = \hat{\boldsymbol{z}}(t-1) - \tilde{\boldsymbol{z}}(t-1) \tag{12}$$

$$\tilde{\boldsymbol{z}}(t) = \lambda_{\boldsymbol{z}}\tilde{\boldsymbol{z}}(t-1) + (1-\lambda_{\boldsymbol{z}})\hat{\boldsymbol{z}}(t-1) \tag{13}$$

where $0 < \lambda_z \leq 1$ is a forgetting factor for smoothing the past states. We find that $\|\hat{e}(t)\|_2$ can serve as a measure of the variation of subspace since, when $\boldsymbol{z}(t)$ rapidly changes, the approximated absolute derivation of $\|\hat{e}(t)\|_2$

$$G_e(t) = \left| \|\hat{\boldsymbol{e}}(t)\|_2 - \|\hat{\boldsymbol{e}}(t-1)\|_2 \right| \tag{14}$$

will be large, and vice versa. Hence, $G_e(t)$ can help estimate how many measurements should be used to update the state. In this brief, we propose to update $L(t)$ at each snapshot as

$$L(t) = round\left\{L_L + \left[1 - g\left(\overline{G}_N(t)\right)\right](L_U - L_L)\right\} \tag{15}$$

where $\overline{G}_N(t) = \overline{G}_e(t)/\overline{G}_{e0}$, with $\overline{G}_e(t)$ averaging from $G_e(t)$ over a time window of length T_s and $\overline{G}_{e0}$ averaging from the first T_s estimates of $\overline{G}_e(t)$. $round\{\cdot\}$ denotes rounding a value to the nearest integer; L_L and L_U are the lower and upper bounds of $L(t)$, respectively; and $g(x) = \min\{\max\{x, 0\}, 1\}$ is a clipping function to keep $\overline{G}_N(t)$ within the interval [0, 1]. Equation (15) differs from that in [7] because a clipping function $g(x)$ is employed to make the estimate more stable and, hence, reduce the variance of the subspace estimate. As expected, more measurements will be used if the subspace variation measure $\overline{G}_N(t)$ is small, and vice versa.

B. KFVNM-Based Subspace Tracking

Associating $\boldsymbol{W}(t)$ with the subspace instead of $\boldsymbol{z}(t)$ in (4) yields the following state-space model for subspace tracking:

$$\boldsymbol{W}^T(t) = \boldsymbol{F}(t)\boldsymbol{W}^T(t-1) + \boldsymbol{\Xi}(t) \tag{16}$$

$$\boldsymbol{x}^T(t) = \boldsymbol{H}(t)\boldsymbol{W}^T(t) + \boldsymbol{\Psi}(t). \tag{17}$$

In (16) and (17), $\boldsymbol{\Xi}(t)$ and $\boldsymbol{\Psi}(t)$ are innovation and residual error matrices, respectively. Moreover, the observation matrix $\boldsymbol{H}(t)$ can be approximated as $\boldsymbol{H}(t) = \boldsymbol{x}^T(t)\hat{\boldsymbol{W}}^*(t-1)$. Alternatively, we propose to employ a better estimate of $\boldsymbol{H}(t)$ as

$$\boldsymbol{H}(t) = \boldsymbol{x}^T(t)\hat{\boldsymbol{W}}^*(t/t-1) \tag{18}$$

where the superscript $*$ denotes the complex conjugate operation. Next, with the help of the proposed KFVNM algorithm and the dynamic model in (16) and (17), one can update and track the subspace $\boldsymbol{W}(t)$ with $L(t)$ measurements. Hence, the resultant KFVNM-based subspace tracking algorithm requires $O(L^3) + (L+1)Nr + 2Lr^2 + L^2r$ operations in each update. For slowly varying subspace, a large L is used to reduce the variance with complexity tending to be $O(L^3)$, whereas for fast-varying subspace, a small L is selected to reduce the bias with considerably smaller complexity. In fact, the KFVNM algorithm can be switched to an RLS-based algorithm (e.g., PAST) to avoid high complexity when L is large [7].

