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<td>Du, JS; Au, FTK</td>
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Estimation of ultimate stress in external FRP tendons

J. S. DU MS, PhD and F. T. K. AU MSc(Eng), PhD, CEng, MICE, FiStructE, MHKIE

In a prestressed concrete beam with external tendons, the tendon stress depends on the member deformation, and it cannot be determined from section analysis alone as in the bonded case. Previous work has been mainly on the ultimate stress in unbonded steel tendons, with little on unbonded fibre-reinforced polymer (FRP) tendons. To account for the relative slip between the unbonded tendon and concrete, the ratio of the equivalent plastic hinge length to the neutral axis depth is analysed using available test results. It is found that this ratio for unbonded partially prestressed concrete (UPPC) beams with external FRP tendons can also be treated as a constant as for those with unbonded steel tendons. A simple method for evaluation of the ultimate tendon stress in either steel or FRP external tendons is therefore proposed. After suitable modifications, the equations currently adopted by various design codes can still be used to predict the ultimate stress in external FRP tendons of UPPC beams.

NOTATION

- $A_p$ cross-sectional area of tendons
- $A_{t1}, A_{t1}’$ cross-sectional areas of tension and compression for non-prestressed steel respectively
- $b, b_w$ widths of flange and web respectively
- $c$ neutral axis depth
- $C_t$ compressive force carried by flange
- $d_p$ depth to centroid of tendons
- $E_p$ Young’s modulus of tendons
- $E_{steel}, E_{FRP}$ Young’s moduli of steel and FRP tendons respectively
- $f$ coefficient dependent on loading type
- $f_{c1}$ cylinder compressive strength of concrete
- $f_{ep}$ effective prestress in tendons
- $f_{ps}$ ultimate stress in unbonded steel or FRP tendons at failure of member
- $\Delta f_{ps}$ tendon stress increment at ultimate
- $f_{y1}, f_{y1}’$ yield and ultimate strength of tendons respectively
- $f_{y1}, f_{y1}’$ yield stresses of tension and compression for non-prestressed steel respectively
- $h_t$ thickness of flange
- $L$ length of unbonded tendons between end anchorages
- $L_a$ span of beam
- $L_p$ equivalent plastic hinge length

- $N$ number of support hinges required to form a failure mechanism crossed by the tendon
- $q_0$ combined reinforcement index
- $\epsilon_{cu}$ ultimate compressive strain in concrete
- $\theta$ rotation of plastic hinge
- $\varphi$ ratio of equivalent plastic hinge length to neutral axis depth
- $\Omega_u$ bond reduction coefficient

1. INTRODUCTION

The use of external prestressing not only leads to simple and economical designs but also enables fast installation and easy replacement of defective tendons. External tendons can be made of high-strength steel or fibre-reinforced polymers (FRP), such as carbon fibre-reinforced polymers (CFRP), aramid fibre-reinforced polymers (AFRP) and glass fibre-reinforced polymers (GFRP). Because of the lack of bonding between the tendons and concrete, the tendon stress upon loading depends on the member deformation, and it cannot be determined from section analysis alone as in the bonded case. Many studies had been carried out within the past five decades for prediction of flexural resistance of prestressed concrete (PC) beams with unbonded tendons, which was closely related to the ultimate tendon stress $f_u$ at failure. Most of the equations suggested for $f_u$ are, however, based on steel tendons and may not apply to FRP tendons without validation.

The ratio $\varphi$ of the equivalent plastic hinge length to the neutral axis depth is analysed using test results of three groups, including the unbonded partially prestressed concrete (UPPC) beams with external CFRP tendons in Beijing Jiao Tong University, the UPPC beams with external AFRP tendons in The University of Hong Kong and those of Ghallab and Beeby. Values of the parameter $\varphi$ for UPPC beams with external FRP tendons are then compared with those for UPPC beams with unbonded steel tendons, with a view to devising a consistent method for evaluation of the ultimate tendon stress.