IV. Robust KFVNM-Based Subspace Tracking

It can be seen from (10) and (11) that the state $\boldsymbol{z}(t)$ is estimated from the past $L(t)$ observations. Hence, any impulse-contaminated measurement $\boldsymbol{x}(k)$ with $t - L(t) + 1 \leq k \leq t$ will impair the estimate of $\boldsymbol{z}(t)$ and the selection of $L(t)$. In M-estimation, an M-estimate cost function is employed instead of the LS objective function in (11) to combat the adverse effect of the outliers. Since we only consider outliers in the measurements, the M-estimate cost function is only applied to the measurement equations to yield the following cost function:

$$J(\boldsymbol{z}(t)) = \frac{1}{2}\|\boldsymbol{e}_o(\boldsymbol{t})\|_2^2 + \sum_{k=t-L(t)+1}^{t} \rho\left(\|\underline{\boldsymbol{e}}(k)\|_2\right) \tag{19}$$

where $\rho(e)$ is an M-estimation function, $\boldsymbol{e}_o(t) = \boldsymbol{S}_o^{-1}(t)(\boldsymbol{F}(t)\hat{\boldsymbol{z}}(t-1) - \boldsymbol{z}(t))$, and $\underline{\boldsymbol{e}}(k) = \boldsymbol{S}_k^{-1}(t)(\boldsymbol{x}(k) - \boldsymbol{H}(k)\boldsymbol{z}(t))$, with $\boldsymbol{S}_o(t)$ and $\boldsymbol{S}_k(t)$ being the Cholesky decomposition of $\boldsymbol{P}(t/t-1)$ and $\boldsymbol{R}_\delta(k)$ in (9), respectively. Usually, the values of $\rho(e)$ are small for large values of $|e|$ to combat the adverse effect of the large measurement outliers. For smaller values of $|e|$, $\rho(e)$ will reduce to the square function to achieve high estimation efficiency. A simple M-estimation function is the modified Huber function [11], [12], i.e.,

$$\rho(e) = \begin{cases} e^2/2, & 0 \leq |e| < \xi \\ \xi^2/2, & \xi \leq |e| \end{cases} \tag{20}$$

where ξ is a threshold parameter used to control the suppression of the outliers, and it needs to be estimated recursively. Following the technique proposed in [10, Sec. III], ξ at the tth time instant can be estimated from the "impulsive-free" error variance $\overline{\sigma}(t)$ as $\xi(t) = 1.96\overline{\sigma}(t)$, where

$$\overline{\sigma}^2(t) = \lambda_\sigma\overline{\sigma}^2(t-1) + C_1(1-\lambda_\sigma)\text{med}\left(A_\sigma(t)\right) \tag{21}$$

$A_\sigma(t) = \{\|\underline{\boldsymbol{e}}(t-N_\sigma+1)\|_2^2, \ldots, \|\underline{\boldsymbol{e}}(t)\|_2^2\}$, med$(\cdot)$ is the median operator, $C_1 = 1.483(1 + 5/(N_\sigma - 1))$ is a finite sample correction factor, and N_σ is the length of the median operation, which is usually chosen to be 5–11 [10]. Since the sensor noise without the outlier is a Gaussian process, it may be simpler to use $\boldsymbol{\delta}(t) = \boldsymbol{x}(t) - \boldsymbol{H}(t)\boldsymbol{z}(t)$ instead of $\underline{\boldsymbol{e}}(t)$ in (19).

A necessary condition for the optimal solution of (19) is $\nabla_z J(\boldsymbol{z}(t)) = \boldsymbol{0}$, i.e.,

$$\left(\boldsymbol{S}_o^{-1}(t)\right)^T \boldsymbol{e}_o(t) + \sum_{k=t-L(t)+1}^{t} \frac{\rho'\left(\|\underline{\boldsymbol{e}}(k)\|_2\right)}{\|\underline{\boldsymbol{e}}(k)\|_2}\left(\boldsymbol{S}_k^{-1}(t)\boldsymbol{H}(k)\right)^T \underline{e}(k) = \boldsymbol{0}. \tag{22}$$

After some manipulation, one gets

$$\hat{\boldsymbol{z}}(t) = \left(\bar{\boldsymbol{H}}^T(t)\boldsymbol{\Omega}(t)\bar{\boldsymbol{H}}(t)\right)^{-1}\bar{\boldsymbol{H}}^T(t)\boldsymbol{\Omega}(t)\bar{\boldsymbol{X}}(t) \tag{23}$$

where $\boldsymbol{\Omega}(t)$ is a weight matrix

$$\boldsymbol{\Omega}(t) = \text{diag}\left\{\boldsymbol{I}, q\left(\|\underline{e}\left(t-L(t)+1\right)\|_2\right)\boldsymbol{I}, \ldots, q\left(\|\underline{e}(t)\|_2\right)\boldsymbol{I}\right\} \tag{24}$$