2. REVIEW OF PREVIOUS WORK

Comprehensive reviews of the ultimate stress in unbonded tendons at flexural failure were reported by Naaman and Alkhari, Allouche et al. and Au and Du. Various groups have also come up with improved methods, and some of the design formulae have been adopted in various codes. The equations fall into two main categories: the bond reduction coefficient approach and the deformation-based approach. Naaman and
Alkhairi\(^7\) have proposed an equation based on the bond reduction coefficient for the ultimate tendon stress \(\sigma_{tu}\), namely

\[
\sigma_{tu} = \frac{f_{pu}}{\Omega_a \epsilon_{cu}} \left( \frac{d_p}{c} - 1 \right) \frac{L_1}{L_2} \leq 0.94 f_{pu} \quad \text{(MPa)}
\]

where the bond reduction factor is taken as \(\Omega_a = k / (L / d_p)\), \(L_1 / L_2\) is the ratio of the length of loaded span(s) in continuous members to the total length of tendon between anchorages, \(d_p\) is the depth to centroid of tendons, \(f_{pu}\) and \(f_{pu}\) are the effective prestress and yield strength of tendon respectively, \(E_p\) is the Young's modulus of tendon, \(\epsilon_{cu}\) is the ultimate concrete compression strain equal to 0.003, \(k\) is the load type factor and \(c\) is the neutral axis depth. Based on the experimental data, \(k\) is found to be 2.6 for mid-span loading and 5.4 for third-point loading. For design purposes, the values of \(k\) are reduced to 1.5 and 3.0, respectively.

The unknowns \(c\) and \(f_{pu}\) in Equation 1 can be solved from the equilibrium equations

\[
A_p f_{pu} + A_s f_{s} - A_t f'_{t} = 0.85 \beta_1 f'_{t} b_w c + C_t \quad \text{(N)}
\]

\[
\begin{cases}
C_t = 0.85 \beta_1 f'_{t} (b - b_w) h_t & \text{if } \beta_1 c > h_t \\
C_t = 0, b_w = b & \text{if } \beta_1 c \leq h_t
\end{cases}
\]

where \(A_p\) is the cross-sectional area of tendon, \(A_s\) and \(f_{s}\) are respectively the cross-sectional area and yield strength of ordinary tension reinforcement, \(A'_t\) and \(f'_{t}\) are respectively the cross-sectional area and yield strength of compression reinforcement, \(f'_{t}\) is the cylinder compressive strength of concrete, \(b\) and \(b_w\) are respectively the breadths of flange and web, \(h_t\) is the thickness of top flange, \(C_t\) is the compressive force carried by the flange if applicable, and \(\beta_1\) is the concrete compression block reduction factor.

Naaman et al.\(^6\) further modified Equation 1 for steel or FRP tendons, and recommended two equations for the ultimate tendon stress at flexural failure. Ghallab and Beeby\(^5\) also revised the bond reduction factor \(\Omega_a\) in Equation 1 taking into account the internal bonded non-pretressed steel and external FRP tendons. Ng\(^10\) suggested a modified bond reduction coefficient independent of the span—depth ratio while accounting for the second-order effect of external tendons.

The bond reduction method of Naaman and Alkhairi, namely Equation 1, was adopted in the 1994 version of the AASHTO LRFD Bridge Code,\(^11\) but the equation was replaced by a deformation-based equation in the 1998 version.\(^12\) As pointed out by Au and Du,\(^8\) Equation 1 is heavily influenced by the load type. For example, the value of \(k\) for mid-span loading is about half that for third-point loading. It is also affected by the arrangement of spans that are loaded. For the ultimate limit state of a highway bridge for instance, it is difficult to judge if one-point or third-point loading should be chosen, and so is the choice of loading arrangements in multi-span beams.

In the deformation-based approach, the beam deformation is assumed to be concentrated in the length of equivalent plastic hinge \(L_p\), and all unbonded tendon elongation is considered to come from the region of equivalent plastic hinge. There are two schools of thought on the determination of equivalent plastic hinge length \(L_p\). The estimate of equivalent plastic hinge length \(L_p\) introduced by Harajli\(^13\) gives

\[
L_p = \frac{L}{f} + 0.5d_p + 0.05Z \quad \text{(mm)}
\]

where \(Z\) is the shear span, \(f\) is a coefficient dependent on the loading type and \(L\) is the length of unbonded tendons between anchorages. The coefficient \(f\) may take different values, namely \(f = \infty\) for single concentrated load, \(f = 3\) for two third-point concentrated loads and \(f = 6\) for uniform loading.

The other approach is to relate \(L_p\) to the neutral axis depth \(c\), namely \(L_p = \varphi c\), where the parameter \(\varphi\) is the ratio of equivalent plastic hinge length to neutral axis depth. It was originally put forward by Pannell,\(^14\) and developed by Tam and Pannell.\(^15\) After analysis of test results from various sources, Au and Du\(^16\) observed that Harajli’s \(L_p\) model placed much emphasis on the effects of loading type on stress increment in unbonded tendons at flexural failure of the beam. In Pannell’s \(L_p\) model, the parameter \(\varphi\) is stable and can be treated as constant. Recently, Roberts-Wollmann et al.\(^17\) presented an equation for the ultimate stress in external tendons that was adopted by the current AASHTO LRFD Code\(^12\) and AASHTO Segmental Bridge Specifications.\(^18\) The equation is actually based on Pannell’s model with \(\varphi\) taken as 10-5. In assessment of the equation of Roberts-Wollmann et al., Harajli\(^19\) gave reasons for scatter in prediction of ultimate stress increase in unbonded tendons, and analysed the measured values of \(L_p\) for 176 specimens from different investigators. He also observed that whenever the values of \(L_p\) were plotted as a linear function of the neutral axis depth \(c\), as proposed by Pannell,\(^14\) and Tam and Pannell,\(^15\) the trend became clearer. He suggested a revised equation for \(L_p\), incorporating the neutral axis depth \(c\) and load type \(f\), as

\[
L_p = \frac{20.7}{f} + 10.5 \cdot c \quad \text{(mm)}
\]

where \(f = \infty\) for single concentrated load, \(f = 3\) for two third-point loads and \(f = 6\) for uniform loading, and \(L\) is the length of unbonded tendons between anchorages.

There are certain problems in incorporating the load type into the calculation of \(L_p\). Investigators have not yet reached a consensus about the load type effect on the ultimate stress in unbonded tendons. It is also difficult to determine the load type at ultimate limit state for structures such as highway bridges. The test data of Harajli and Kanji\(^20\) indicated that the stress increase in unbonded tendons at ultimate was not consistently higher for beams with mid-span loading compared to beams with third-point loading. From analysis of these data,
it is also found that the experimental values of \( L_p \) for members under mid-span loading are comparable in magnitude to those of their counterparts tested under third-point loading.\(^{16}\)

Pannell’s deformation-based model for determination of the ultimate stress \( f_{ps} \) in unbonded tendons also formed the basis of the British Code BS 8110,\(^{21}\) the Canadian Code A23-3-94,\(^{22}\) and the draft Chinese Code for Strengthening of Highway Bridges.\(^{23}\) Although the above codes, together with the AASHTO LRFD and AASHTO Segmental Bridge Code, are all based on Pannell’s model, there are some differences in the values of parameter \( \phi \), calculation of neutral axis depth \( c \), and how to account for the effect of continuous beams.

### 3. FURTHER DEVELOPMENT OF DEFORMATION-BASED MODEL

According to Pannell’s deformation-based model, the failure of a UPPC beam can be modelled as a series of rigid members connected by plastic hinges at critical sections, as shown in Figure 1. If the rotation of the plastic hinge is denoted by \( \theta \), and the distance from the neutral axis to the tendon is \( z_p = d_p - c \), then the tendon elongation can be written as

\[
\delta = z_p \theta = (d_p - c) \theta \quad \text{(mm)}
\]

The corresponding tendon strain increase is

\[
\Delta \varepsilon_{ps} = \frac{\delta}{L} = \frac{(d_p - c) \theta}{L}
\]

where \( L \) is the length of unbonded tendons between anchorages. From strain considerations, the rotation of the plastic hinge can be approximated as

\[
\theta = L_p \frac{\varepsilon_{ps}}{c}
\]