$q(e)$ is the weight function

$$q(e) = \frac{\rho'(e)}{e} = \begin{cases} 1, & 0 \leq |e| < \xi \\ 0, & \xi \leq |e| \end{cases} \tag{25}$$

and $\rho'(e)$ is the derivative of $\rho(e)$. It can be seen that (23) is a system of nonlinear equation, and its solution usually requires iterative method such as the iteratively reweighted LS (IRLS)

TABLE I
ROBUST KFVNM-BASED SUBSPACE TRACKING ALGORITHM

Stage 1. Initialize $\boldsymbol{W}(0)$, $\boldsymbol{R}_\delta(0)$, $\boldsymbol{Q}_w(0)$ and $L(0)$.

Stage 2. For $t = 1, 2, \ldots, T_s$ do

- i). Calculate $\hat{\boldsymbol{e}}(t)$ as (12) and (13).
- ii). Calculate $G_e(t)$ as (14) and $\overline{G}_{e0}$ by averaging first t estimates $G_e(t)$.
- iii). Calculate $L(t)$ as (15) and update as $\min\{L(t), t\}$.
- iv). Examine the estimating error $\hat{\sigma}(t)$ and update the weight matrix $\boldsymbol{\Omega}(t)$ as (24).
- v). Estimate $\boldsymbol{W}(t)$ using the robust KFVNM as (23).
- vi). Update the noise covariance matrix $\boldsymbol{Q}_w(t)$ and $\boldsymbol{R}_\delta(t)$ as (26) and (27).

End t

At $t = T_s$, obtain $\overline{G}_{e0}$ by averaging first T_s estimates $G_e(t)$.

Stage 3. For $t = T_s+1, T_s+2, \ldots$ do

- i). Calculate $\hat{\boldsymbol{e}}(t)$ as (12) and (13).
- ii). Calculate $G_e(t)$ as (14) and obtain $\overline{G}_N(t)$ by normalizing $\overline{G}_e(t)$ with $\overline{G}_{e0}$.
- iii). Update $L(t)$ as (15).
- iv). Examine the estimating error $\hat{\sigma}(t)$ and update the weight matrix $\boldsymbol{\Omega}(t)$ as (24).
- v). Estimate $\boldsymbol{W}(t)$ using the robust KFVNM as (23).
- vi). Update the noise covariance matrix $\boldsymbol{Q}_w(t)$ and $\boldsymbol{R}_\delta(t)$ as (26) and (27).

End t

method, in which the previous iteration is used to compute the weight matrix. This process is repeated until convergence. Fortunately, the *a priori* state estimate $\boldsymbol{F}(t)\hat{\boldsymbol{z}}(t-1)$ is usually a good estimate of $\boldsymbol{z}(t)$, and hence, it can be used to compute the weight at (24). Hence, a satisfactory result can be obtained usually after one iteration. Basically, the modified Huber function assigns a weight of one to ordinary samples and a zero weight to those samples with a very large prediction error.

We now consider the estimation of $\boldsymbol{Q}_w(t)$ and $\boldsymbol{R}_\delta(t)$. In [7], they are assumed to be known *a priori*, whereas in this work, they are estimated in the presence of possible outliers. In particular, we shall extend the method in [8] to the proposed KFVNM algorithm, as shown in the following:

$$\boldsymbol{Q}_w(t) = \lambda_w \boldsymbol{Q}_w(t-1) + (1-\lambda_w)\tilde{\boldsymbol{Q}}_w(t) \tag{26}$$

$$\boldsymbol{R}_\delta(t) = \lambda_\delta \boldsymbol{R}_\delta(t-1) + (1-\lambda_\delta)\tilde{\boldsymbol{R}}_\delta(t). \tag{27}$$