According to Pannell,\(^{14}\) and Tam and Pannell,\(^{15}\) the ratio of equivalent plastic hinge length \( L_p \) to neutral axis depth \( c \), namely \( \phi = L_p/c \), can be treated as a constant for PC beams with unbonded tendons even for different span/depth ratios. Assuming the unbonded tendons to remain elastic and further making use of Equations 6 and 7, the tendon stress at ultimate \( f_{ps} \) appears as

\[
f_{ps} = f_{pe} + \Delta f_{ps} = f_{pe} + E_p \Delta \varepsilon_{ps} = f_{pe} + \frac{\phi E_p \varepsilon_{ps} (d_p - c)}{L} \quad \text{(MPa)}
\]

where \( \Delta f_{ps} \) is the tendon stress increment at ultimate.

In the AASHTO LRFD Code\(^{12}\) and AASHTO Segmental Bridge Code,\(^{18}\) the associated parameters are: \( \phi = 10-5, E_{cu} = 0-003 \) and \( E_p = 200 \text{ kN/mm}^2 \), giving \( \phi E_p E_{cu} = 6300 \text{ N/mm}^2 \). The tendon stress at ultimate \( f_{ps} \) is

\[
f_{ps} = f_{pe} + \frac{6300 (d_p - c)}{L} \quad \text{(MPa)}
\]

where \( L_e = L/(1 + N/2) \), \( L \) is the length of tendon between anchorages or fully bonded deviators, and \( N \) is the number of support hinges required to form a failure mechanism crossed by the tendon.

Eliminating the neutral axis depth \( c \) between Equation 8 and Equations 2a to 2c for equilibrium at the critical section, a general equation of \( \phi \) can be obtained as

\[
\phi = \frac{\Delta f_{ps}}{E_p \varepsilon_{ps} (L_p - l_p)} \left[ 1 - \left( \frac{f_{pe}}{f_{ps}} \right) \frac{L}{L_p} \right] \quad \text{(MPa)}
\]

The tendon stress at ultimate \( f_{ps} \) can be obtained by rearranging Equation 10 as

\[
f_{ps} = f_{pe} + \Delta f_{ps} = f_{pe} + \frac{\phi E_p \varepsilon_{ps} (d_p - c)}{L} \quad \text{(MPa)}
\]

\[
c = c_p + \frac{A_p \Delta f_{ps}}{0.85 \beta_1 f_s b_w} \quad \text{(mm)}
\]

Assuming \( E_{cu} = 0-003 \) and using the measured \( f_{ps} \) and \( c \) calculated from Equation 2, the values of \( \phi \) can be obtained from Equation 10. Au and Du\(^{16}\) found that \( \phi \) tended to be constant in a series of 148 simply supported specimens, in which the mean, standard deviation and coefficient of variation of \( \phi \) are 16-1, 6-8 and 0-42 respectively. Taking \( \phi \approx 10 \) is therefore considered as conservative in most cases of PC members with unbonded tendons. In their study, however, almost all unbonded tendons are made of steel. Whether the conclusion applies to FRP tendons needs further validation.

### 4. EVALUATION OF PARAMETER \( \phi \) BASED ON EXPERIMENTS

#### 4.1. Experimental work by Au et al.\(^2\) at The University of Hong Kong (HKU)

Two groups of simply supported UPPC beams, each having two external tendons, were tested to failure. Group S (i.e. SSS1 to SSS3) used 7-wire steel strands of 12.9 mm nominal diameter, while group P (i.e. SPS1 to SPS3) used AFRP tendons of 10.5 mm nominal core diameter. Figure 2
shows the dimensions of the specimens while its material properties and the measured ultimate tendon stress $f_{ps}$ are shown in Table 1. The amounts of prestressing force, non-prestressed reinforcement and concrete strength of specimens in group S roughly correspond to those in group P. For convenience, the specimens were divided into three subgroups each having roughly the same nominal partial prestressing ratio (PPR) defined as $PPR = A_p f_{pe}/(A_p f_{pe} + A_s f_y)$. Specimens SSS1 and PSS1 had nominal parameter PPR = 0.25. Specimens SSS2 and PSS2 had nominal parameter PPR = 0.3, while specimens SSS3 and PSS3 had nominal parameter PPR = 0.5.