Here, λ_w and $\lambda_\delta \in (0,1]$ are the forgetting factors; $\tilde{\boldsymbol{Q}}_w(t)$ is estimated from the errors with a window length of N_w, i.e., $\hat{\boldsymbol{w}}(k) = \hat{\boldsymbol{z}}(k/k) - \boldsymbol{F}(k)\hat{\boldsymbol{z}}(k-1/k-1)$, $t - N_w + 1 \le k \le t$; and $\tilde{\boldsymbol{R}}_\delta(t)$ is estimated from the errors with a window length of N_δ, i.e., $\hat{\boldsymbol{\delta}}(k) = \boldsymbol{x}(k) - \boldsymbol{H}(k)\hat{\boldsymbol{z}}(k/k), t - N_\delta + 1 \le k \le t$. Typical values of N_w and N_δ are 1–5. Whenever $q(\|\underline{\boldsymbol{e}}(k)\|_2)$ is found to be zero, $\hat{\boldsymbol{\delta}}(k)$ is likely to be corrupted by impulsive noise, and these corrupted observations will be excluded from updating the covariance matrices $\boldsymbol{Q}_w(t)$ and $\boldsymbol{R}_\delta(t)$ in (26) and (27). By applying the Kalman filter to each row of $\boldsymbol{W}(t)$, one gets the proposed robust KFVNM subspace tracking algorithm in Table I. It should be noted that, at the beginning of the algorithm, the selected $L(t)$ may be larger than the number of available samples. Hence, $L(t)$ should be chosen as $\min\{L(t), t\}$.

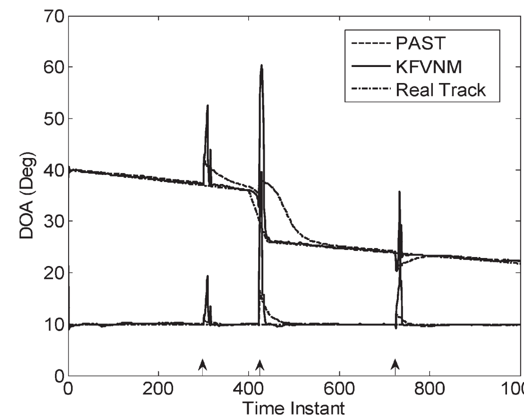

Fig. 1. DOA tracking in impulsive noise using KFVNM and PAST. Arrows indicate the time instants when the impulses are applied.

V. SIMULATION RESULTS

To evaluate the performances of the proposed robust KFVNM-based subspace tracking algorithm, computer simulations of a DOA tracking problem is performed. A ULA with $N = 10$ sensors separated by a half-wavelength is considered, and the conventional ESPRIT algorithm is utilized to estimate the DOAs from the subspace. Two uncorrelated narrow-band signals with SNR $= 10$ dB impinge on the array from the far field. The first DOA is time invariant at $\theta_1 = 10^\circ$, whereas the second DOA is time varying, as specified by

$$\theta_2(t) = \begin{cases} 40 - 1 \times 10^{-2}t, & 0 \le t < 400 \\ 36 - 2.5 \times 10^{-1}(t-400), & 400 \le t \le 440 \\ 26 - 7.5 \times 10^{-3}(t-440), & 440 < t \le 1000 \end{cases}$$

where θ_2 slowly changes at the time intervals [0, 400] and [440, 1000] and rapidly changes at the time interval [400, 440]. The ambient noise is assumed to be CG [9, Sec. II]. For simplicity, we assume that the nominal noise is zero-mean white Gaussian distributed with a power of 0 dB, whereas the impulsive noise has the same distribution with a power of 30 dB occurring at the time intervals [301, 303], [424, 425], and [726, 727]. The locations are fixed so that their effects can be visualized more clearly. The conventional PAST [3] and robust PAST [9] algorithms are also tested for comparison, and the forgetting factor of these two algorithms is set to be 0.98 according to [3] and [10]. The parameters of the KFVNM algorithm are $\lambda_z = \lambda_w = \lambda_\delta = \lambda_\sigma = 0.99$, $L_L = 1$, $L_U = 50$, $T_s = 100$, $N_w = N_\delta = 5$, and $N_\sigma = 10$.