Specimens SSS1, SSS2, PSS1 and PSS2 were cast of grade 60 concrete while specimens SSS3 and PSS3 were cast of grade 85 concrete. The actual cube strengths on the day of testing are shown in Table 1. The parameter PPR for specimens B1 to B4 is between 0.47 and 0.62.

4.2. Experimental work by Du1 at Beijing Jiao Tong University (BJTU)

Four simply supported UPPC beams (i.e. B1 to B4) each with two external tendons were tested to failure under third-point loading. Specimen B2 used 7-wire steel strands of 15.2 mm nominal diameter, while specimens B1, B3 and B4 used CFRP tendons. Each tendon consisted of three CFRP bars of 7 mm diameter and nominal tensile strength of 2400 MPa. The prestressing level of tendon, defined as the ratio of effective prestress $f_{pe}$ to ultimate strength $f_{pu}$, of the specimens tested ranged from 18.2% to 25.8% for steel tendons and from 30.4% to 40.3% for AFRP tendons.

4.3. Experimental work by Ghallab and Beeby3

Sixteen PC beams with the same cross-section were tested under either mid-span or third-point loading up to failure after being strengthened using two external AFRP tendons with one or two deviators. Except for specimen PC12, all specimens were internally prestressed using a 7 mm diameter high tensile steel strand.

![Figure 2. Dimensions of specimens by Au et al. at HKU (unit: mm)](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>L₁: mm</th>
<th>L₂: mm</th>
<th>$f_{c}':$ MPa</th>
<th>$A_s$: mm²</th>
<th>$f_{y}':$ MPa</th>
<th>$A_p$: mm²</th>
<th>$f_{pe}':$ MPa</th>
<th>$A_p$: mm²</th>
<th>$d_p$: mm</th>
<th>$E_p$: kN/mm²</th>
<th>$f_{pu}':$ MPa</th>
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<td>58.3</td>
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<td>142.0</td>
<td>702-5</td>
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</table>

Table 1. Details of specimens by Au et al.² and Du¹
wire with yield strength of 1470 MPa about one week after casting. The experiment was carried out to study the effect of several factors on the ultimate stress in external AFRP tendons, including the ratio of effective stress to ultimate tendon strength \( f_{pe} / f_{pu} \), depth to external tendons \( d_p \), number of deviators, ratio of distance between deviators to the span length, concrete strength, span/depth ratio, ratio of internal bonded prestressed steel to non-prestressed steel, and the load type. Figure 4 shows the dimensions of the specimens, with their material properties shown in Table 3. The parameter PPR for specimens PC1 to PC12 is between 0.74 and 1.00.

### 4.4. Evaluation of parameter \( \varphi \)

The parameter \( \varphi \) is further studied for UPPC members mainly with FRP tendons. In using Equation 10, \( e_{cu} \) is taken as 0.003, and the other parameters have adopted the specific values as listed in Tables 1 and 2. The total length of specimen or the length of tendon between anchorages \( L \) is adopted but not the net span \( L_n \). As the Young's modulus of tendons affects \( \varphi \), the values of \( \varphi \) of specimens with FRP tendons are converted to those of the corresponding specimens with steel tendons for comparison on the same basis. In the conversion, the right side of Equation 10 for specimens with FRP tendons is divided by \( \frac{E_{steel}}{E_{FRP}} \), where \( E_{steel} \) and \( E_{FRP} \) are Young's moduli of steel and FRP tendons, respectively. The values of \( \varphi \) for each specimen before and after conversion are calculated for the above sets of experimental results and shown in Table 3.