The DOA tracking results of the proposed KFVNM-based subspace tracking algorithm and the PAST algorithm are shown in Fig. 1. We can notice that both methods are significantly affected by the impulsive noise, and it takes a long time to forget these adverse effects. Conversely, due to the robustness of the robust KFVNM algorithm, the impulsive noise can be satisfactorily suppressed, and the subspace is accurately tracked, as shown in Fig. 2. Furthermore, the proposed robust KFVNM algorithm is able to offer a better tracking performance than the robust PAST algorithm, since a dynamic model of fast-varying subspace is employed. Fig. 3 shows the number of measurements selected by the proposed algorithms. It shows that the selection in the LS-based KFVNM algorithm is significantly affected by the impulsive noise, whereas the robust KFVNM

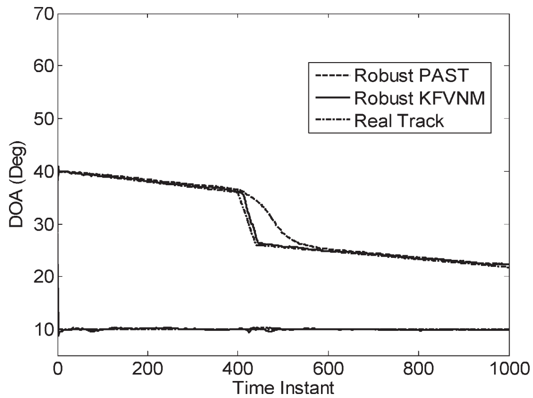

Fig. 2. DOA tracking in impulsive noise using robust KFVNM and robust PAST.

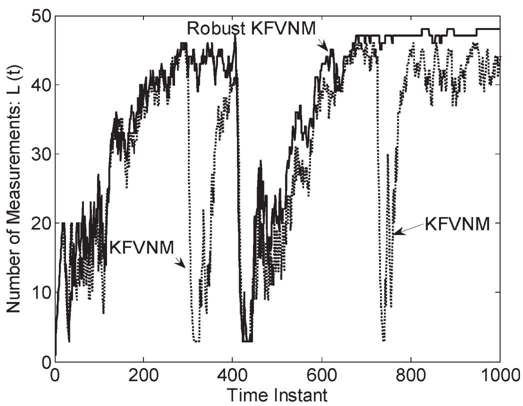

Fig. 3. Number of measurements $L(t)$ used in (dashed line) KFVNM and (solid line) robust KFVNM in an impulsive noise environment.

algorithm is able to suppress the impulsive noise effectively and select the measurements more appropriately.

Next, impulsive noise is added to the measurements at three randomly selected time instants, and each impulsive noise lasts for two or three time instants. Five hundred Monte Carlo simulations are run, and the SNR is 10 dB. The root mean squared error (RMSE) at each time instant is calculated as RMSE $= \sqrt{\sum_{i=1}^{K}\sum_{n=1}^{r}(\theta_n - \hat{\theta}_{i,n})^2/(Kr)}$, where K is the total number of Monte Carlo experiments, r is the number of signals, θ_n is the nth DOA, and $\hat{\theta}_{i,n}$ denotes the nth estimated DOA in the ith Monte Carlo experiment. The RMSEs of the DOA at each time instant estimated by the proposed robust KFVNM and robust PAST are illustrated in Fig. 4. The proposed robust method is seen to achieve a better performance than PAST when the subspace is fast-varying. In addition, the impulsive noise can be satisfactorily suppressed by robust algorithms.

VI. Conclusion

A robust Kalman filter-based subspace tracking algorithm in an impulsive noise environment has been presented. It is based on a new adaptive KFVNM to improve the tracking of time-varying subspaces. By incorporating M-estimation into this algorithm, a new robust subspace tracking algorithm in impulsive noise is obtained. Its effectiveness is illustrated by computer simulation of the DOA tracking problem in ULAs.

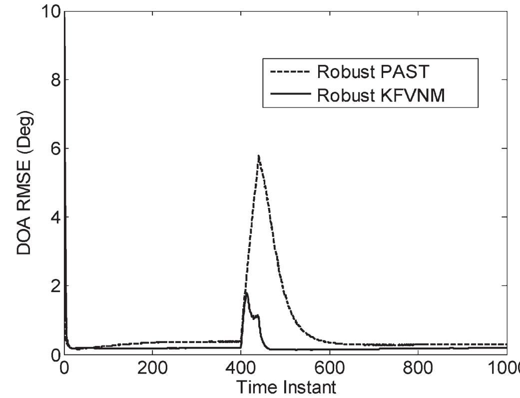

Fig. 4. DOA RMSEs in impulsive noise using robust KFVNM and robust PAST.

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