In the tests of Au et al., the mean, standard deviation and coefficient of variation of \( \varphi \) are respectively 37-9, 3-71 and 0-10 for the three specimens with AFRP tendons before conversion. After conversion they are 26-9, 3-60 and 0-13,
respectively, for all six specimens. In the tests of Du,¹ the mean, standard deviation and coefficient of variation of \( j \) are respectively 20.6, 2.58 and 0.13 for the three specimens with CFRP tendons before conversion. After conversion they are 14.9, 1.59 and 0.11, respectively, for all four specimens. In the tests of Ghallab and Beeby,³ the mean, standard deviation and coefficient of variation of \( j \) are respectively 24.1, 6.47 and 0.27 for all 15 specimens with AFRP tendons before conversion. After conversion, they are 15.3, 4.09 and 0.27, respectively. In the 25 specimens from the three groups, the mean, standard deviation and coefficient of variation of \( j \) are respectively 18.0, 6.23 and 0.35 after conversion. The statistics for 25 specimens are comparable to the 148 specimens studied by Au and Du,¹⁶ in which the mean, standard deviation and coefficient of variation of \( j \) are 16.1, 6.8 and 0.42, respectively.

Despite minor variations, the values of \( j \) are generally stable and tend to be constant in tests having tendons of the same materials. As pointed out by Au and Du,¹⁶ the difference in measurement methods, failure criteria adopted, and variation of material properties may have caused the scatter of results. The difference between the total length of specimen \( L \) and the net span \( L_n \) used by different investigators also have some effect on the variation of \( j \). For example, the tests of Au et al.² had \( L/L_n \) as large as 1·12, while those of Du¹ had \( L/L_n \) of 1·07. The tests of Ghallab and Beeby³ had \( L/L_n \) of only 1·04 or less. After rearranging the right side of Equation 10, Au and Du¹⁶ qualitatively explained why the parameter \( j \) could be constant. Hence it is reasonable to take \( j \) as a constant in PC members with unbonded FRP tendons. The parameter \( j \text{FRP} \) for specimens with FRP tendons can be expressed as

\[
j \text{FRP} = \lambda j \text{steel}
\]

where \( j \text{steel} \) is the parameter \( j \) for specimens with steel tendons.

As taking the value of \( j \text{steel} \) around 10 is conservative in most cases,¹⁶ \( j \text{steel} = 10 \) is suggested here for regular design purposes. In Table 3, it can also be found that only one of the values of \( j \) is less than 10 after the conversion from FRP to steel. Figure 5 compares the values of \( j \) for the above-

![Figure 4. Dimensions of specimens by Ghallab and Beeby (unit: mm)](image)

<table>
<thead>
<tr>
<th>No.</th>
<th>Tendon materials</th>
<th>( \varphi ) before conversion</th>
<th>( \varphi ) after conversion</th>
<th>No.</th>
<th>Tendon materials</th>
<th>( \varphi ) before conversion</th>
<th>( \varphi ) after conversion</th>
</tr>
</thead>
<tbody>
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<td>29.3</td>
<td>PC4</td>
<td>AFRP</td>
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Table 3. Value of \( \varphi \) before and after conversion
mentioned 25 specimens after conversion against the rule of \( \varphi_{\text{steel}} = 10 \). Equation 13 shows that if Young’s modulus of FRP is less than that of steel, \( \varphi_{\text{FRP}} \) is larger than \( \varphi_{\text{steel}} \) and vice versa. As the parameter \( \varphi \) is directly related to the equivalent plastic hinge length \( L_p \) and ultimate tendon stress \( f_{pu} \), the influence of Young’s moduli of FRP and steel tendons on \( L_p \) and hence on \( f_{pu} \) would be roughly the same as that on the parameter \( \varphi \).

5. PROPOSED EQUATION FOR ULTIMATE STRESS IN EXTERNAL TENDONS

With the determination of \( \varphi_{\text{FRP}} \) and substituting \( \varphi_{\text{FRP}} = 10 \lambda \) and \( \varepsilon_{pu} = 0.003 \) into Equation 8, the ultimate stress in external FRP tendons can be expressed as

\[
 f_{pu} = f_{pc} + \frac{0.03 \lambda E_{\text{FRP}} (d_p - c)}{L_p} \leq (f_{pu})_{\text{FRP}} \quad \text{(MPa)}
\]

where \( (f_{pu})_{\text{FRP}} \) is the ultimate tensile strength of FRP tendons, \( \lambda = E_{\text{steel}} / E_{\text{FRP}} \), \( L_p = L/(1 + N/2) \), and \( N \) is the number of support hinges required to form a failure mechanism crossed by the tendon. The practice of AASHTO LRFD Code\(^{12} \) for continuous beams is adopted because some investigation\(^{17} \) showed that the rotation at a supported plastic hinge was only half of that at a mid-span plastic hinge in some cases. It means that the equivalent plastic hinge length at an interior support is half of that at mid-span in continuous beams.

In Equation 14, \( 0.03 \lambda E_{\text{FRP}} (d_p - c) \) is actually equal to \( 0.03 E_{\text{steel}} (d_p - c) \) or 6000 \( (d_p - c) \) that is independent of \( E_{\text{FRP}} \). It shows that Pannell’s deformation-based design equations can predict the ultimate stress in unbonded FRP tendons, although they have been established for unbonded steel tendons without explicitly including their Young’s moduli. Ghallab and Beeby\(^3 \) experimentally found that the equation in BS 8110 generally gave good agreement with the actual results for AFRP tendons after changing the limit of \( f_{ps} \leq 0.7 f_{pu} \) for steel to \( f_{ps} \leq (f_{pu})_{\text{FRP}} \) for FRP. The present systematic analysis of experimental data from various sources shows that Equation 14 can be rewritten in a unified form as

\[
 f_{ps} = f_{pe} + \frac{6000 (d_p - c)}{L_p} \leq (f_{pu})_{\text{FRP}} \quad \text{(MPa)}
\]

where the neutral axis depth \( c \) can be solved from Equations 2a to 2c for section equilibrium, \( L_p = L/(1 + N/2) \), \( L \) is the length of tendon between anchorages or fully bonded deviators, and \( N \) is the number of support hinges required to form a failure mechanism crossed by the tendon in continuous beams.

To demonstrate the validity of Equation 15, the computed values of tendon stress \( f_{ps} \) at ultimate are compared with the above experimental results, and the results of correlation analyses are plotted in Figure 6 giving a correlation coefficient of 0.89. Most of the predicted values of tendon stress \( f_{ps} \) at ultimate are on the safe side. Note that Equation 15 is similar in form to the AASHTO equation (i.e. Equation 9) except that the use of \( \varphi = 10 \) resulting in a higher coefficient 6000 will be slightly less safe in view of the experimental results.

The unbonded tendon stresses \( f_u \) at ultimate are also computed for the above specimens using the equation in the ACI 318-08 building code\(^{24} \), namely

\[
f_{ps} = f_{pe} + 70 + \frac{f_c}{\lambda \rho_p} \quad \text{(MPa)}
\]

where \( \rho_p = A_p / b d_p \); for span-to-depth ratio \( L_a / d_p \) of 35 or less, \( \lambda = 100 \), and \( f_{ps} \) shall not be taken as greater than the lesser of \( f_{pe} \) and \( (f_{pe} + 420) \); for \( L_a / d_p \) greater than 35, \( \lambda = 300 \), and \( f_{ps} \) shall not be taken as greater than the lesser of \( f_{pe} \) and \( (f_{pe} + 210) \). Figure 7 plots the computed values of \( f_{ps} \) using Equation 16 against the experimental results, giving a correlation coefficient of 0.80. For these specimens, the correlation coefficient based on the ACI 318-08 building code is less than that based on Equation 15.

6. CONCLUSIONS

This paper has evaluated the validation of Pannell’s deformation-based method to predict the ultimate stress in FRP tendons of UPPC beams. The ratio \( \varphi \) of equivalent plastic hinge
length to neutral axis depth at critical section is analysed based on the experimental results from three research groups. The parameter $\varphi$ can be regarded as a constant for UPPC members with FRP tendons but it is also related to the Young’s modulus of FRP. A modified equation similar in form to those adopted by various design codes is proposed, and it is applicable to both FRP and steel tendons.

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**REFERENCES**


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